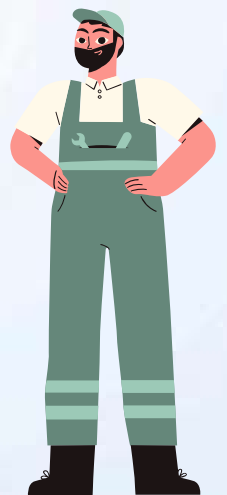


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FLUID MECHANICS

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BY JASPAL SIR

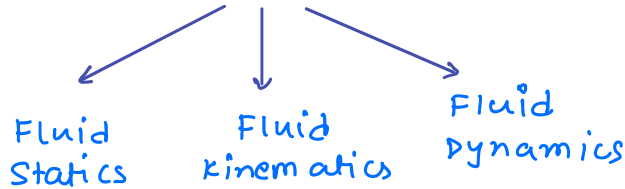
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FLUID MECHANICS

- Fluid Mechanics is a branch of engineering, that deals with the properties of the fluid at rest as well as in motion.

- It is further analysed in following forms.



- Fluid Statics :- It deals with fluid in rest condition.

- Fluid Kinematics :- It deals with fluid in motion. without considering the force responsible for motion.

- Fluid Dynamics :- It deals with fluid in motion by considering the forces responsible for motion.

- In general matter exist in following states

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(A) SOLID

(B) FLUID $\begin{cases} \rightarrow \text{LIQUID} \\ \rightarrow \text{GASES} \end{cases}$

- A fluid is a substance which exists in LIQUID and GASEOUS phase.

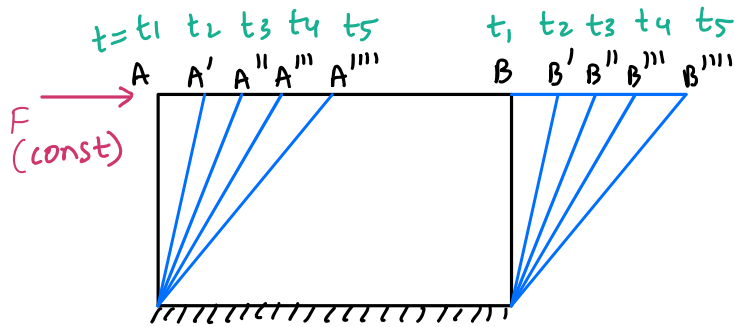
- Fluid is capable of deforming continuously under the action of shear force, shear stress or tangential stress. However small the magnitude of stress may be.

- Hence in solids where stress is proportional to strain, in fluid stress is proportional to rate of deformation or rate of strain.

NOTE :- Here continuous deformation of the fluid means that when a constant shear stress is applied over the fluid, it will continue to deform as long as stress is present.

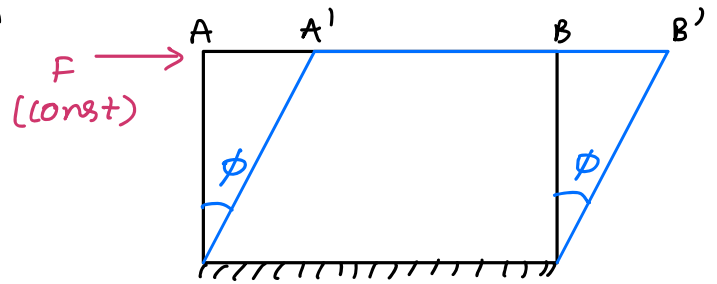
But in case of SOLID, deformation eventually stops at fixed strain angle.

FLUID



$$\tau \propto \frac{d\phi}{dt}$$

SOLID



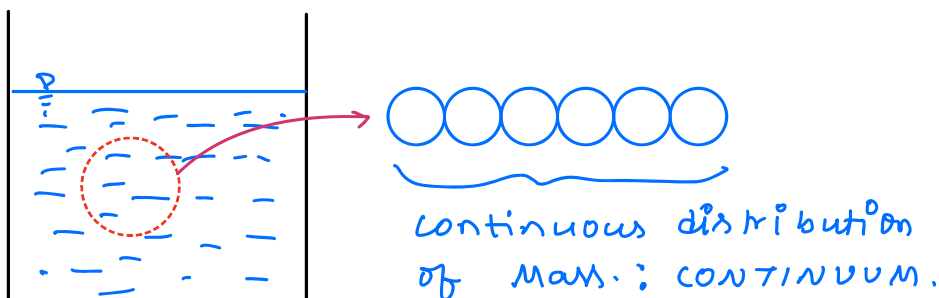
ϕ = shear strain.

$$\tau \propto \phi$$

- Hence in fluid, we are more concerned with rate of deformation and in solids we are more concerned with its magnitude.

- Fluid can also be stated to be in CONTINUUM for its analysis.

In macro system, when the intermolecular distances are very small as compared to dimensions of the medium (system), we can assume that the adjacent to one molecule, there is another molecule without any space in between. Hence the entire fluid mass can be considered as continuous distribution of mass, which is termed as continuum.



GLASS/BOTTLE etc.

- Fluid may also be defined as the substance that is capable of flowing and it has no definite shape of its own and conforms to the shape of container.

- Fluid can be classified as :-

(a) LIQUID :- It is a fluid which has a definite volume, and it changes slightly with temp and pressure.

(For practical purposes it is considered to be incompressible i.e. donot show change in volume.)

(b) GAS :- It is a fluid, which is compressible & posses no definite volume & always expands upto its volume is equal to the volume of container, it is placed in.

NOTE :- A vapour is a gas, temp and pressure of which is very near to liquid.
Eg → steam.

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(c) IDEAL FLUID :- This are the fluids which have no VISCOSITY, SURFACE TENSION, and are INCOMPRESSIBLE.

- They donot offer resistance against flow.

- It is an imaginary or hypothetical fluid, considered only for simplicity in mathematical calculation.

Eg:- Those fluids having very low viscosity may be considered as ideal fluid. Eg. water, air.

(d) REAL / PRACTICAL :- All fluid that exist in nature are Real fluids.

- They offer resistance against flow

- posses viscosity, surface Tension and compressibility.

PROPERTIES OF FLUID :-

(A) MASS DENSITY / SPECIFIC MASS :-

- It is defined as ratio of mass of fluid and its volume
OR
- It is the mass of the fluid which possesses per unit volume.

$$\rho = \frac{M}{V}$$

UNIT
 kg/m^3 , g/cm^3
(MKS) (CGS)

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- Mass density of the fluid depends upon its Temperature and pressure.
- With increase in Temperature, molecular activity or Randomness ^{Souvik 2021} increases thus spacing b/w molecules increases hence lesser number of molecules would pack in same volume that results in reduced mass density.

$$\rho \propto \frac{1}{T}$$

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- With increase in pressure, large number of molecules can be forced into given volume, that results in higher mass density.

$$\rho \propto T$$

- At STP, $\rho_{\text{water}} = 999.9 \text{ kg/m}^3 \approx 1000 \text{ kg/m}^3$
 $T = 0^\circ\text{C}$
 $P = \text{Atm.}$ $\rho_{\text{air}} = 1.292 \text{ kg/m}^3$

(B) SPECIFIC WEIGHT / WEIGHT DENSITY (γ)

- It is the weight of the fluid per unit volume.

$$\gamma = \frac{W}{V}$$

units:- N/m^3 , kgf/m^3
 dyne/cm^3

$$1 \text{ dyne/cm}^3 = 10 \text{ N/m}^3$$

$$1 \text{ kgf/m}^3 = 9.81 \text{ N/m}^3$$

- It signifies the force exerted by gravity over the unit volume of fluid.

- It varies from location to location as "g" varies with location

$$\gamma = \rho g$$

- Thus, "γ" also depend upon Temperature and pressure as "ρ" varies.

$$\gamma \propto P, \quad \gamma \propto \frac{1}{T}$$

At STP, $\gamma_{\text{water}} = 9807 \text{ N/m}^3 \simeq 9810 \text{ N/m}^3$

$$\gamma_{\text{air}} = 12.670 \text{ N/m}^3$$

(c) SPECIFIC VOLUME (S_v) :-

- It is the volume of the fluid per unit wt or mass.

$$S_v = \frac{V}{M \text{ or } W} \quad S_v = \frac{1}{\rho} \text{ or } \frac{1}{\gamma}$$

- For problems involving gas $\rightarrow$ use $S_v = \frac{V}{M}$
- " " " liquid $\rightarrow$ use $S_v = \frac{V}{W}$

All the above properties (P, γ, S_v) for fluid depends on temperature and Pressure but its impact on gases is considerably more than in liquids.

(d) SPECIFIC GRAVITY (G)

- It is the ratio of fluid of given volume to the weight of standard fluid of same volume

$$G = \frac{W_{\text{fluid}}}{W_{\text{st. fluid}}} \quad V_f = V_{sf}$$

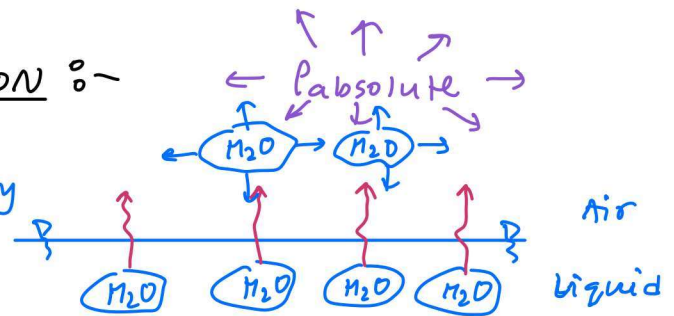
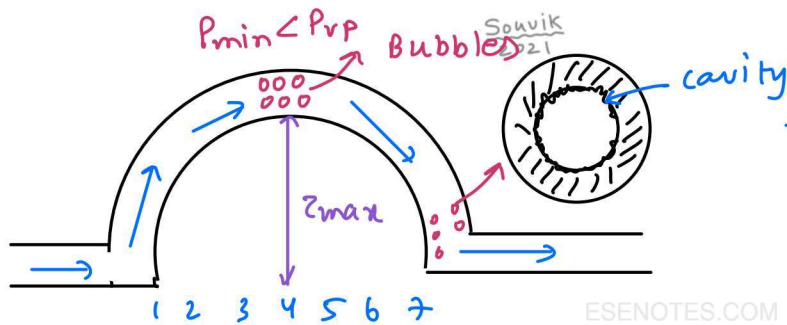
$$G = \frac{W_f}{W_{sf}} \cdot \frac{V_f}{V_{sf}} = \frac{W_f}{W_{sf}} \cdot \frac{V_f}{V_f}$$

$$G = \frac{W_f}{V_f} \cdot \frac{1}{\left(\frac{W_{sf}}{V_{sf}}\right)} = \frac{\gamma_f}{\gamma_{sf}}$$

$$G = \frac{\gamma_f}{\gamma_{sf}}$$

- Hence, γ can also be defined as ratio of specific weight or specific density of fluid to the specific weight or specific density of the standard fluid
- For LIQUID, standard fluid is taken as **PURE WATER at 4°C .**
- For GASES, std. fluid is considered as **HYDROGEN or AIR at std. temp & press.**
- Since weight density and mass density varies with temp. must also be reported when γ is reported.

(e) VAPOUR PRESSURE & CAVITATION :-



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- At liquid air interface, continuous exchange of molecules takes place as liquid molecule escape from the surface in to gaseous form termed as vapour, due to the energy poses by them.
- This vapour molecules exert partial pressure over the liquid surface (in general space) termed as vapour pressure.
- On increase in temperature as molecular activity increases, vapour pressure also increases.
- If absolute pressure impose over the liquid becomes less than vapour pressure of liquid boiling of the liquid will start.
- Hence it can be stated that the boiling of liquid can start at any given temperature, if the pressure above the liquid surface becomes less than vapour pressure.
- Liquid with a ^{high} vapour pressure evaporates readily and are termed as volatile liquid. eg. Benzene.

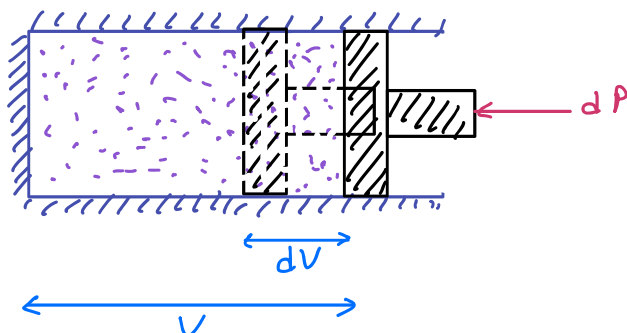
- Hg has very low VP, hence it doesn't vapourise even at very low pressure, thereby it is used in equipments for pressure measurements.
- V.P of water at different temp. is as follows

$T(^{\circ}C)$	VP (kPa)
10	1.2
20	2.3
100	101.3 = 1 atm. $\rightarrow$ Boiling start.

cavitation :- If in any flow system pressure at any point in the liquid becomes less than V.P, vapourisation starts resulting in pockets of dissolved gases and vapours.

- These bubbles ^{Souvik} move with flow of liquid into the high pressure region where they burst or collapse giving rise to high impact pressure.
- Due to which VIBRATION, NOISE, EROSION, SURFACE PITTING and FATIGUE FAILURE of the material takes place.
- This phenomenon is termed as cavitation.
- With increase in temp., VP increases, tendency of vap. increases hence tendency of cavitation also increases.

(f) COMPRESSIBILITY AND ELASTICITY :-



- All fluids can be compressed by application of the pressure and when this pressure is removed fluid expands to their original volume (similar to that of elastic solid)

- This property of fluid to undergo volume change on application of pressure is termed as COMPRESSIBILITY.
- which is quantitatively related with Bulk modulus of elasticity (K).
- BMOE (K) is defined as change in pressure per unit change in volume to the original volume.

$$K = \frac{\text{volumetric stress}}{\text{volumetric strain}} = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

unit :- N/mm^2 , Kgf/m^2 , gmf/cm^2

At NTP, $K_{\text{water}} = 2.06 \times 10^9 N/m^2$

$K_{\text{air}} = 1.03 \times 10^5 N/m^2$

* Air is 20000 or 2×10^4 more compressible than water.

(*) - K is not const., for fluids it increases with increase in pressure, as when a fluid mass is compressed its molecule becomes close together and the resistance to further compression increases.

- For eg. K_{water} almost doubles when pressure is raised from 1 atm to 3500 atm

(*) - "K" also varies with Temperature

- For "LIQUID", with increase in temperature, inter molecular bond b/w the particles decreases, hence resistance against vol. change decreases, thus more volume change occurs. Hence "K" decreases.
- For "GASES", with increase in temperature random motion of particles increases, that further increases resistance to vol. change, thereby less volume change is observed. Hence "K" increases.

- Now, compressibility is quantitatively related to bulk modulus as follows.

compressibility $\leftarrow \beta = \frac{1}{K}$ units :- $\frac{m^2}{N}$

- In general,

$$m = \text{const.}$$

$$m = \rho v$$

$$dm = \rho dv + v d\rho$$

$$0 = \rho dv + v d\rho$$

$$-\frac{dv}{v} = \frac{d\rho}{\rho}$$

$$K = -\frac{dP}{\left(\frac{dv}{v}\right)} = -\frac{dP}{\left(\frac{d\rho}{\rho}\right)}$$

$$K = -\rho \frac{dP}{d\rho}$$

$$\therefore \beta = \frac{d\rho}{\rho dP}$$

$\rightarrow$ If $\frac{d\rho}{dP} = 0 \Rightarrow \beta = 0 \rightarrow$ Fluid is incompressible.

$\rightarrow$ If $\frac{d\rho}{dP} \neq 0 \Rightarrow \beta \neq 0 \rightarrow$ Fluid is compressible.

For liquids (water) at 20°C

$$P = 1 \text{ atm}, \rho_{\text{water}} = 998 \text{ kg/m}^3$$

$$P = 100 \text{ atm}, \rho_{\text{water}} = 1003 \text{ kg/m}^3$$

$$\% \text{ change in } \rho = \frac{1003 - 998}{998} \times 100 = 0.49\% \quad (\text{negligible})$$

Hence, liquids are considered to be incompressible.

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For Gases :-

(a) ISOTHERMAL COMPRESSIBILITY OF GASES :-

$$PV = mRT \quad (T = \text{const})$$

$$P = \frac{mRT}{V}$$

$$\frac{dP}{dP} = RT$$

$$\text{we know, } K = P \frac{dP}{dP} = PRT = P$$

$$\boxed{K_{\text{isothermal}} = P}$$

compressibility,

$$\boxed{\beta = \frac{1}{K_{\text{iso}}} = \frac{1}{P}}$$

(b) ADIABATIC COMPRESSIBILITY :-

Heat = const.

$$PV^{\gamma} = \text{const}$$

γ = adiabatic const.

$$\gamma = \frac{C_P}{C_V} = \frac{\text{specific heat at const Press.}}{\text{specific heat at const. vol.}}$$

note:- Here sp. heat is quantity of heat req. to raise the temp. of 1g of a substance by 1°C. (J/gm/°C)

now,

$$P \left(\frac{m}{P} \right)^{\gamma} = \text{const.}$$

$$P = C P^{\gamma}$$

$$\frac{dP}{dP} = \gamma P^{\gamma-1}$$

$$\frac{dP}{dP} = P \frac{\gamma}{P}$$

$$K = P \frac{dP}{dP} = P \frac{P\gamma}{P} = P\gamma$$

$$\boxed{K_{\text{adia}} = P\gamma}$$

$$\boxed{\beta = \frac{1}{K_{\text{adia}}} = \frac{1}{\gamma P}}$$

- Q3) Compute at what head of water, vapourisation will start if vapour pressure of water at 20°C is 2.5 kPa .

$$1 \text{ atm} = 100 \text{ kPa} \quad \text{and} \quad 1 \text{ m head of water} = 10 \text{ kPa}.$$

Sol vapourisation starts when, $P_{\text{absolute}} < P_{\text{vapour}}$.

$$\text{Atm pressure} + P_{\text{gauge}} < P_{\text{vapour}}.$$

$$100 + 10x < 2.5$$

$$x < -9.75 \text{ m}$$

Thus vacuum pr.

$$P_{\text{absol}} = P_{\text{gauge}} + P_{\text{ref.}}$$

$$P_{\text{gauge}} = x \times 10$$

$$P_{\text{ref. pr.}} = 1 \text{ atm}$$

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- Q3) Bulk mod. of water is $210 \times 10^4 \text{ kN/m}^2$. What pres. is req. to reduce the vol. of water in a closed container by 2% , also find the increase in mass density.

Sol

$$K = 2 \times 10^4$$

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$$K = - \frac{dp}{\left(\frac{dv}{v}\right)}$$

$$210 \times 10^4 \times (-0.02) = -dp$$

$$dp = 42000 \text{ kN/m}^2$$

$$\frac{dv}{v} = - \frac{dp}{p}$$

$\therefore$ increase in mass density = 2%

- Q3) The density of sea water at free surface where pressure is 98 kPa is approx. 1030 kg/m^3 . Take K of sea water as $2.34 \times 10^9 \text{ N/m}^2$ and $dp = \rho g dz$. Determine the density and pressure at the depth of 2500 m [$T = \text{const}$].

Sol

$$P_0 = 1030 \text{ kg/m}^3$$

$$P_0 = 98 \text{ kPa}$$

$$K = 2.34 \times 10^9 \text{ N/m}^2$$

$$\} \quad P_0, \rho_0$$

$$K = \frac{dp}{\left(-\frac{dv}{v}\right)} = \frac{dp}{\left(-\frac{d\rho}{\rho}\right)}$$

$$K = \frac{\rho g dz}{\frac{d\rho}{\rho}}$$

$$\int_{P_0}^{P_2} \frac{d\rho}{\rho^2} = \int_{z=0}^z \frac{g}{K} dz$$

$$\left[\frac{-1}{\rho} \right]_{P_0}^{P_2} = \frac{g}{K} (z)$$

$$-\frac{1}{P_2} + \frac{1}{P_0} = \frac{gz}{K}$$

at $z = 2500$

$$-\frac{1}{P_2} + \frac{1}{2500} = \frac{9.81 \times 2500}{2.34 \times 10^9}$$

$$\frac{1}{P_2} = \frac{1}{2500} - \frac{9.81 \times 2500}{2.34 \times 10^9}$$

$$P_{2.5 \text{ km}} = 1041.3 \text{ KJ/m}^3$$

$$P_2 = \frac{1}{\frac{1}{P_0} - \frac{gz}{K}}$$

(b) $dP = \rho g dz$

$$\int_{P_0}^{P_2} dP = \int_{z=0}^{z=2} \rho g dz$$

$$\int_{P_0}^{P_2} dP = \int_{z=0}^{z=2} \frac{g dz}{\frac{1}{P_0} - \frac{gz}{K}}$$

on solving.

$$P - P_0 = K \left[\ln \left(\frac{1}{P_0} \right) - \ln \left(\frac{1}{P_0} - \frac{gz}{K} \right) \right]$$

$$P - P_0 = K \ln \left(\frac{1}{1 - \frac{P_0 g z}{K}} \right)$$

$$P = 25495.2 \text{ KPa}.$$

8) An empirical pressure density relationship for water

$$\frac{P}{P_a} = 3001 \left[\frac{\rho}{\rho_a} \right]^7 - 3000$$

P_a = atm. pressure

ρ_a = atm. density.

Determine the bulk mod. of elasticity & compressibility of water at 5 atm. pressure.

sol

$$K = \frac{dP}{-\frac{dv}{v}} = \frac{dP}{-\frac{d\rho}{\rho}} = -\rho \frac{dP}{d\rho}$$

$$P = 3001 \text{ Pa} \left[\frac{P}{P_a} \right]^7 - 3000 \text{ Pa}$$

$$dP = \frac{3001 \text{ Pa}}{(P_a)^7} \times 7 P^6 dP$$

$$\frac{dP}{dP} = \frac{3001 \text{ Pa} \times 7 P^6}{(P_a)^7}$$

$$K = 3001 \times 7 \times P_a \left(\frac{P}{P_a} \right)^7$$

$$\text{Now, } 3001 \text{ Pa} \left(\frac{P}{P_a} \right)^7 = P + 3000$$

$$K = 7 (P + 3000 \text{ Pa})$$

$$K = 7 P_a \left(\frac{P}{P_a} + 3000 \right)$$

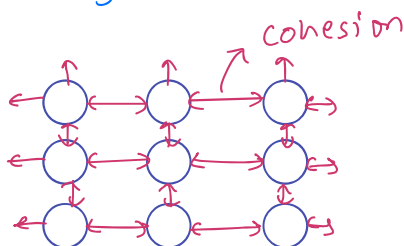
$$= 7 \times 101.3 (5 + 3000) = 2.13 \times 10^6 \text{ KN/m}^2$$

$$B_{\text{air}} = \frac{1}{K} = 4.69 \times 10^{-7} \text{ m}^2/\text{KN}$$

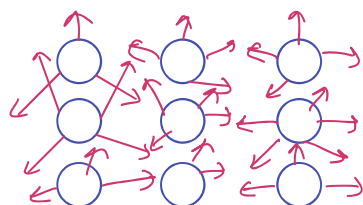
June 28 18) VISCOSITY

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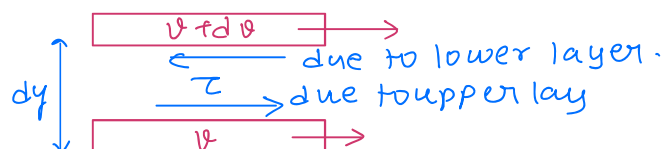
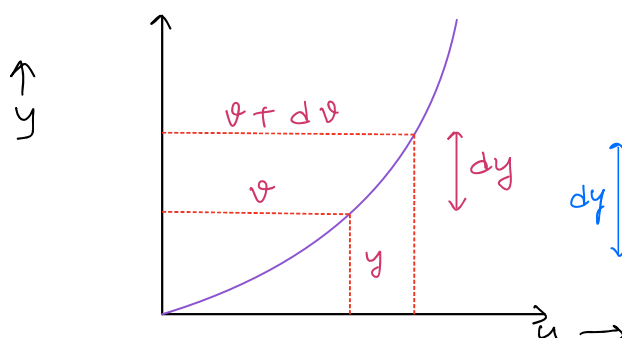
- It is the property of the fluid by virtue of which it offers resistance to the movement of one layer over the adjacent layer of fluid.
- It is due to cohesion b/w the molecules in case of liquid and momentum transfer b/w fluid layers in case of gases.



Liquid

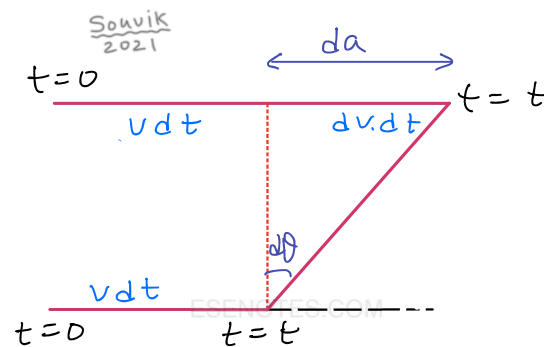


Gases.



- Suppose one layer of fluid is moving w.r.t other layer by a viscosity of μ and distance b/w them is dy .
- Since upper layer is moving with higher velocity, would force the lower slow moving layer along itself leading to development of resistance/shear on it and lower layer would try to retard the upper layer by applying same shear on it in opposite direction.
- In time " dt ",

Distance travelled by upper layer = $(v + dv) dt$
 " " " lower = $v dt$.



Here shear strain = θ

If θ is small, $\tan \theta \approx \sin \theta \approx \theta$

$$\tan \theta = \frac{da}{dy}$$

$$\text{Shear strain} = d\theta = \frac{da}{dy} = \frac{dv \cdot dt}{dy}$$

$$\frac{d\theta}{dt} = \frac{dv}{dy} \quad \text{--- (i)}$$

Hence, rate of change of shear strain or

Rate of angular deformation, $\frac{d\theta}{dt} = \text{velocity gradient} \left(\frac{dv}{dy} \right)$

- Newton's law of viscosity :-

- According to Newton's law of viscosity, shear stress acting on the layer of fluid at a distance of y from the surface is proportional to rate of change of shear strain / Rate of Angular deformation.

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu \frac{d\theta}{dt}$$

$$\tau = \mu \frac{dv}{dy}$$

μ = dynamic viscosity
Absolute " "
coeff. of " "
viscosity.

unit :- $N-s/m^2$, kg/ms , $Pa-s$, $gm/cm-s$
(poise)

$$\frac{kg}{ms} = \frac{1000 gm}{100 cm-s} = 10 gm/cm-s$$

$$\therefore 10 \text{ poise} = 1 kg/m-s \text{ or } N-s/m^2$$

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$$1 \text{ poise} = 0.1 kg/m-s \text{ or } N-s/m^2$$

- Fluid which obeys newton's law of viscosity are termed as "Newtonian fluid".

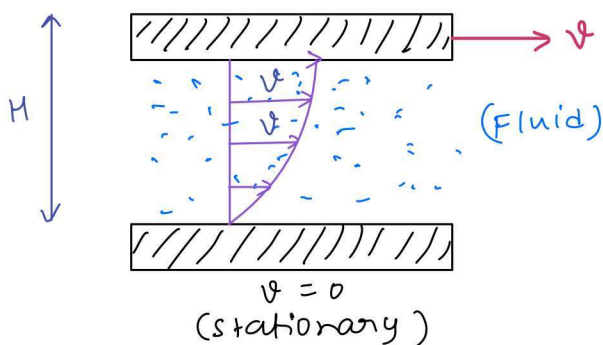
Eg :- Air, mercury, petrol, gasoline etc.

At NTP, $\mu_{\text{water}} = 1.002 \text{ centi poise}$

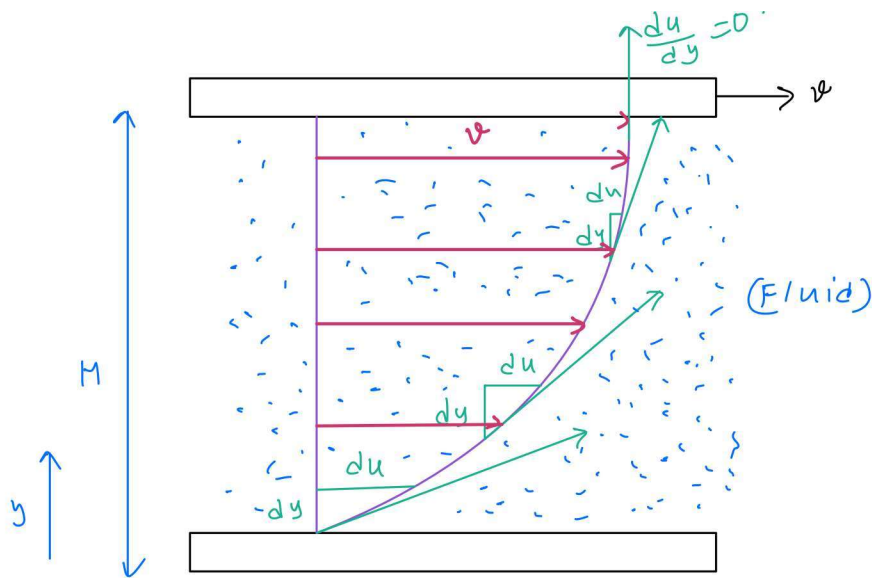
$\mu_{\text{air}} = 1.83 \times 10^{-2} \text{ centi-}$

From here it can be referred that water is about 55 time more viscous than air.

- If flow is taking place b/w the two bodies eg parallel plates, the variation of the velocity b/w the plates is not linear in general, but if thickness of the fluid is considered to be small, then this variation may be assumed to be linear. (termed as linearisation of Newton's law of viscosity).



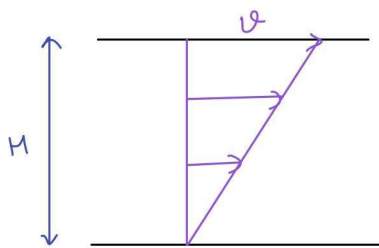
$$\tau = \mu \frac{dv}{dy}$$



- At $y=0$, $\frac{du}{dy} = \max$, Hence $\tau = \max$.

- At $y=H$, $\frac{du}{dy} = 0$, ^{Souvik 2021} Hence, $\tau = \tau_{\min} = 0$.
 $\Rightarrow \frac{du}{dy}$ i.e. velocity gradient decreases from stationary to the moving plate.

- If H is small, then variation of velocity is assumed to be linear.



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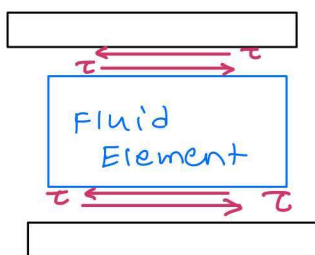
here, $\frac{du}{dy} = \text{const.}$

In this case, velocity gradient = const.

$$\tau = \mu \frac{du}{dy}$$

$$\text{at } y=0, \frac{du}{dy} = \frac{v}{H}$$

$$\therefore \tau = \mu \frac{v}{H}$$



NOTE:- when a real fluid flows over a solid body, the fluid particles at the surface of the body flow with the same velocity as that of surface of the solid, so as to satisfy no slip condition, hence relative velocity of the fluid particle at the surface of solid body is zero.

(NO SLIP CONDITION)

June 29

- In problems of Flow, viscosity is also sometimes represented in terms of **KINEMATIC VISCOSITY**, in which only flow of fluid is under consideration but the force causing the flow is not considered

$$\nu = \frac{\mu}{\rho}$$

ν = kinematic viscosity
 μ = dynamic viscosity

Units:- $\frac{\text{kg}}{\text{m-s}} \frac{\text{m}^3}{\text{kg}} = \underline{\underline{\text{m}^2/\text{s}}}$ $\rightarrow$ only length & time is involved.

• cm^2/s (**Stokes**)

• $1 \text{ Stokes} = 10^{-4} \text{ m}^2/\text{s}$

At NTP :- $\nu_{\text{water}} \stackrel{\text{Soniak 2021}}{=} 10^{-2} \text{ stokes (cm}^2/\text{s)}$

$\nu_{\text{air}} = 1.5 \times 10^{-1} \text{ stokes}$

$\rightarrow \nu_{\text{air}}$ is approx 15 times more than that of water.

$$\begin{aligned} \nu_w &= \frac{\mu_w}{\rho_w} \\ \nu_w &= \frac{55 \mu_a}{\rho_w} \\ &= 55 \nu_a \frac{\rho_a}{\rho_w} \\ \nu_w &= 55 \nu_a \rho_a \end{aligned}$$

VARIATION OF VISCOSITY:-

(i) WITH TEMPERATURE :-

LIQUIDS

(i) Dynamic

- with incr. of temp, molecules get energized, bonds get weaker, hence dynamic viscosity **decreases**.

$$\mu_L \propto \frac{1}{T}$$

Here, $\mu_{\text{liquid}} = a \cdot 10^{\left(\frac{b}{T-c}\right)}$
(Exact variation)
 a, b, c are constants.

GASES

Viscosity.

- In gas with incr. in temp. mean free path increases, molecular collision per unit vol per unit time also increases, hence dynamic viscosity **increases**.

$$\mu_g \propto T$$

$$\mu_{\text{gas}} = \frac{a \sqrt{T}}{1 + \frac{b}{T}}$$

a, b are constants.

Kinematic viscosity (ν)

- with incr in temp dynamic viscosity and density both decreases. But decrease in D.V comes out to be more than decrease in density. Hence Kinematic viscosity also **decreases** with incr. in Temp.

$$\nu = \frac{\mu \downarrow \downarrow}{\rho \downarrow} \downarrow$$

$$\boxed{\nu_L \propto \frac{1}{T}}$$

- with inc. in temp. dynamic viscosity of gas increases and at const. pressure, its density decreases. ($P = \rho R T$, $\rho \propto 1/T$), hence

Kinematic viscosity of gases increases with incr in temp. (Even at faster rate than dynamic viscosity).

$$\nu = \frac{\mu \uparrow}{\rho \downarrow} \uparrow \uparrow$$

$$\boxed{\nu_{\text{gas}} \propto T}$$

(ii) PRESSURE :-

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LIQUIDS

GASES

Dynamic viscosity

- If liquid is considered to be incompressible, then with change of pressure, there is no change in intermolecular cohesion b/w particles hence dynamic visc. remains unchanged

$$\boxed{\mu_L = \text{const}} \quad P \uparrow$$

- dynamic visc. is generally independent with pressure particularly at low to moderate pressure (3-4 atm). But if pr. is increased exceptionally high, in that d. viscosity increases.

$$\boxed{\mu_{\text{gases}} = \text{const}} \quad \text{pressure} \uparrow$$

(valid up to 3-4 atm)

$$\boxed{\mu_{\text{gases}} \propto \text{pressure}}$$

(For high pressure)

Kinematic viscosity.

- with incr. of pressure there is no change in μ and ρ of liquid, if it is considered to be incompressible. Hence K.V remains const.

- with incr in pressure, dynamic viscosity of gases remains const. for low to medium pressure but its density increases significantly ($P = \rho R T$, $\rho \propto P$)

$$\nu = \frac{\mu}{\rho} = \text{const}$$

$$\nu = \text{const}$$

Pressure

Hence, kinematic viscosity decreases.

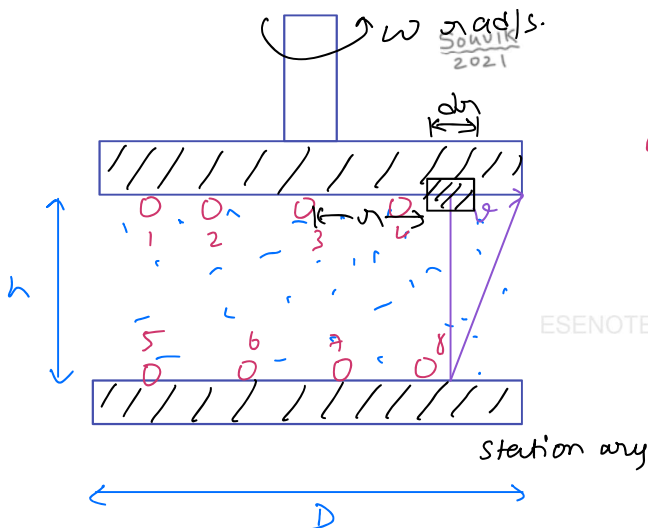
$$\nu_{PT} = \frac{\mu}{\rho_T} \downarrow$$

$$\nu \propto \frac{1}{P}$$

June 30

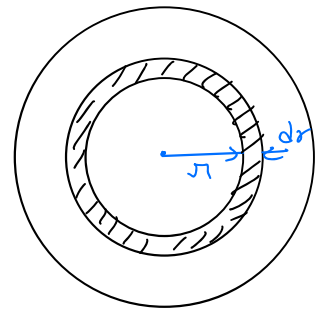
- Q) consider a fluid of viscosity μ b/w two circular parallel plates of radius " R " separated by a distance " h ". upper plate is rotated at an angular velocity of " ω " where as bottom plate is held stationary. The velocity profile b/w two plate is linear. compute the Torque experienced by bottom plate.

Sol



$$\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega$$

$$\omega_5 = \omega_6 = \omega_7 = \omega_8 = 0$$

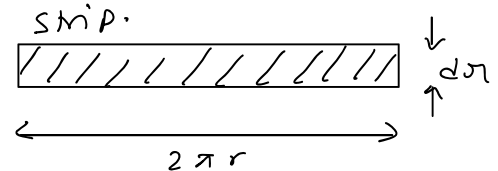


$$\text{Shear stress, } \tau = \mu \frac{du}{dy}$$

$$\text{As variation is linear, } \frac{du}{dy} = \text{const} = \frac{v}{h} = \frac{\omega r}{h}$$

$$\text{shear stress on } \therefore \text{ elemental ring } \tau = \mu \frac{\omega r}{h}$$

$$\begin{aligned} \text{force on ring} &= \tau \times A \\ &= \mu \frac{\omega r}{h} \times A \\ &= \mu \frac{\omega r}{h} \times 2\pi r dr \end{aligned}$$



$$\text{Torque on elemental ring, } T = F \cdot r$$

$$dT = \frac{2\pi\mu\omega r^3}{h} dr$$

$$dT = \frac{2\pi\mu\omega}{h} r^3 dr$$

$$\text{Total torque } \tau = \int_0^R \frac{2\pi\mu\omega}{h} r^3 dr$$

$$\tau = \frac{2\pi\mu\omega}{h} \frac{R^4}{4}$$

$$\tau = \frac{\pi\mu\omega R^4}{2h} = \frac{\pi\mu\omega D^4}{32h}$$

This is the total torque transferred by upper plate to the lower plate.

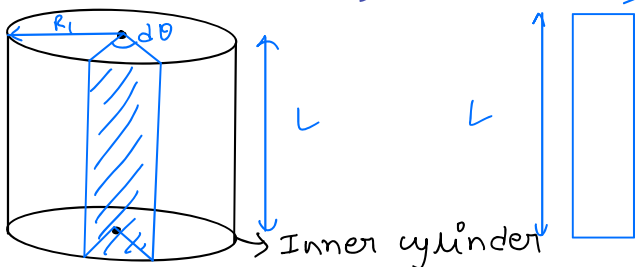
**) (b)

A rotating viscometer has two cylinders the radius of inner fixed cylinder is R_1 and the radius of outer rotating cylinder is R_2 . The viscometer is used for the measurement of viscosity of the fluid. Find the expression for the viscosity in terms of torque acting on inner cylinder of height L , gap b/w the bottom of the two cylinders is " a " and angular speed is " ω ".

Sol

Plan of side

Take the element very carefully



Total torque, $T = T_{\text{side}} + T_{\text{bottom}}$.

T_{side}

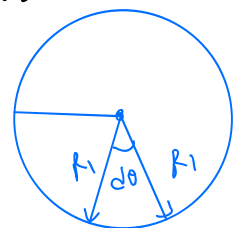
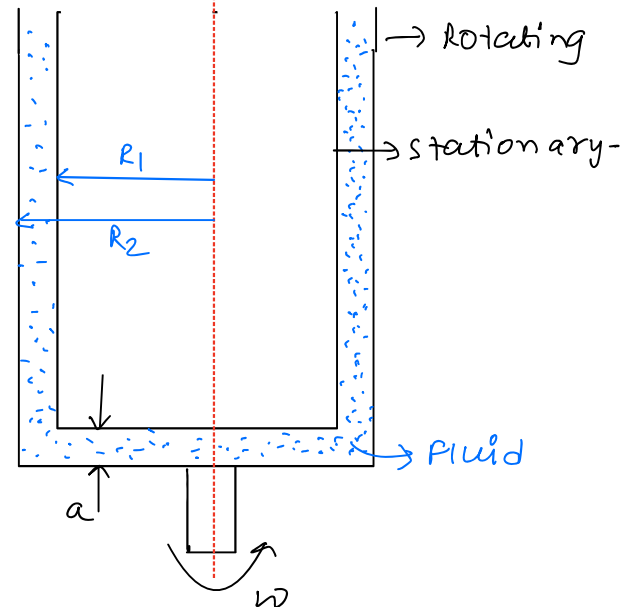
Shear stress on elemental strip, $\tau = \mu \frac{dv}{dy}$

Assuming velocity is uniformly varying.

$$\tau = \mu \frac{\omega R_2}{R_2 - R_1}$$

$$\text{Force, } F = \mu \frac{\omega R_2}{R_2 - R_1} \times (R_1 d\theta \cdot L)$$

$$\text{Torque, } T = \frac{\mu\omega R_2}{R_2 - R_1} \times (R_1 d\theta \cdot L) \cdot R_1$$



$$\therefore T = \int_0^{2\pi} \frac{\mu \omega R_2 L}{R_2 - R_1} R_1^2 d\theta$$

$$T_s = \frac{\mu \omega R_2 R_1 L (2\pi R_1)}{R_2 - R_1}$$

T_{bottom}

Shear stress, $\tau = \mu \frac{\omega r}{a}$

Force, $F = \mu \frac{\omega r}{a} (2\pi r \cdot dr)$

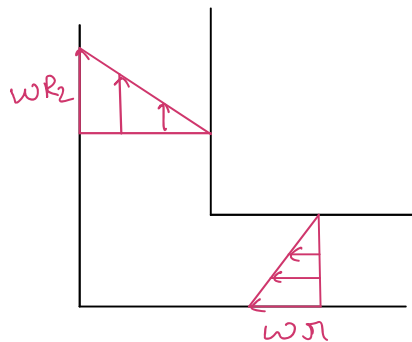
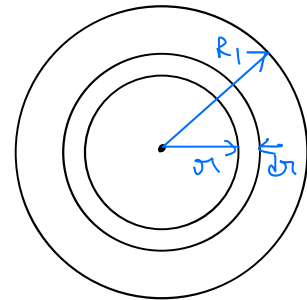
Torque, $dT = \frac{\mu \omega r}{a} (2\pi r \cdot dr) \cdot r$

$$dT = \frac{2\pi \mu \omega}{a} r^3 \cdot dr$$

$$T_{bot} = \int_0^{R_1} \frac{2\pi \mu \omega}{a} r^3 dr$$

$$T_{bot} = \frac{2\pi \mu \omega}{a} \frac{R_1^4}{4} = \frac{\pi \mu \omega R_1^4}{2a}$$

Total torque, $T = \frac{\mu \omega R_1 R_2 (2\pi R_1) L}{R_2 - R_1} + \frac{\pi \mu \omega R_1^4}{2a}$



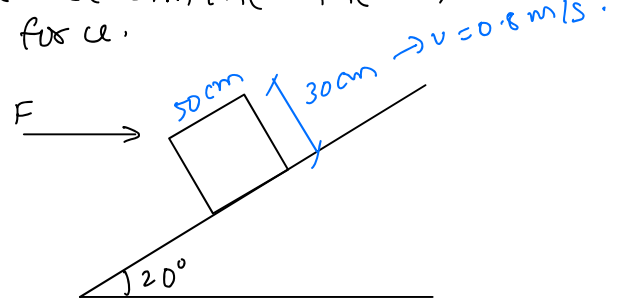
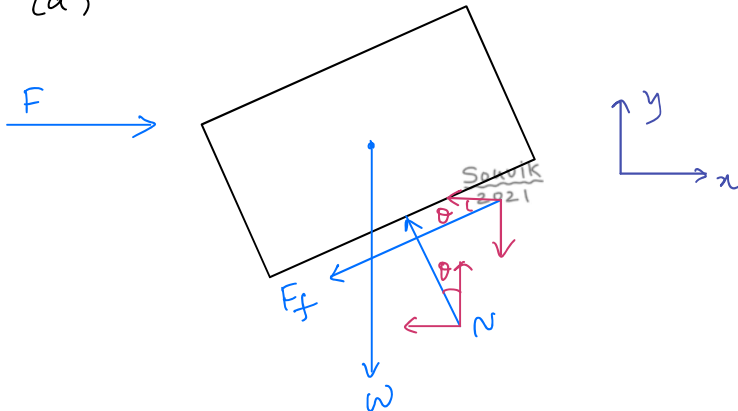
Q) A $50\text{cm} \times 30\text{cm} \times 20\text{cm}$ block weighing 150N is to be moved up an inclined plane at constant velocity of 0.8 m/s . Coeff of friction $f = 0.27$.

(a) Determine the force F required to be required in horizontal direction.

(b) If a 0.4mm thick oil film with dynamic viscosity, $\mu = 0.012\text{ Pa}\cdot\text{sec}$ is applied b/w the block and inclined surface. determine the % reduction in required force.

Sol

(a)



$$\Sigma F_y = 0:-$$

$$N \cos \theta - W - F_f \sin \theta = 0.$$

$$N \cos \theta - W - f N \sin \theta = 0.$$

$$N (\cos \theta - f \sin \theta) = W$$

$$N = \frac{W}{\cos \theta - f \sin \theta} = \frac{150}{\cos 20^\circ - 0.27 \sin 20^\circ} = 177\text{ N}.$$

$$\Sigma F_x = 0:-$$

$$F - N \sin \theta - F_f \cos \theta = 0.$$

$$F = 177 \sin 20^\circ + 0.27 \times 177 \cos 20^\circ$$

$$F = 105.4\text{ N}$$

(b)

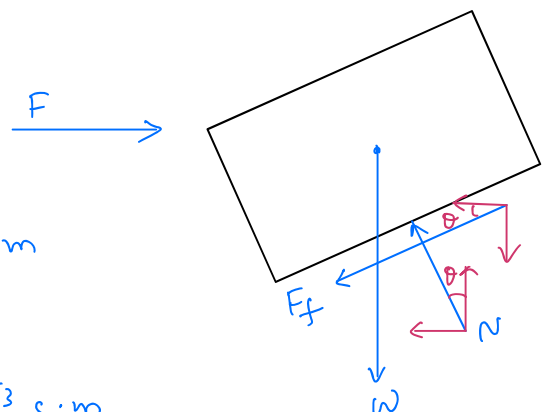
$$\text{Here, } F_f = \tau \cdot A$$

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{v}{h}$$

$$= 0.012 \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times \frac{0.8\text{ m}}{0.4 \times 10^{-3}\text{ s}\cdot\text{m}}$$

$$h = 0.4\text{ mm}$$



$$\tau = 24 \text{ N/m}^2$$

$$F_f = 24 \times (50 \times 20) \times 10^{-4}$$

$$F_f = 2.4 \text{ N}$$

$$\therefore N \cos \theta - W - F_f \sin \theta = 0.$$

$$N \cos 20^\circ - 150 - 2.4 \sin 20^\circ = 0.$$

$$N = 160.5 \text{ N}$$

$$\therefore F - N \sin \theta - F_f \cos \theta = 0.$$

$$F = 160.5 \sin 20^\circ + 2.4 \cos 20^\circ$$

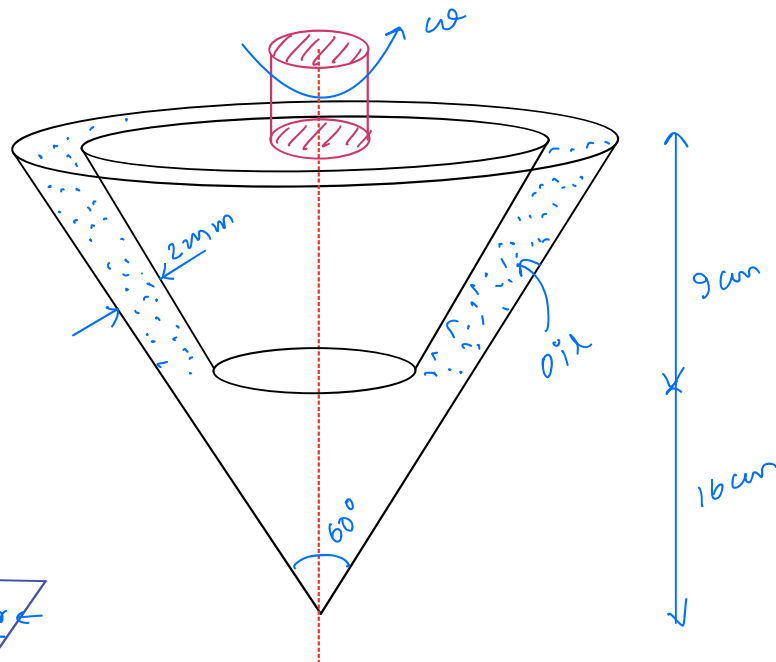
$$F = 57.15 \text{ N}$$

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$$\% \text{ change} = \frac{105.4 - 57.15}{105.4} \times 100 = 45.83 \% \text{ reduction in force.}$$

July 1

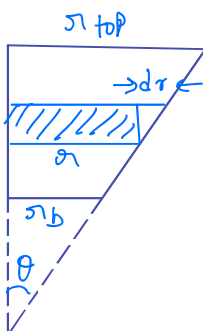
- Q) For the truncated cone as shown, compute the torque required if the cone is rotated at 200 rpm. viscosity of the oil in 2mm gap b/w the cone and housing is 2 poise. Find the torque transmitted.



shear stress on the sloping surface of the strip.

$$\tau = \mu \frac{du}{dy} = \mu \frac{v}{h} = \mu \frac{\omega r}{h}$$

Sol



shear force on the strip, $df = \tau dA = \frac{\mu \omega r}{h} 2\pi r dr$

$$df = \tau dA = \frac{\mu \omega r}{h} 2\pi r \frac{dr}{\sin \theta}$$

Torque on strip, $dT = df \cdot r = \frac{\mu \omega r}{h} 2\pi r \frac{dr}{\sin \theta} \cdot r$

$$\text{Total Torque, } T = \int dT = \int_{r_b}^{r_t} \frac{\mu \omega 2\pi}{h \sin \theta} r^3 dr$$

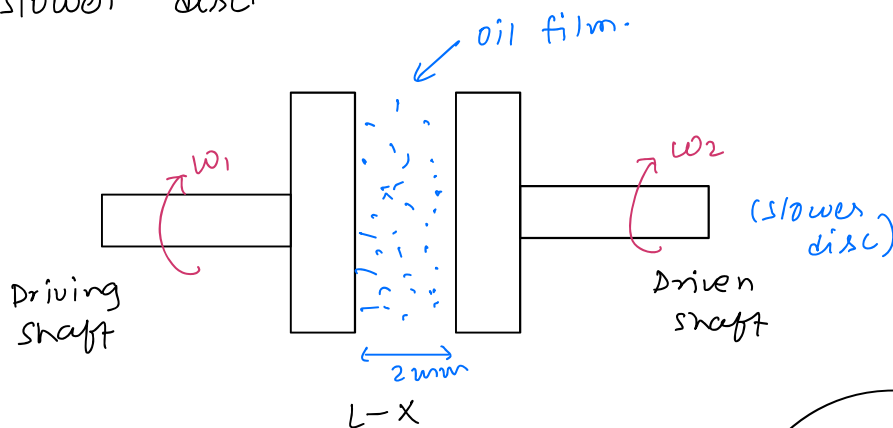
$$T = \frac{\mu \omega 2\pi}{h \sin \theta} \frac{1}{4} (r_t^4 - r_b^4)$$

$$T = \frac{\pi \omega \mu}{2 h \sin \theta} (r_t^4 - r_b^4)$$

putting the values.

$$T = 2.35 \text{ N-m}$$

- Q) The clutch system as given is used to transmit torque through a 2mm thin oil film of $\mu = 0.3 \text{ Ns/m}^2$, between two identical 300mm dia disc when the driving shaft rotates at speed 1450 rpm, the driven shaft to rotate 1398 rpm. Assuming the velocity profile to be linear, find the torque applied on slower disc.



Sol/

$$\omega_1 = 1450 \text{ rpm}$$

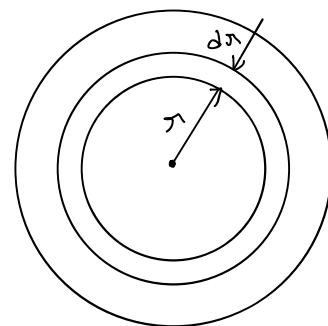
$$\omega_2 = 1398 \text{ rpm}$$

shear stress over the disc element

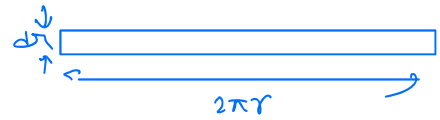
$$\tau = \mu \frac{du}{dy} = \mu \times \frac{v}{h}$$

$$\tau = \mu \times \frac{(\omega_1 - \omega_2) r}{h}$$

$v = \text{relative velocity}$
 $v = (\omega_1 - \omega_2) r$



Shear force, $dF = \tau \cdot dA$



$$dA = 2\pi r \cdot dr$$

$$dF = \frac{\mu(\omega_1 - \omega_2)}{h} r \cdot 2\pi r dr$$

Torque, $dT = dF \cdot r$

$$\int_0^T dT = \int_0^R \frac{2\pi\mu(\omega_1 - \omega_2)}{h} r^3 dr$$

(Given)
 $\omega = \frac{2\pi N}{60}$

$$T = \frac{2\pi\mu(\omega_1 - \omega_2)}{4h} R^4 \cdot dr$$

$$T = \frac{\pi\mu(\omega_1 - \omega_2)}{2h} R^4$$

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$$T = \frac{\pi \times 0.3 \frac{Ns}{m^2} (1450 - 1398) \times 0.3^4}{2 \times 2 \times 10^{-3} m}$$

$$T = 0.64 \text{ N-m}$$

Q) A thin square plate $1m \times 1m$ is placed horizontally in a horizontal gap of ht $2m$, filled with the oil of viscosity 10 poise and pulled at a constant velocity of $0.1m/sec$. Find the force acting on the plate.

- If the oil of viscosity μ_1 & μ_2 are placed on the either side of the plate, then compute the position of the plate for shear force on both side of the plate to be same and for pull required to drag the plate to be

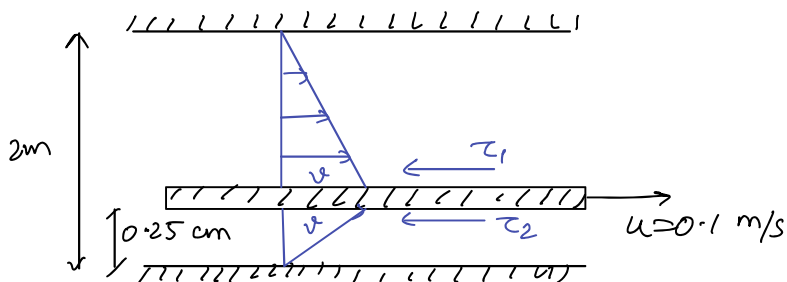
Sol
 $\mu = 10 \text{ poise} \left(\frac{g}{cm-s} \right) = 1 \text{ N-s/m}^2$
 $v = 0.1 \text{ m/sec}$

(a) $P = F_1 + F_2$

$$= \tau_1 A + \tau_2 A$$

$$= A(\tau_1 + \tau_2)$$

$$= A \left(\mu \frac{du}{dy_1} + \mu \frac{du}{dy_2} \right) = A\mu \left[\frac{v}{h_1} + \frac{v}{h_2} \right]$$



$$= 1 \text{ m}^2 \times 1 \frac{\text{N-s}}{\text{m}^2} \times 0.1 \frac{\text{m}}{\text{s}} \left[\frac{100}{1.75} + \frac{100}{0.25} \right]$$

$$= 0.1 \times 457.14 = 45.714 \text{ N.}$$

(b) let μ_1 be placed above and μ_2 below.

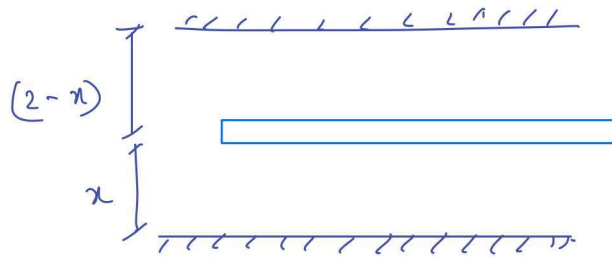
Suppose the plate is kept at a distance x from bottom plate

Force due to top fluid

$$F_1 = \tau_1 \cdot A$$

$$= \mu_1 \frac{du}{dy_1} \cdot A$$

$$F_2 = \tau_2 A = \mu_2 \frac{du}{dy_2} \cdot A$$



$$\mu_1 \frac{v}{(2-x)} \cdot A = \mu_2 \frac{v}{x} \cdot A$$

$$\frac{\mu_1}{2-x} = \frac{\mu_2}{x}$$

$$x \mu_1 = 2\mu_2 - \mu_2 x$$

$$x(\mu_1 + \mu_2) = 2\mu_2$$

$$x = \frac{2\mu_2}{\mu_1 + \mu_2}$$

$$\Rightarrow x = \frac{\mu_2 h}{\mu_1 + \mu_2} \quad (*) \rightarrow \text{objective}$$

(i) Pull req is min.

pull req to drag = $F_1 + F_2$

$$P = \left[\mu_1 \frac{v}{(h-x)} + \mu_2 \frac{v}{x} \right] \times A$$

for P to be minimum

$$\frac{dP}{dx} = vA \left[\frac{\mu_1}{(h-x)^2} + \left(-\frac{\mu_2}{x^2} \right) \right] = 0$$

$$\frac{\mu_1}{(h-x)^2} = \frac{\mu_2}{x^2}$$

$$\frac{\mu_1}{\mu_2} = \left(\frac{h-x}{x} \right)^2 = \left(\frac{h}{x} - 1 \right)^2$$

$$\frac{h}{x} - 1 = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\frac{h}{x} = \sqrt{\frac{\mu_1}{\mu_2}} + 1$$

$$x = \frac{h}{\sqrt{\frac{\mu_1}{\mu_2}} + 1}$$

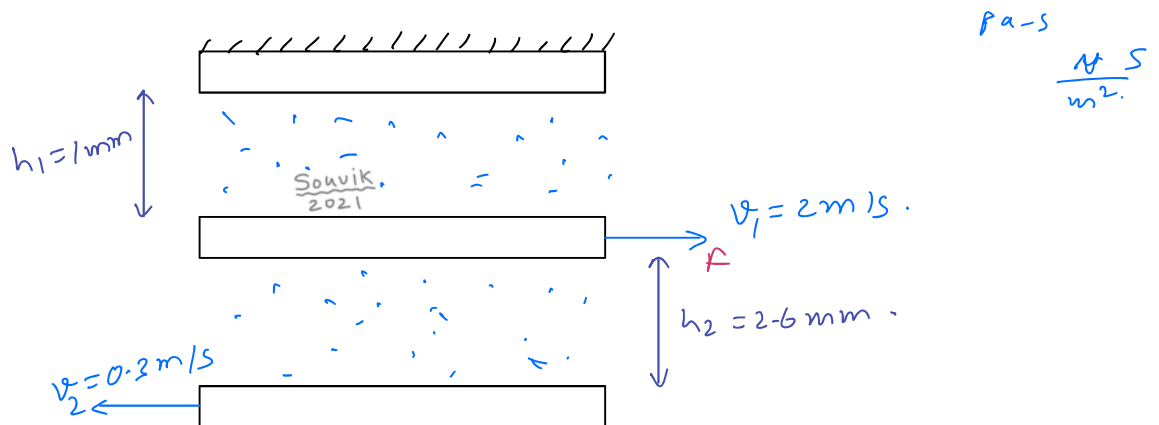
for this distance of plate, the pull req will be minimum.

- Q) A $40\text{cm} \times 40\text{cm}$ flat plate is pulled at 2m/sec horizontally through a 3.6mm thick oil layer sandwiched b/w two plates one stationary and other moving at a constant speed of 0.3m/s as shown.

The dynamic viscosity of oil = $0.027\text{ Pa}\cdot\text{s}$.

Assume velocity in each layer to vary linearly.

- (a) Plot velocity profile and find the location where oil velocity is zero.
- (b) Determine the force req to maintain this motion.



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Sol (a) velocity profile

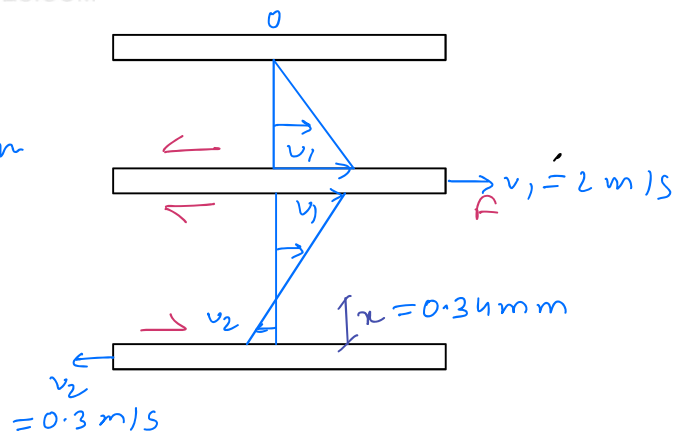
let oil velocity is zero at a distance x from bottom plate.

$$\frac{v_1}{h_2 - x} = \frac{v_2}{x}$$

$$v_1 x = v_2 h_2 - v_2 x$$

$$x(v_1 + v_2) = v_2 h_2$$

$$x = \frac{v_2 h_2}{v_1 + v_2} = 0.34\text{ mm from bottom}$$



- (b) Force req to maintain this.

$$F = F_{\text{top}} + F_{\text{bottom}}$$

$$= \tau_1 A + \tau_2 A = A \left(\mu \frac{du}{dy_1} + \mu \frac{dv}{dy_2} \right)$$

$$= A \mu \left(\frac{v_{01}}{h_1} + \frac{v_{02}}{h_2} \right)$$

$$v_{n1} = y - 0 = y$$

$$v_{n2} = v_1 + v_2 = 2.3 \text{ m/s}$$

$$F = 1600 \times 10^{-4} \text{ m}^2 \times 0.027 \frac{\text{N-s}}{\text{m}^2} \left(\frac{2}{10^{-3} \text{ m}} \text{ m/s} + \frac{2.3 \text{ m/s}}{2.6 \times 10^{-3}} \right)$$

$$F = 0.16 \times 0.027 \frac{\text{N}}{\text{s}} \left(2884.61 \frac{1}{\text{s}} \right)$$

$$F = 12.46 \text{ N}$$

RHEOLOGY OF FLUID :-

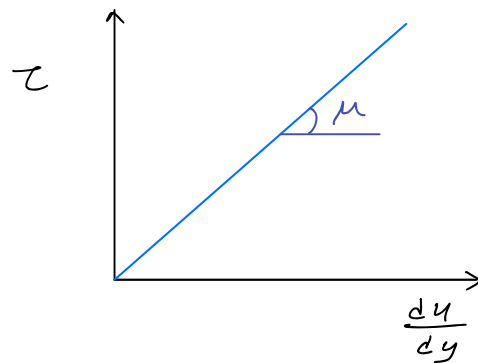
- Rheology is the study of different types of fluid in flow.
- Fluids may be classified as follows:-

(a) NEWTONIAN FLUID ^{Souvik} - The fluid which obeys Newton's law of viscosity is termed as Newtonian fluid i.e. for which shear stress on the layer of the fluid is proportional to rate of change of shear strain/velocity gradient.

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy}$$

$$y = \tau, \quad x = \frac{du}{dy}, \quad m = \mu$$

Eg:- water, air, mercury.



July 6 (b) Non-Newtonian fluid :-

- These are the type of fluids which do not follow Newton's law of viscosity. i.e. in this case shear stress b/w the layer of the fluid is not directly proportional to rate of change of shear strain/Rate of Angular deformation/velocity gradient.

- In general for any fluid, shear stress varies as follows.

Shear stress, $\frac{du}{dy}$ = velocity gradient

$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

A = consistency index
B = yield stress.
n = flow behaviour index

- For Newtonian fluid, yield stress $B=0$.
consistency index, $A=\mu$.
 $n=1$

Here,

$$\frac{d\tau}{d\left(\frac{du}{dy}\right)} = An \left(\frac{du}{dy}\right)^{n-1} = \mu_{\text{apparent}}$$

Rate of change of shear stress with change in velocity gradient.

It signifies slope of shear stress and $\frac{du}{dy}$ curve
also termed as APPARENT VISCOSITY

- For Newtonian fluid, $A=\mu$, $n=1$

$$\therefore \mu_{\text{app}} = \mu \quad (**)$$

apparent viscosity is constant i.e independent of change in velocity gradient

- Non-newtonian fluids ^{Sauvik} are further classified as follows.

- (a) Time-independent (b) Time dependent.

(a) TIME-INDEPENDENT NNF :-

- It is the type of non-newtonian fluid apparent viscosity of which does not depend on time.

- For them yield stress, $B=0$ i.e curve will start from origin.

$$\tau = A \left(\frac{du}{dy}\right)^n$$

- These are further classified as :-

- (i) DILATANT FLUID (ii) PSEUDO PLASTIC FLUID.

(i) DILATANT FLUID :-

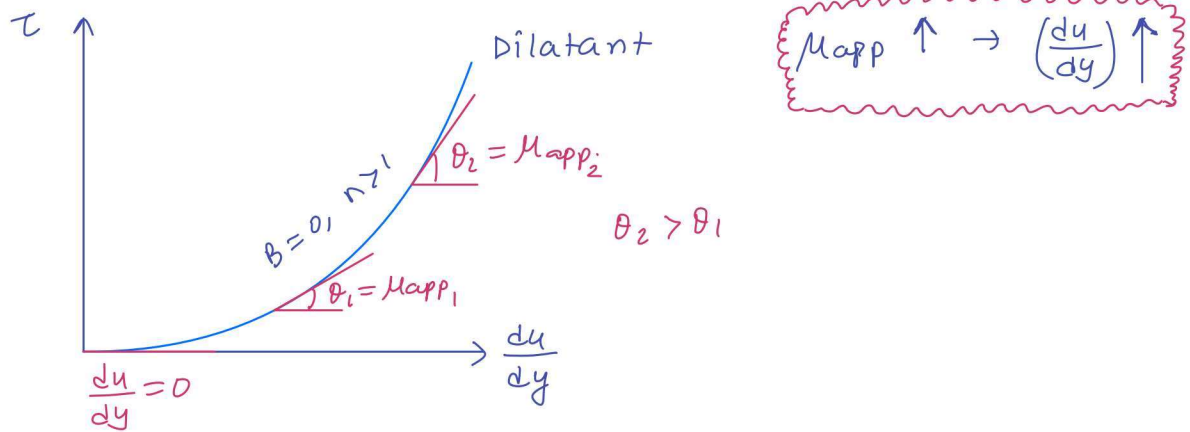
- These are also termed as shear thickening fluid

- These are the type of fluids for which apparent increases with velocity gradient.

$$\mu_{\text{app}} = An \left(\frac{du}{dy}\right)^{n-1}$$

Here, $n > 1 \Rightarrow$ for dilatant fluid.

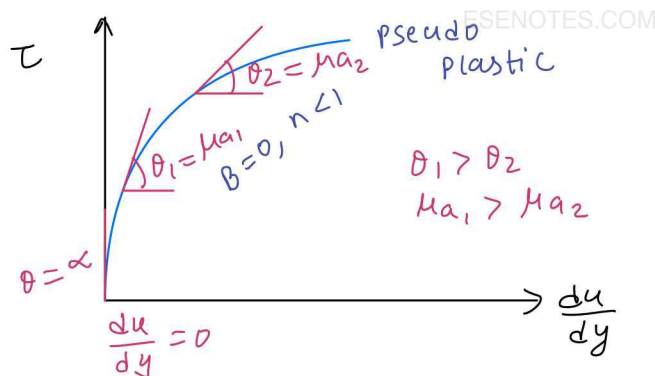
Eg:- Solution of suspended starch, sand, sugar in water.



(ii) PSEUDO PLASTIC FLUID :-

- These are also termed as **shear thinning fluid**.
- These are the type of fluids apparent viscosity of which decreases with velocity gradient. i.e. **$n < 1$**
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- Eg:- blood, paper pulp, syrup, milk, gelatine, paint, polymer solution.

For them, $\mu_{app} \downarrow \rightarrow \left(\frac{du}{dy}\right)^{n-1}$



NOTE:- BINGHAM PLASTIC FLUID (Does not fall in any of the category above)

- It is also termed as **ideal plastic fluid**.
- It is a type of fluid which requires a certain min shear stress termed as **yield stress**. τ_0 . before they start flowing.
- For these fluids $B = \tau_0$, $A = \mu$, $n = 1$.

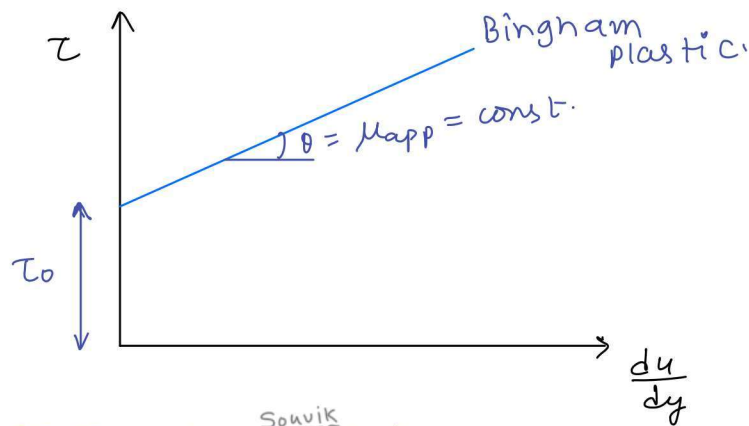
$$\therefore \tau = \mu \left(\frac{du}{dy} \right) + \tau_0$$

($y = mx + c$)

$$\mu_{app} = \frac{\tau}{\frac{du}{dy}} = A n \left(\frac{du}{dy} \right)^{n-1}$$

$$\mu_{app} = \mu = \text{constant}$$

Eg → Sewage sludge, drilling mud, tooth paste.



(b) TIME DEPENDENT ^{Souvik} ~~Fluid~~ :-

- These are the type of fluids, apparent viscosity of which depends on time.
- For this fluids yield stress is not zero.

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$$\tau = A \left(\frac{du}{dy} \right)^n + B$$

- These are further classified as :-

(i) Thixotropic fluid

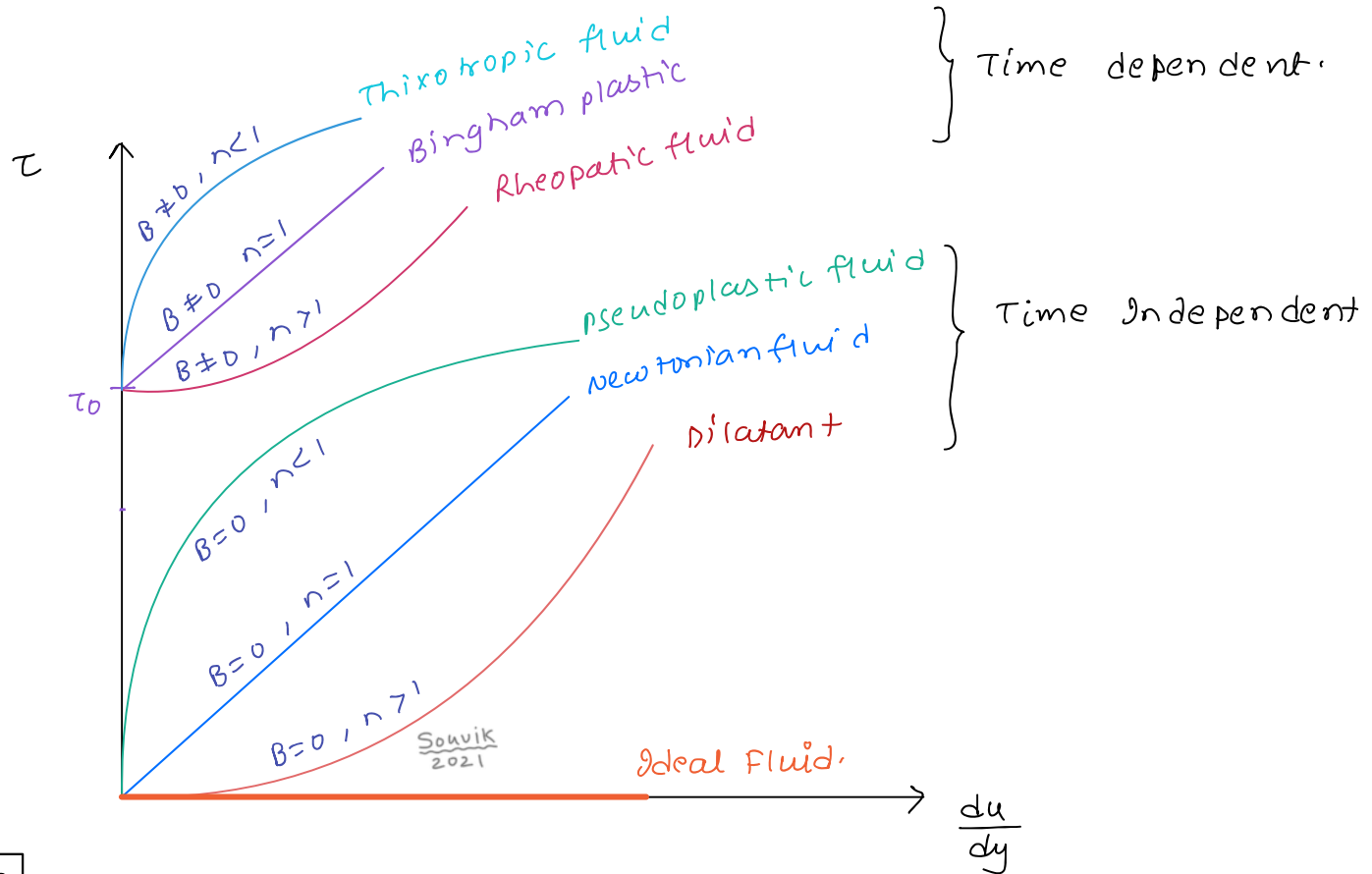
(ii) Rheotropic fluid/
Rheopetic fluid.

(i) THIXOTROPIC FLUID :-

- These are the type of fluid apparent viscosity of which decreases with time.
- For them flow behaviour index, $n < 1 \rightarrow$ slope will decrease
Eg!- some paints, enamel, ketchup, printer ink.

(ii) RHEOPETIC FLUID :-

- These are the type of fluid, app. viscosity of which increases with time.
- For them, $n > 1 \rightarrow$ slope will increase.
Eg!- Lubricants, gypsum paste.



July 7

(C) IDEAL FLUID :-

It is an imaginary fluid which does not possess surface tension, viscosity (invicid) and is incompressible.

Q) Identify the fluids for which velocity gradient and shear stress variation is as follows.

(i) $\frac{du}{dy}$ (rad/sec) 0 1 2 3 4

τ (kPa) 2 3 4 5 6

$$\text{sol} \Rightarrow \frac{du}{dy} = 0, \quad \tau = 2 \quad \text{and} \quad \frac{d\tau}{d\left(\frac{du}{dy}\right)} = \frac{4-3}{2-1} = 1 \quad \left. \vphantom{\frac{d\tau}{d\left(\frac{du}{dy}\right)}} \right\} \text{const.}$$

$$= \frac{3-2}{5-4} = 1$$

Here $\tau_0 = 2 \text{ kPa}$, and $\frac{d\tau}{d\left(\frac{du}{dy}\right)} = \text{const.}$

i.e Bingham plastic.

(ii)	$\frac{du}{dy}$ (rad/s)	0	0.5	1.0	1.8
	τ (kPa)	0	2	4	6

Sol = $\frac{d\tau}{d(\frac{du}{dy})} = \frac{2}{0.5} = 4$
 $\frac{2}{0.6} = 3.33$
 $\frac{2}{0.7} = 2.85$

$\theta = \frac{d\tau}{d(\frac{du}{dy})} = \text{decreasing}$
 $\tau_0 = 0$

$$\tau = A \left(\frac{du}{dy} \right)^n + \tau_0$$

Hence, $n < 1$ i.e. pseudo plastic fluid.

(iii)	$\frac{du}{dy}$ (rad/s)	0	1	3	5
	τ (kPa)	15	20	30	40

Sol = $\frac{d\tau}{d(\frac{du}{dy})} = \frac{20-15}{1} = 5$, $\frac{30-20}{2} = 5$, $\frac{40-30}{2} = 5$

$$\tau_0 = 15 \text{ kPa}$$

Map is const., $n = 1$.

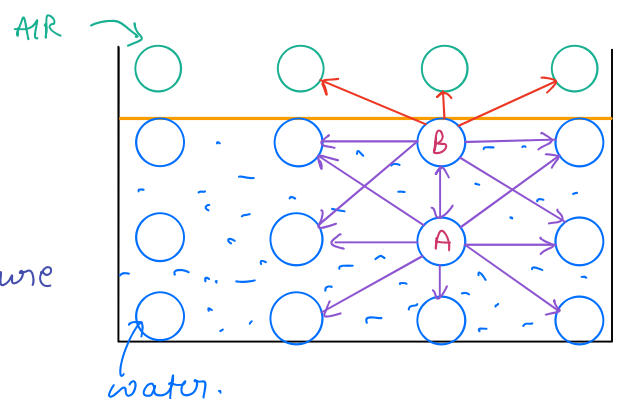
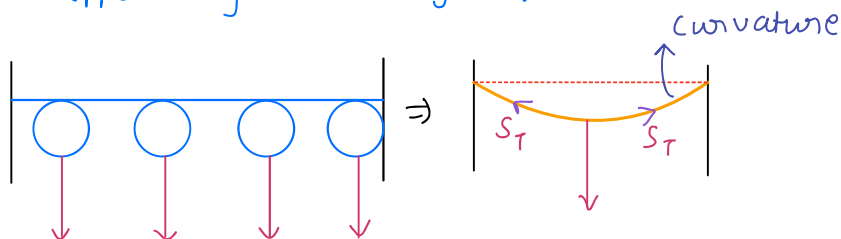
BINGHAM PLASTIC.

(iv) SURFACE TENSION :-

- Net force on molecule A = 0.

$$\sum F_B \neq 0 \downarrow$$

upper layer will get pulled.



- At the interface of liquid and gas or between any two immiscible fluid a special layer is formed due to attraction of fluid molecules on the surface from below the surface.

- As a result a net downward force is observed which pulls the surface down causing a curvature to the surface which in turn develops a tension in the surface termed as surface tension. (ST)

- It is force per unit length. $\sigma = \frac{\text{force}}{\text{length}}$ (N/m)

- Since surface tension is primarily due to cohesion, all the factors affecting cohesion affects surface tension also.

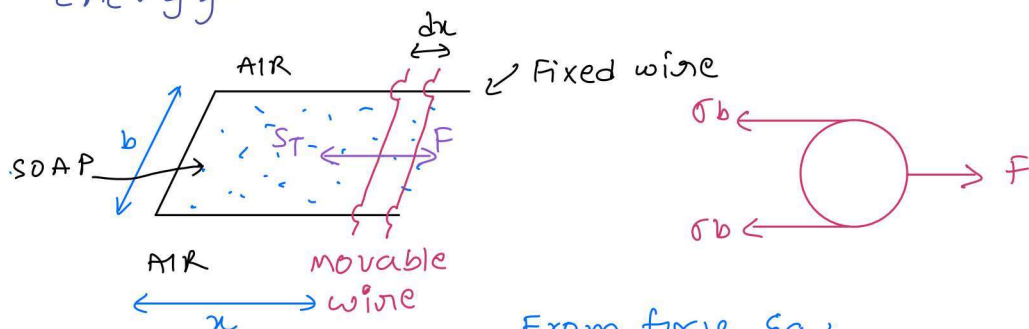
Eg:- with increase in temp. of liquid, cohesive forces between the molecules reduces, hence surface tension also reduces.

- Due to this surface ^{Sapvik} tension, an object can be supported over the surface of the liquid
Eg. Insect, pin, coin.
and formation of droplet over the leaves after rain fall.

- The ST also depends on the type of interface i.e type of liquid and air.

$\sigma_{\text{water in contact with air at } 20^\circ\text{C}} = 0.0736 \text{ N/m}$

- S.T can also be defined as work done per unit increase in the surface area of liquid surface which is stored in the form of surface energy.



From force Eq.,

$$F = 2\sigma b$$

work done in moving the wire by distance "dx"

$$dW = F \times dx = 2\sigma b dx = \sigma (2b dx) = \sigma dA$$

$$\sigma = \frac{dW}{dA}$$

Area in both side.

- 8) spherical droplets of radius "R" is split into n-number of smaller droplets of equal size. Find the work required to be done.

Sol

Mass of big droplet of radius "R" = mass of n-droplet of radius r

Since, $P = \text{const}$

$\therefore$ vol. of big droplet = vol. of n-small droplet

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$r = \left(\frac{R^3}{n}\right)^{1/3} \quad n = \frac{R^3}{r^3}$$

Increase in SA of water droplets = $4\pi r^2 \cdot n - 4\pi R^2$

$$= 4\pi r^2 \times \left(\frac{R}{r}\right)^3 - 4\pi R^2$$

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$$dA = 4\pi R^2 \left(\frac{R}{r} - 1\right)$$

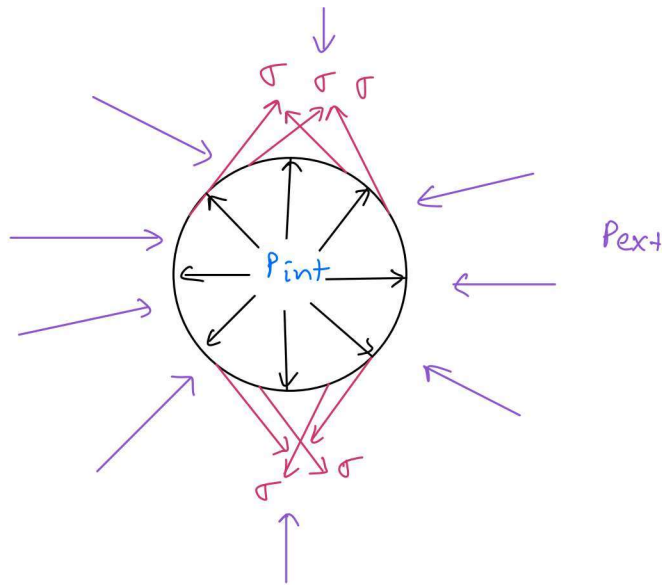
$$\sigma = \frac{dW}{dA}$$

Work done, $dW = \sigma \cdot dA$

$$= \sigma \times 4\pi R^2 \left(\frac{R}{r} - 1\right)$$

$$W = \sigma 4\pi R^2 (n^{1/3} - 1)$$

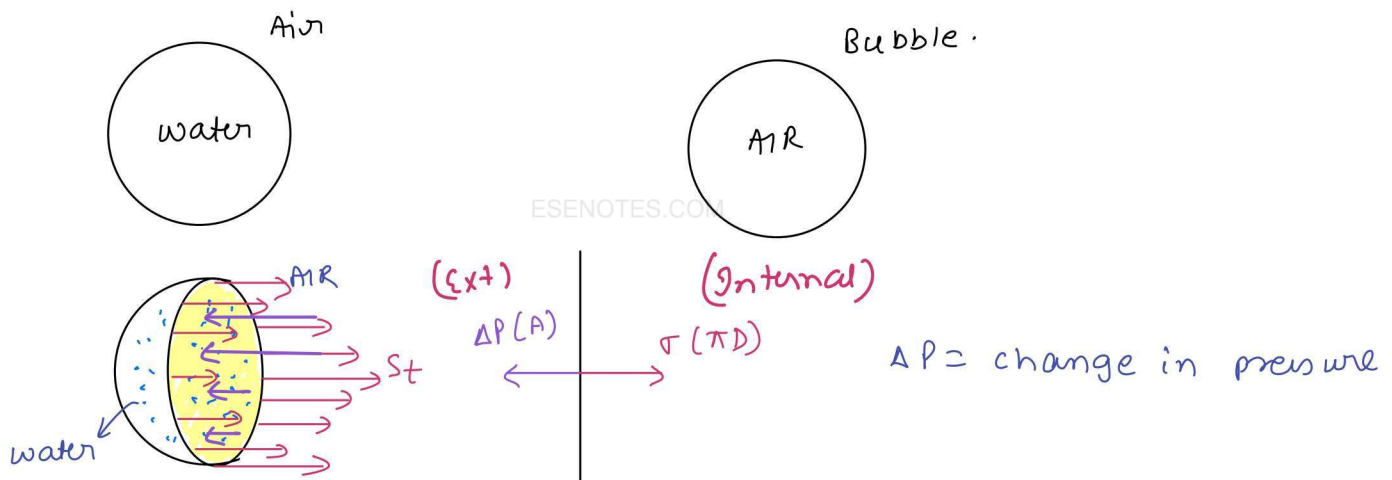
- The effect of the ST can be understood in case of droplet and liquid jet.
- When the droplet separates from main body of liquid it is subjected to inward force or pressure all over the surface due to surface tension, which causes liquid droplet or jet to compress or contract, resulting in increase of internal pressure upto an extent it balances external inward force at which liquid droplet assumes spherical geometry. (Geometry having min. surface area).



— This concept can be used to find the pressure intensity in excess of outside pressure in droplets.

case-(i)

- (a) Pressure inside (i) droplet of liquid in air or
(ii) ~~air~~ bubble in water.



From Force eq:-

$$\sigma \pi D = \Delta P (A)$$

$$\Delta P = \frac{\sigma \pi D}{\pi \frac{D^2}{4}} = \frac{4\sigma}{D}$$

$$\Delta P = \frac{4\sigma}{D}$$

$$\Delta P = P_{int} - P_{ext} = \frac{4\sigma}{D}$$

NOTE:- Since $\Delta P \propto \frac{1}{D}$, smaller is the bubble/droplet higher is the pressure difference.

OR The above analysis can also be found using work concept.

work done by σ $dw = \sigma \cdot dA = \sigma d(4\pi R^2) = \sigma 8\pi R dR$ — (i)

Also, $dw = \underbrace{\Delta P \cdot A}_{\text{Force}} \cdot \underbrace{dR}_{\text{distance}} = \Delta P \cdot (4\pi R^2) \cdot dR$ — (ii)

(i) = (ii)

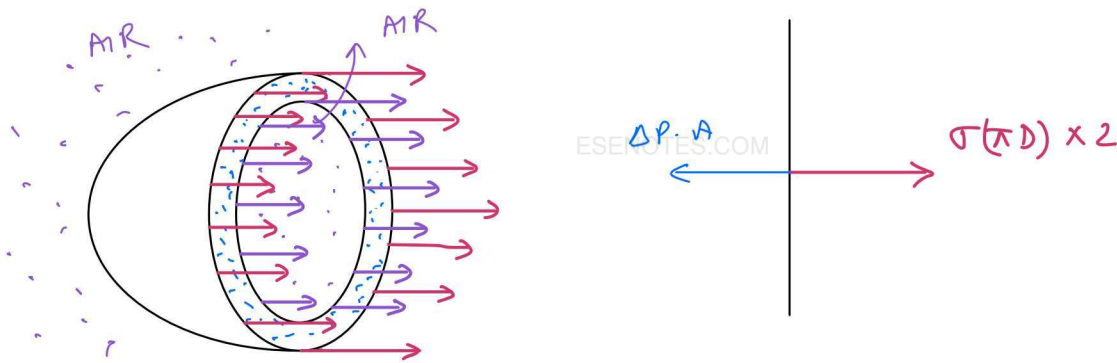
$\sigma 8\pi R dR = \Delta P \cdot 4\pi R^2 \cdot dR$

$\Delta P = \frac{8\sigma\pi R}{4\pi R^2} = \frac{2\sigma}{R} = \frac{4\sigma}{D}$

$\therefore \Delta P = \frac{4\sigma}{D}$

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case (ii) :- Pressure inside soap bubble in Air.



Thickness is negligible

From force eq :- $\sigma \pi D \times 2 = \Delta P \cdot \frac{\pi D^2}{4}$

$\Delta P = \frac{8\sigma}{D}$

From work done concept

$dw = \sigma dA = \sigma d(4\pi R^2 \times 2)$
 $= \sigma \cdot 16\pi R \cdot dR$

Also, wd by pressure force

$dw = \Delta P \cdot (4\pi R^2) \times dR$

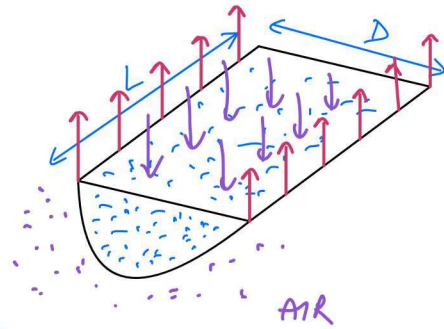
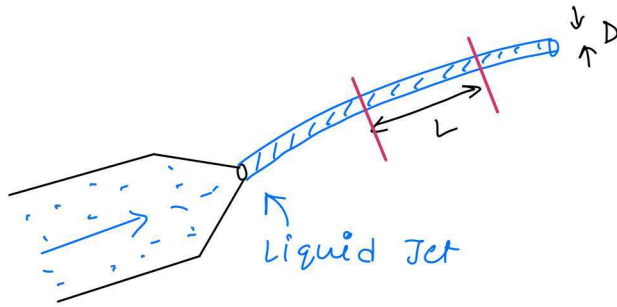
$\sigma \cdot 16\pi R \cdot dR = \Delta P \cdot 4\pi R^2 \cdot dR$

$\Delta P = \frac{4\sigma}{R}$

$\Delta P = \frac{8\sigma}{D}$

σ is acting on two surface

case (iii) :- Pressure inside liquid jet :-



→ Only one interface.

$$\sigma(2L) = \Delta P \cdot (L \cdot D)$$

$$\Delta P = \frac{2\sigma}{D}$$

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From WD concept.

$$\text{WD by } \sigma, \quad dW = \sigma \cdot dA = \sigma \times d(2\pi R \cdot L) = \sigma \cdot 2\pi L$$

$$\text{WD by pressure force, } dW = \Delta P \cdot A \cdot dR = \Delta P \times (2\pi R \cdot L) \cdot dR$$

$$\therefore \sigma(2\pi L) = \Delta P \cdot (2\pi R L)$$

$$\Delta P = \frac{\sigma}{R}$$

$$\therefore \Delta P = \frac{2\sigma}{D} \quad \checkmark$$

Eg:- compute P_{int} or pressure req to form the air bubble.

Sol one interface.

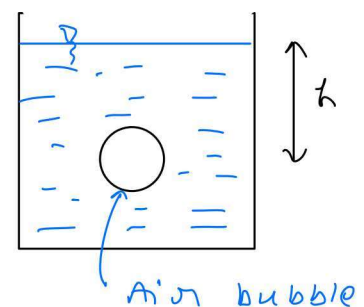
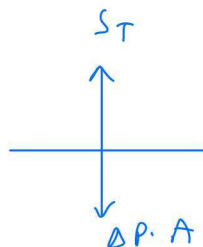
$$\sigma \cdot (\pi D) = \Delta P \cdot \frac{\pi D^2}{4}$$

$$\Delta P = \frac{4\sigma}{D}$$

$$P_{int} - P_{ext} = \frac{4\sigma}{D}$$

$$P_{int} - \rho_w h = \frac{4\sigma}{D}$$

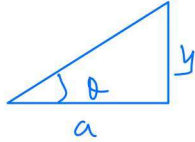
$$P_{int} = \frac{4\sigma}{D} + \rho_w h //$$



- Q) A container of width $2a$ is filled with liquid. A thin wire of weight per unit length " λ " is gently placed over the liquid surface in the middle of surface. As a result the liquid surface depresses by a distance y ($y \ll a$). Find the surface tension of liquid.

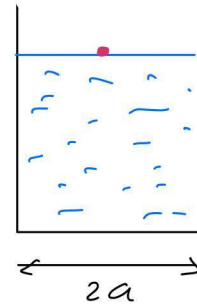
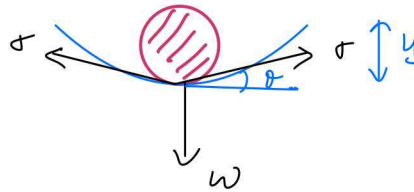
Sol

$$\lambda = \frac{w}{L}$$



$$\tan \theta = y/a \quad y \ll a$$

$$\tan \theta \approx \theta \approx \sin \theta$$

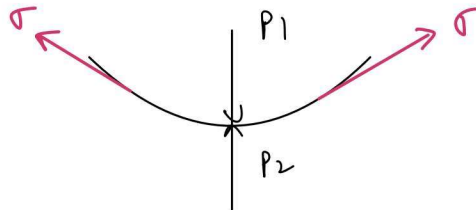


$$2(\sigma L) \sin \theta \quad \text{Sourvik 2021} \quad \lambda \cdot L$$

$$\sigma = \frac{\lambda}{2 \sin \theta} = \frac{\lambda}{2 \times y/a} = \frac{a\lambda}{2y}$$

NOTE:- Pressure on concave side of interface b/w two immiscible liquid is always more.

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→ P_1 needs to resist both P_2 and σ . Hence

$$P_1 > P_2$$

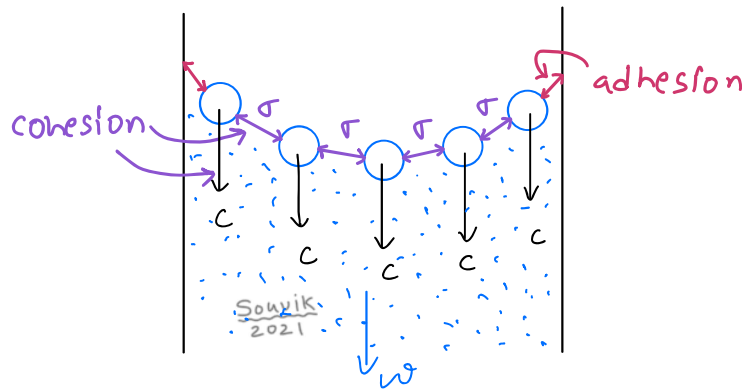
Bubble : Droplet : liquid jet

$$\frac{8\sigma}{d} : \frac{4\sigma}{d} : \frac{2\sigma}{d}$$

$$4 : 2 : 1$$

(i) CAPILLARITY :-

- It is defined as rise or fall of a liquid in a small diameter tube inserted into it.
- The rise is called "CAPILLARY RISE" and fall is called "CAPILLARY FALL"



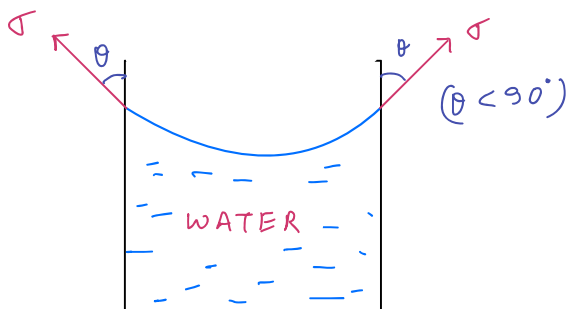
- Adhesion of liquid to the walls of the vessel/tube can cause an upward force on liquid at edges and results in meniscus turning upward.
- Since S-T acts to hold the surface intact, so instead of just the edge moving upward the whole liquid surface is dragged up to an extent it is balanced by weight/pressure of liquid.

Jul 9

CAPILLARY RISE

It occurs when adhesion is more than cohesion.

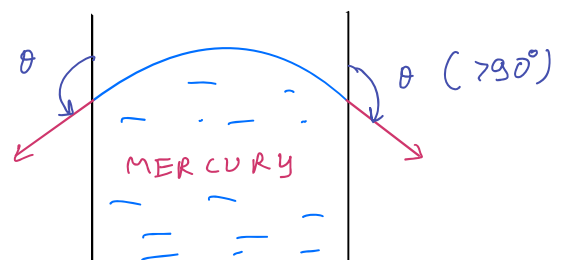
- Here contact angle θ is less than 90°

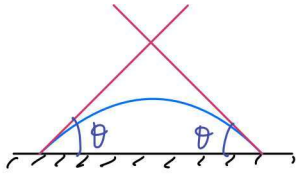


CAPILLARY FALL

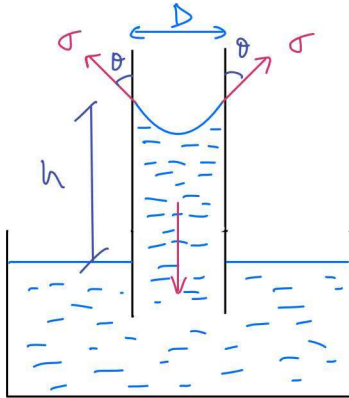
It occurs when cohesion is more than adhesion.

- Here contact angle $\theta > 90^\circ$.

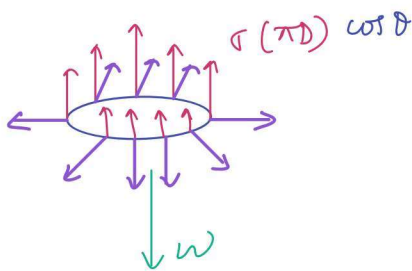




- Here liquid wets the solid surface.
- Here top surface is concave upwards



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Horizontal component cancel each other.

From force equil.

$$\sigma \pi D \cos \theta - W_w = 0$$

$$\sigma \pi D \cos \theta = W_w r_w$$

$$\sigma \pi D \cos \theta = A h r_w$$

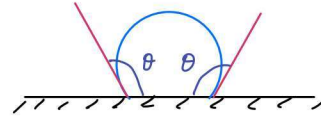
$$\sigma \pi D \cos \theta = \frac{\pi D^2}{4} h r_w$$

$$h = \frac{4 \sigma \cos \theta}{r_w D}$$

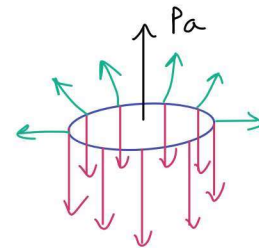
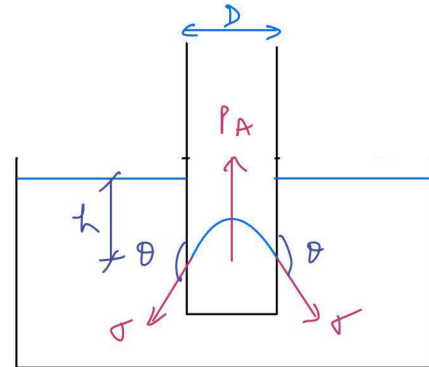
OR

$$\sigma \pi D \cos \theta = A \times P \quad (P = r_w h)$$

$\sigma \rightarrow$ Force



- Here liquid don't wet the solid surface.
- Here top surface is convex upward.



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Horizontal comp. will be cancelled out.

$$\sigma (\pi D) \cos(180 - \theta) = P_a \times A$$

$$\sigma \pi D \cos \theta = P_a \times A$$

$$\sigma \pi D \cos \theta = (h r) \times \frac{\pi D^2}{4}$$

$$h = \frac{4 \sigma \cos \theta}{r D}$$

$\theta < 90^\circ \rightarrow$ fluid is wetting fluid

cohesive < adhesive $\rightarrow$ — " —

$\theta > 90^\circ \rightarrow$ non-wetting fluid

cohesive > adhesive $\rightarrow$ — " —

NOTE:- capillary rise or fall inversely depends on size of tube, hence capillary effect of water is usually negligible in tubes whose size is greater than 10 mm.

- Due to this only pressure measuring devices are made with large tubes of size greater than 10 mm to minimize capillary effect.

- Here the contact angle b/w the surface of the tube and liquid depends on material of the tube and gases present above the liquid.

if salt is present θ will increase

(a) water glass air interface,	$\theta \approx 0^\circ$
(b) mercury glass air "	$\theta \approx 130^\circ$
(c) kerosene glass air "	$\theta \approx 26^\circ$
(d) water paraffin air "	$\theta \approx 107^\circ$
(e) Mercury ^{Solvik 2025} sodalime glass,	$\theta \approx 140^\circ$

SPECIAL CASES

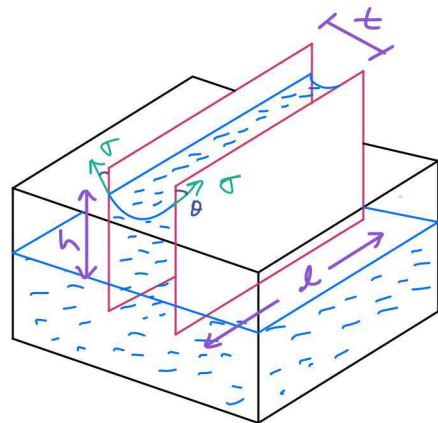
(a) capillary rise b/w parallel plate :-

From force equilibrium. ESENOTES.COM

$$2\sigma \cos \theta l = P \times A$$

$$2\sigma \cos \theta l = h \gamma (t \times l)$$

$$h = \frac{2\sigma \cos \theta}{\gamma \cdot t}$$



(b) capillary rise in Annular Space :-

R_o = outer dia

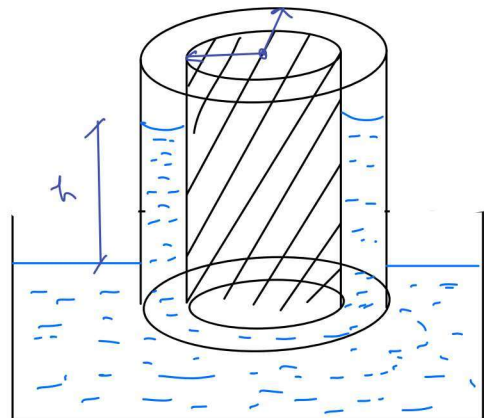
R_i = inner dia.

By force equilibrium

$$\sigma (2\pi R_i) \cos \theta + \sigma (2\pi R_o) \cos \theta = P \times A$$

$$2\pi \sigma \cos \theta (R_i + R_o) = h \gamma A$$

$$A = \frac{\pi}{4} D_o^2 - \frac{\pi}{4} D_i^2$$



$$\pi r \cos \theta (D_i + D_o) = \gamma h \frac{\pi}{4} (D_o^2 - D_i^2)$$

$$h = \frac{4 \gamma \cos \theta}{\gamma (D_o - D_i)}$$

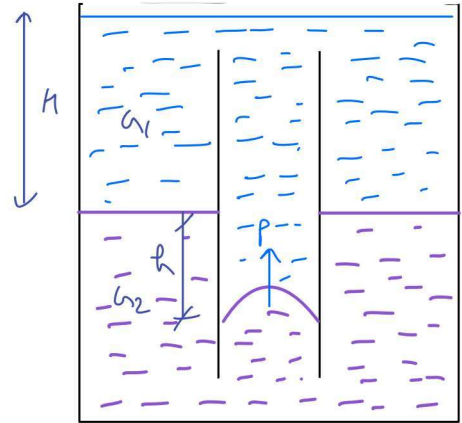
Q) Compute the capillary depression for the following case.

Sol $\gamma_1 = \text{sp. gravity of upper liq}$
 $\gamma_2 = \text{sp. gravity of lower liq}$

$$\sigma (\pi D) \cos \theta - PA + \gamma \omega \gamma_1 (H+h) A = 0$$

$$P = \gamma \omega \gamma_1 H + \gamma \omega \gamma_2 h$$

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$$\therefore \sigma (\pi D) \cos \theta - [\gamma \omega \gamma_1 H + \gamma \omega \gamma_2 h - \gamma \omega \gamma_1 (H+h)] \frac{\pi D^2}{4} = 0.$$

$$\sigma (\pi D \cos \theta) - \frac{\pi D^2}{4} [\gamma \omega \gamma_2 h - \gamma \omega \gamma_1 h] = 0.$$

$$4 \sigma \cos \theta - D \gamma \omega \gamma_2 h + D \gamma \omega \gamma_1 h = 0$$

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$$h = \frac{4 \sigma \cos \theta}{D \gamma \omega (\gamma_2 - \gamma_1)}$$

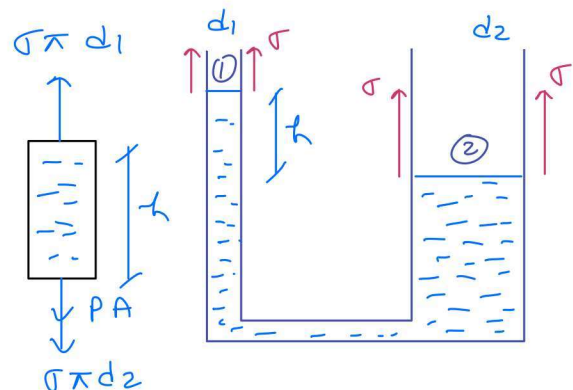
July 10 Q) A U-tube is made up of two capillaries of diameter d_1 and d_2 respectively ($d_2 > d_1$). The U-tube is kept vertically and partially filled with water of ST γ and zero contact angle. Compute the difference in the level of meniscus caused by capillarity.

Sol ST at pt ① = $\sigma \pi d_1$
 ST at pt ② = $\sigma \pi d_2$

$$\sigma \pi d_1 - \sigma \pi d_2 - \frac{\pi d_1^2}{4} h \gamma \omega = 0.$$

$$\sigma (d_1 - d_2) = \frac{d_1^2}{4} h \gamma \omega$$

$$h = \frac{4 \sigma (d_1 - d_2)}{d_1^2 \gamma \omega}$$



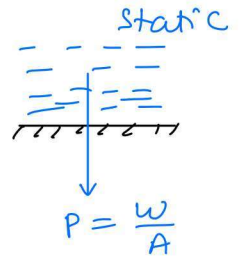
Fluid pressure and its measurement

- Pressure is defined normal force exerted by a fluid per unit area.

Units : N/m^2 , N/mm^2 , Bar
 (Pa) (MPa) (10^5 N/m^2)

$$1 \text{ N/m}^2 = 10^{-6} \text{ N/mm}^2$$

$$1 \text{ atm} = 101.325 \text{ KPa} = 1.01325 \text{ bar}$$



$\text{kg/cm}^2 \rightarrow$ is not a unit of pressure
 kgf/cm^2 is unit of pressure.

$$1 \text{ kgf/cm}^2 = 9.81 \frac{\text{N}}{\text{cm}^2} \text{ or } 9.81 \times 10^4 \frac{\text{N}}{\text{m}^2} = 9.81 \times 10^4 \text{ Pa}$$

Solved 2021

psi = (pound force per square inch)

Note:-

$$\text{psi} = 0.453 \frac{\text{kgf}}{(\text{inch})^2} = 0.453 \times \frac{9.81 \text{ N}}{(2.54 \times 10^{-2})^2 \text{ m}^2}$$

$$1 \text{ psi} = 6888.1 \text{ N/m}^2$$

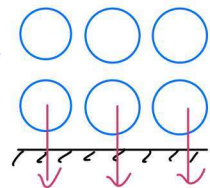
$$\frac{\text{Atm}}{\text{psi}} = \frac{101325}{6888.1} = 14.7$$

$$1 \text{ atm} = 14.7 \text{ psi}$$

ATMOSPHERIC PRESSURE :-

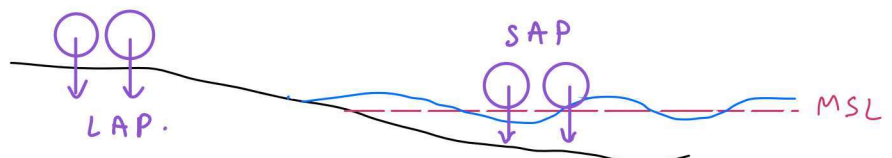
AIR

- It is the pressure exerted by atmosphere.
- Atmospheric pressure at mean sea level is termed as standard atmospheric pressure.
- But atmospheric pressure at any other location is termed as local atmospheric pressure.



- Its value at MSL is

1 atm or 1.013 bar



- It is measured with the help of Barometer.

NOTE :-

press ure head in terms of water = P/ρ_w

$$\rho = \frac{P}{r_{Hg}}$$

hence at MSL

pressure head in terms of water (hw) = $\frac{p}{\gamma_w} = \frac{101325 \text{ N/m}^2}{9810 \text{ N/m}^3}$

$$h\omega = 10.3 \text{ m}$$

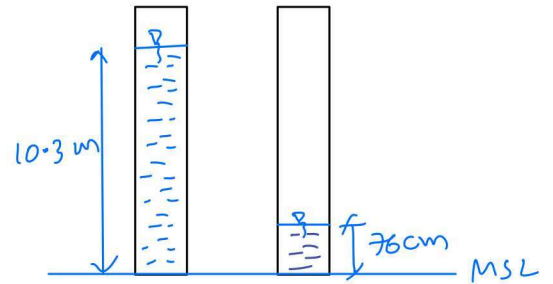
- At MSL press ure head in terms of mercury (h_{Hg}) = $\frac{P}{\rho_{Hg}}$

$$h_{wg} = \frac{101325}{13.6 \times 9810} = 0.76 \text{ m} = 76 \text{ cm}$$

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2021

ABSOLUTE PRESSURE :-

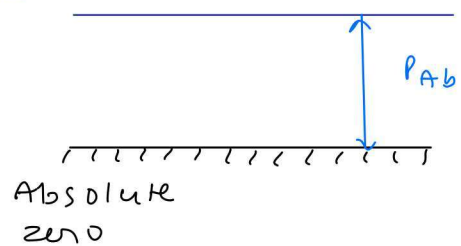
- Pressure measured w.r.t absolute zero or complete vacuum pressure is called as absolute pressure.



- It is also termed as actual pressure at a given location.

- For eg. at MSL absolute pressure is $1 \text{ atm} = 1.013 \text{ Bar}$.

- It is measured with the help of ANEROID BAROMETER.



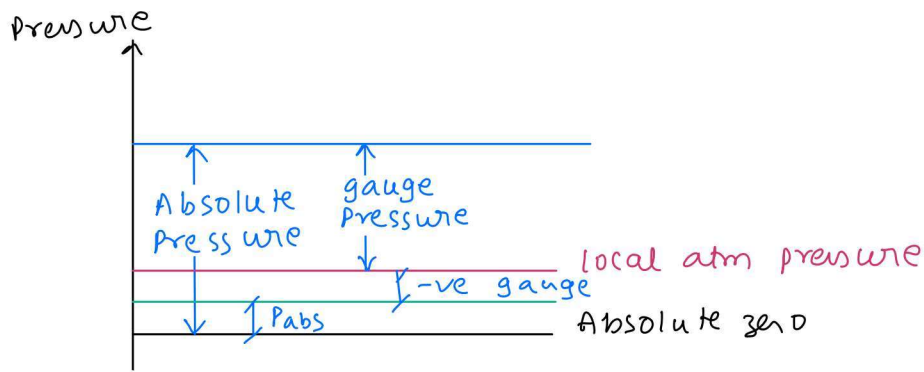
GAUGE PRESSURE

- It is pressure measured w.r.t local atmospheric pressure.

- It may be +ve or -ve.

- -ve gauge pressure is also termed as vacuum pressure.

- Hence gauge pressure is the pressure above ^{local} atmospheric pressure and vacuum pressure may be termed as pressure below atmospheric pressure.



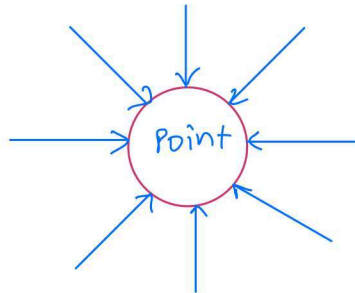
$$\therefore P_{gauge} = P_{abs} - P_{atm}$$

$$P_{vacuum} = P_{atm} - P_{absolute}$$

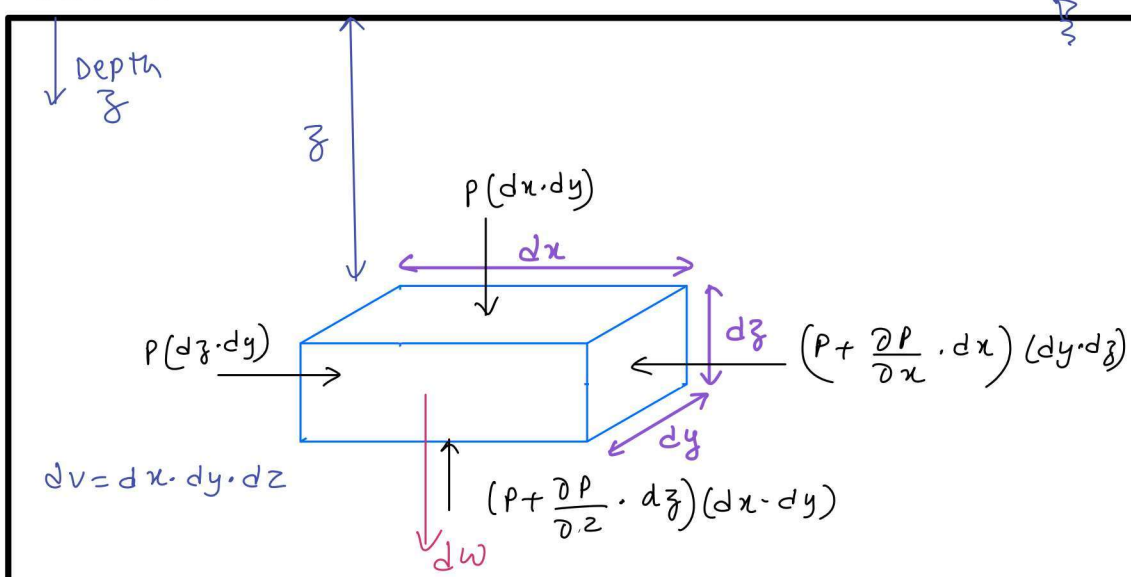
- It is measured with the help of **manometer**, or with **BOURDON GAUGE**

PRESSURE AT A POINT^{Souvik} AND ITS VARIATION WITH DEPTH

- pressure at a point in static fluid is same in all directions.
- It will always act normal to the surface in contact.
- It is a scalar quantity i.e. magnitude but no direction.



Fluid Element



From static equilibrium

$$\Sigma F_z = 0 :-$$

$$P(dx \cdot dy) + dW - (P + \frac{\partial P}{\partial z} \cdot dz)(dx \cdot dy) = 0$$

$$dW - \frac{\partial P}{\partial z} \cdot (dz \cdot dx \cdot dy) = 0$$

$$\frac{\partial P}{\partial z} dz \cdot dx \cdot dy = \gamma \cdot (dx \cdot dy \cdot dz) \quad \uparrow \text{volume}$$

$$\frac{\partial P}{\partial z} = \gamma$$

$\Rightarrow$ Pressure gradient wrt depth is constant provided unit weight (γ) of fluid is constant.

Integrating both side

$$\int \partial P = \int_0^z \gamma dz$$

$$P = \gamma z + C$$

$$@ z=0, P_{\text{gauge}} = 0$$

$$\therefore C = 0$$

$$\therefore P_{\text{gauge}} = \gamma z$$

$$@ z=0, P_{\text{abs}} = P_{\text{atm}}$$

$$\therefore P_{\text{atm}} = C$$

$$P_{\text{abs}} = \gamma z + P_{\text{atm}}$$

$$P_{\text{abs}} - P_{\text{atm}} = P_{\text{gauge}} = \gamma z$$

both are same.

$\Rightarrow$ Hence, Pressure increases with increase in depth from top and decreases in depth from bottom

Also,

$$\Sigma F_x = 0 :-$$

$$P dy dz - (P + \frac{\partial P}{\partial x} dx) dy dz = 0$$

$$\frac{\partial P}{\partial x} (dx dy dz) = 0$$

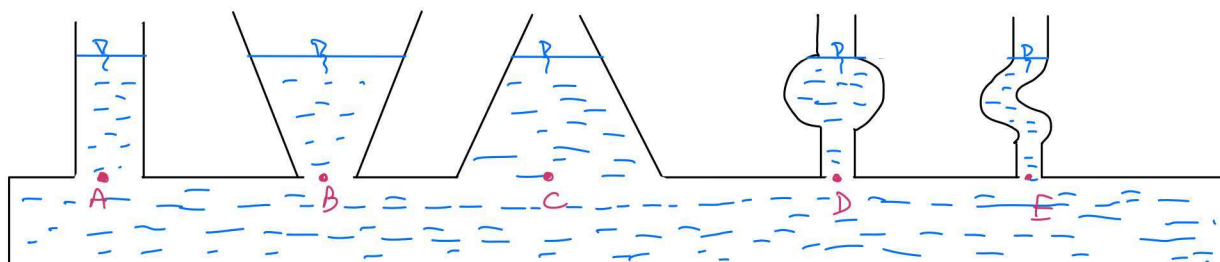
$$\text{Since } dx dy dz = dv \neq 0,$$

$$\therefore \frac{\partial P}{\partial x} = 0.$$

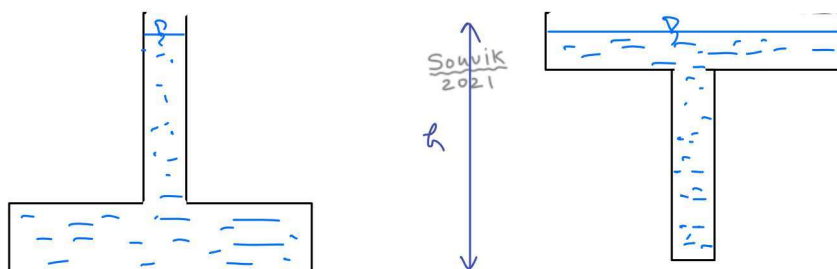
i.e Pressure doesnot vary in horizontal plane.

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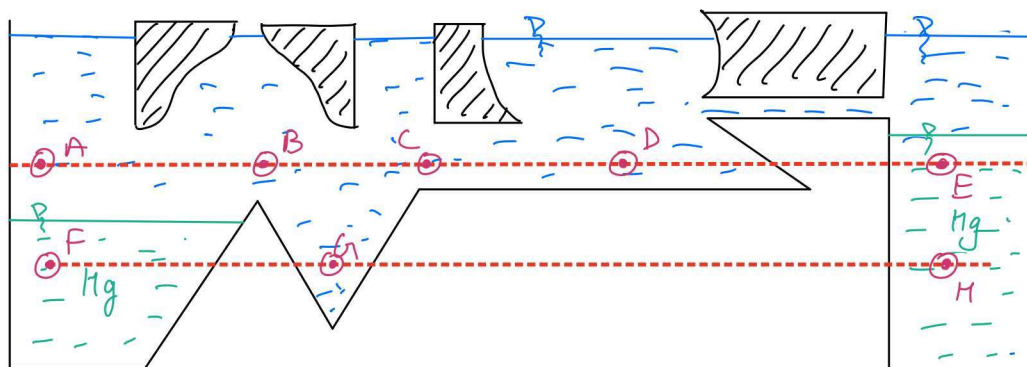
- Pressure at a point in a fluid at rest is independent of the shape and cross-section of the container in which it is kept and it changes with vertical distance only and remains constant in another direction.



$$P_A = P_B = P_C = P_D = P_E \text{ (doesn't depend on shape).}$$



Also it doesnot depend upon cross-section



{ EK point se dusre point mai agar ja sakte ho, same fluid }
 { K through then pressure will be same else, it will vary }

→ same fluid, same level

$$P_A = P_B = P_C = P_D$$

$$\neq P_E \rightarrow \text{different fluid}$$

$$P_F \neq P_G$$

→ Diff. fluid

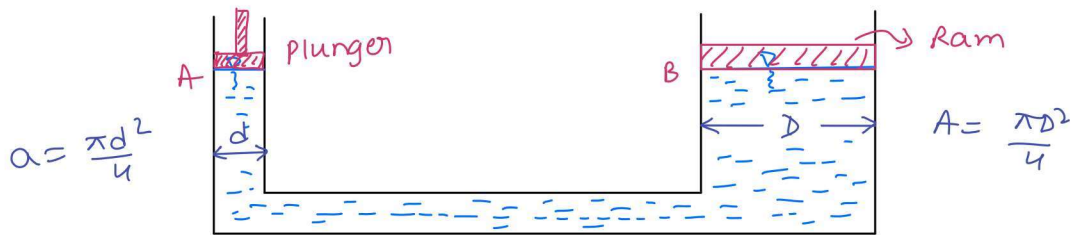
$P_F \neq P_H$ (Diff. fluid cross)
 karna padega.

- If the points are located at same horizontal level and we can move one point to another point while moving through same liquid, pressure at all these points will be same.

NOTE:- For small to moderate change in elevation in gases the variation in pressure is negligible as their densities are comparatively very low.

PASCAL'S LAW :-

If pressure is applied to a confined fluid it increases the pressure throughout by the same amount.



- If force F is applied over the plunger increase in pressure

$$\Delta P = \frac{f}{a} \rightarrow \text{force applied}$$

Now,

$$P_A = P_B$$

$$\Delta P_A = \Delta P_B$$

$$\Delta P_B = \frac{F}{A}$$

Force developed at B, $F_B = \frac{F}{a} \times A$ $\frac{A}{a} > 1$

$$\therefore F_{\text{applied}} < F_{\text{developed}}$$

Ex: 1000 kg
0.1 mg
 $\frac{1000}{0.1 \times 10^{-6}} = 10^{10} = A/a$

work done concept :-

$$WD \text{ by applied force} = F_a \times x \quad \text{--- (A)}$$

vol. displaced in plunger = vol. displaced in Ram.

$$ax = Ay \quad \text{--- (B)}$$

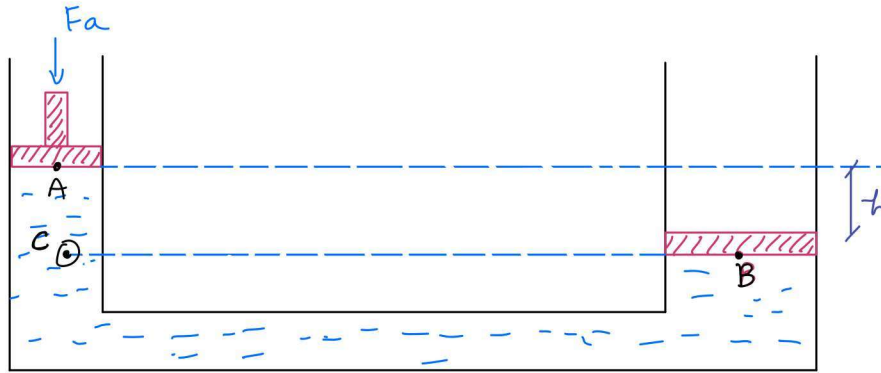
$$WD \text{ by developed force} = F_d \cdot y$$

$$= \frac{F_a}{a} A y = \frac{F_a}{a} A \frac{ax}{A} = F_a x$$

$$WD \text{ by developed force} = WD \text{ by applied force.}$$

Hence Energy principle is also valid in this case.

- If the plunger and Ram are located at different levels then

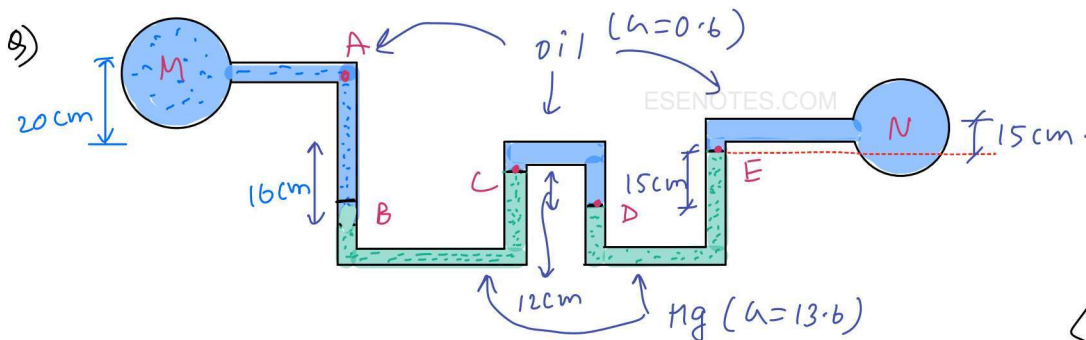


$$\Delta P_A = \frac{F_a}{a} \quad P_C = \gamma h + \Delta P_A = \gamma h + \Delta P_C = \gamma h + \frac{F_a}{a}$$

$$P_C = P_B = \gamma h + \frac{F_a}{a}$$

$$F_d = P_B \cdot A$$

hence, $F_d = \left[\gamma h + \frac{F_a}{a} \right] A$



compute $P_M - P_N$.

so start from M and proceed to N

$$P_M + \left[0.6 \times 1000 (16 + 20) \times 10^{-2} - 12 \times 10^{-2} \times 13.6 \times 10^3 + 0.6 \times 10^3 \times 0.12 - 0.15 \times 13.6 \times 10^3 - 0.6 \times 10^3 \times 0.15 \right] g = P_N$$

$$P_M - P_N = 34.077 \text{ kPa.}$$

Why multiply by g?

$$\begin{aligned} \Rightarrow \frac{\text{kg}}{\text{m}^3} \times \text{m} &= \text{kg/m}^2 \\ \frac{\text{kg}}{\text{m}^2} \times \frac{\text{m}}{\text{s}^2} &= \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{1}{\text{m}^2} \\ &= \frac{\text{KN}}{\text{m}^2} = \text{kPa} \end{aligned}$$

PRESSURE MEASUREMENT DEVICE :-

Measurement of the pressure can be done by any of the following devices.

- (a) BAROMETER.
- (b) ANEROID BAROMETER.
- (c) MANOMETERS.
- (d) MECHANICAL GAUGES.

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(a) BAROMETER :-

- It is used to measure atmospheric pressure.

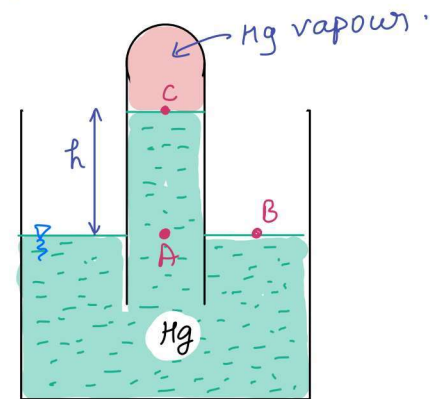
$$P_A = P_B = P_{\text{atm}}$$

$$P_A = \rho_{\text{Hg}} \times h + P_{\text{Hg vapour.}}$$

Negligible

$$P_A = \rho_{\text{Hg}} h$$

$$P_{\text{atm}} = \rho_{\text{Hg}} h$$



Torricelli Apparatus.

- In this mercury filled tube is inverted into the mercury container which is open to the atmosphere.
- Here Hg is used because of its low vapour pressure and high density.
- If point D, a small hole is made in the tube the pressure at point C would go down.

$$P_{\text{atm}} = \rho_{\text{Hg}} h + P_{\text{atm}}$$

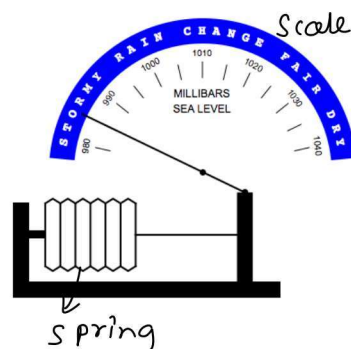
$$\rho_{\text{Hg}} h = 0$$

$$h = 0$$

(b) Aneroid Barometer :-

- It is used to measure Absolute pressure.

- As torricellian barometer consists of mercury which is poisonous, it is not safe to use it in day to day purpose.



Aneroid Barometer.

- In this barometer as the pressure rise or fall the box either compress or expand and this motion of the box is calibrated with pressure.

(C) MANOMETER :-

- These are based on principle of balancing a column of fluid by the same or other column of fluid.
- These are further classified as

(i) Simple Manometer.

(ii) Differential Manometer.

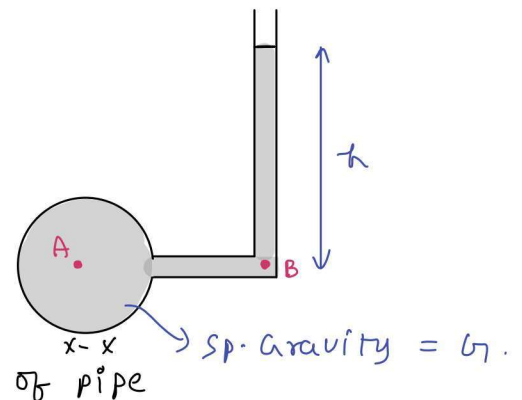
(1) SIMPLE MANOMETER :-

- These are used to measure the pressure at a given point. These are further classified as

(1) PIEZOMETER :-

$$P_A = P_B = \rho g h$$

- This piezometer cannot be used for measuring negative pressure.

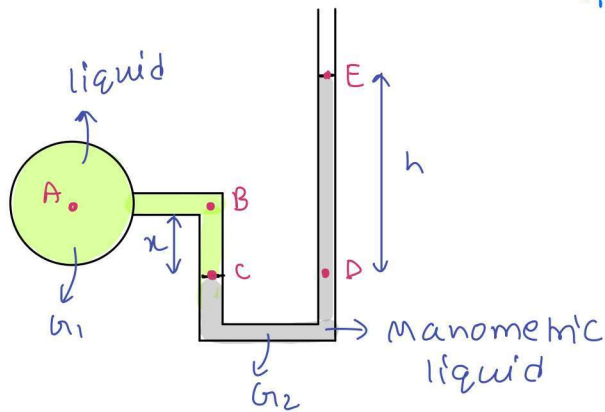


- This device cannot be used for large pressure measurement as long tube cannot be used.
- It cannot be used to measure pressure of gas as no free surface will be formed.

(2) U-tube Manometer :-

- For large pressure measurement, for measuring gas pressure at a point U-tube manometers are used.
- Normally U-tube manometers contains the liq of sp. gravity more than that for liquid whose pressure is to be measured.
- The liquid used should be immiscible with other liquid, and should have low thermal sensitivity and vapour pressure.

case (i) measurement of large pressure



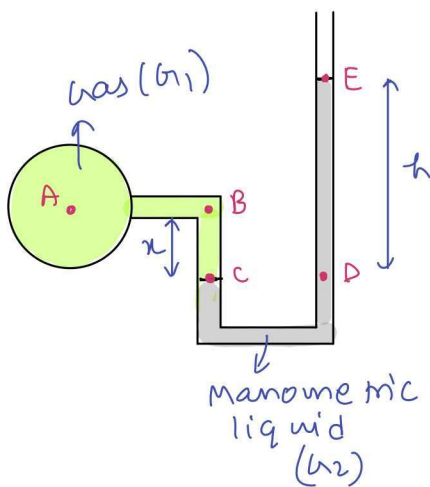
$$P_A + \rho_1 \rho_w x - \rho_2 \rho_w h = P_E$$

Now, $P_E = P_{\text{gauge}} = 0$.

$$P_A = \rho_2 \rho_w h - \rho_1 \rho_w x$$

$$P_A = \rho_w (\rho_2 h - \rho_1 x)$$

case (ii) :- measurement of gas pressure



$$P_A + \rho_1 \rho_w x - \rho_w \rho_2 h = 0$$

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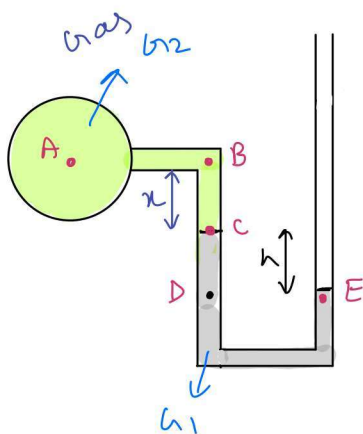
$$P_A = \rho_w (\rho_2 h - \rho_1 x)$$

$\rho_1 = \text{negligible}$.

$$P_A = \rho_2 h \rho_w$$

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case (iii) measurement of -ve pressure



$$P_A + \rho_2 \rho_w x + \rho_1 \rho_w h = 0$$

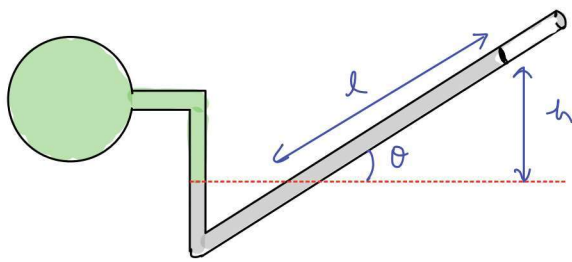
$$P_A = -\rho_w (\rho_2 x + \rho_1 h)$$

$\rho_2 \rightarrow \text{negligible (gas)}$

$$P_A = -\rho_1 \rho_w h$$

NOTE:- If small pressure is to be measured and manometric liquid is heavier, then differential height in U-tube manometer will be small and difficult to read, hence in such case

- Lighter manometric liquid can be used.
- One leg of manometer is made inclined such that the readings to be measured become larger.

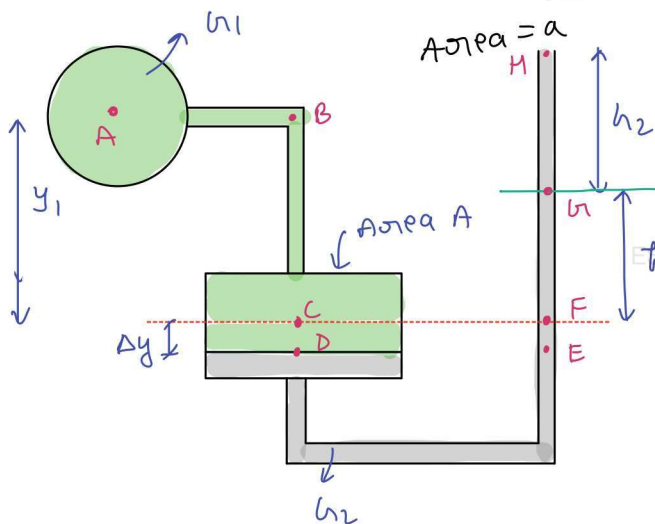


$$l = \frac{h}{\sin \theta}$$

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(3) SINGLE COLUMN Manometer :-

- In this case only one reading is required to be taken as against two readings on two limbs in case of u-tube manometer.



Initial position of manometric liquid when not connected to pipe A

From initial condition.

$$P_C = P_F$$

$$\rho_1 r_w y_1 = \rho_2 r_w h_1 \quad - (i)$$

From mass conservation

$$\Delta y \cdot A = h_2 \cdot a \quad - (ii) \quad \Delta y = \frac{h_2 a}{A}$$

from M:- $0 + \rho_2 r_w (h_2 + h_1 + \Delta y) - \rho_1 r_w (\Delta y + y_1) = P_A$

$$0 + \rho_2 r_w h_2 + \cancel{\rho_2 r_w h_1} + \rho_2 r_w \Delta y - \rho_1 r_w \Delta y - \cancel{\rho_1 r_w y_1} = P_A$$

From eqn (i) and putting value of Δy

$$\rho_2 r_w h_2 + \rho_2 r_w \frac{h_2 a}{A} - \rho_1 r_w \frac{h_2 a}{A} = P_A$$

$$P_A = \left[\rho_2 \left(1 + \frac{a}{A} \right) - \rho_1 \frac{a}{A} \right] h_2 r_w$$

$$P_A = h_2 r_w \left[\rho_2 + \frac{a}{A} (\rho_2 - \rho_1) \right]$$

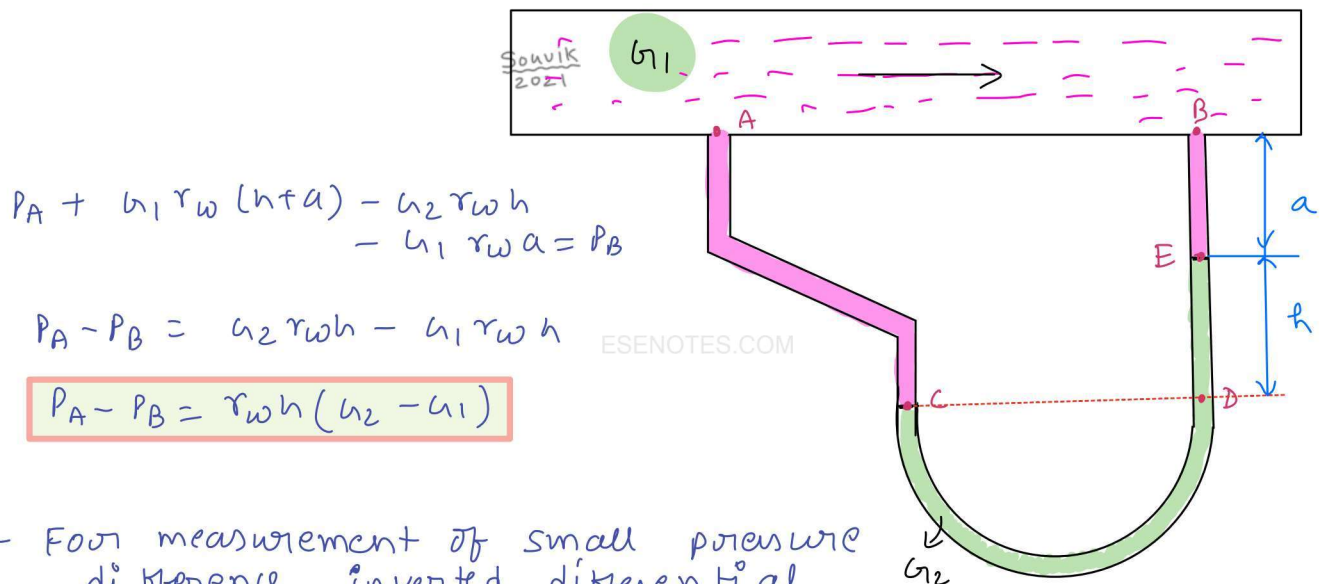
Since $\frac{a}{A}$ is very small $\approx \frac{1}{100}$ so $\frac{a}{A} (h_2 - h_1) \ll h_2$
thus can be neglected.

$$P_A = h_2 \rho_w h_2$$

Here only one reading h_2 is required for measurement of the pressure at a point.

(d) DIFFERENTIAL MANOMETER :-

- It is used to measure pressure difference b/w two points in a pipe or b/w two different pipes.



$$P_A + \rho_1 \rho_w (h+a) - \rho_2 \rho_w h - \rho_1 \rho_w a = P_B$$

$$P_A - P_B = \rho_2 \rho_w h - \rho_1 \rho_w h$$

$$P_A - P_B = \rho_w h (\rho_2 - \rho_1)$$

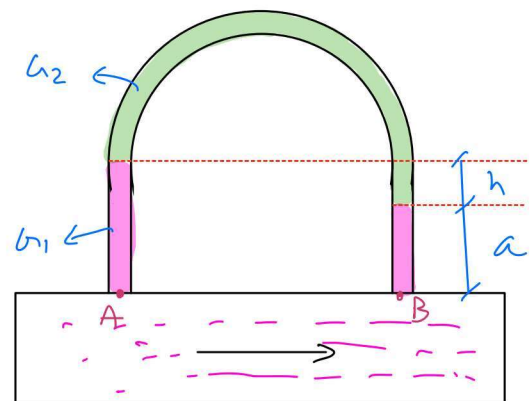
- For measurement of small pressure difference inverted differential manometer can be used.

$$P_A - \rho_1 \rho_w (h+a) + \rho_2 \rho_w h + \rho_1 \rho_w a = P_B$$

$$P_A - P_B = \rho_1 \rho_w h - \rho_2 \rho_w h$$

$$P_A - P_B = (\rho_1 - \rho_2) \rho_w h$$

obviously $\rho_2 > \rho_1$

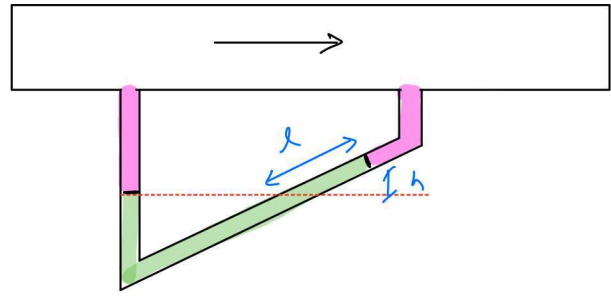


Note:- If $(\rho_1 - \rho_2)$ is small, large value of h is obtained which increases its sensitivity.

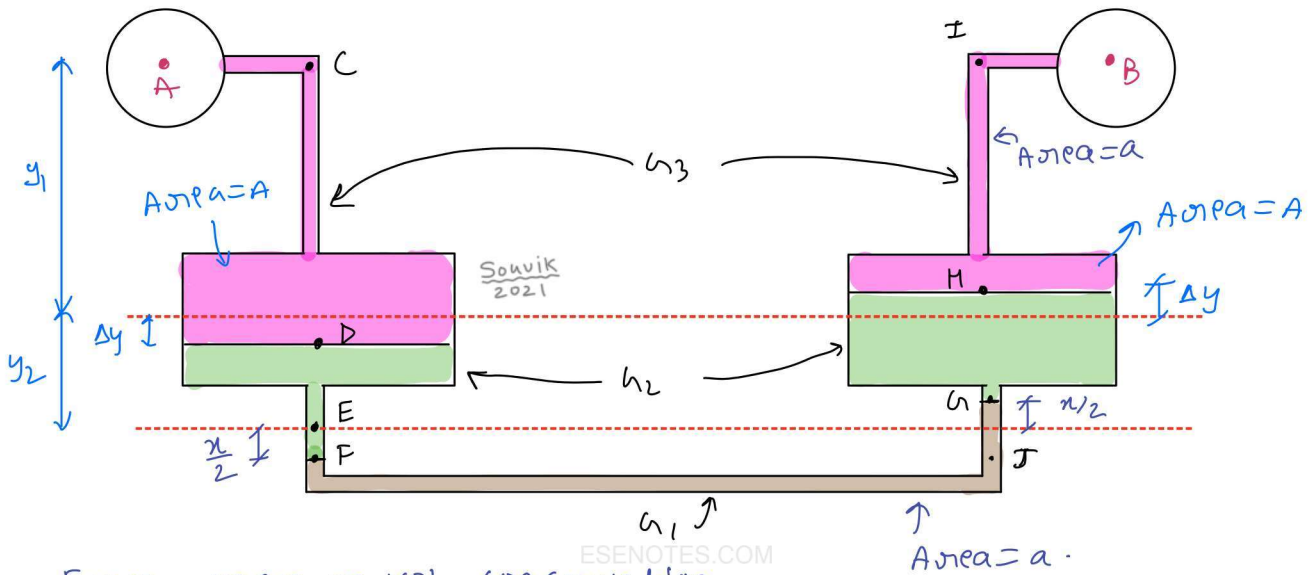
Sensitivity can also be increased by inclining one limb of the manometer. (Thus reading is increased by $\gamma \sin \theta$).

$$l > h$$

$$l = \frac{h}{\sin \theta}$$



note:- Sometimes more precise reading are to be measured, hence **micromanometers** are used.



From mass or vol. conservation

left limb :- $A \Delta y = a \cdot \frac{x}{2}$

Now, $P_A + h_3 \rho g (y_1 + \Delta y) + h_2 \rho g (y_2 - \Delta y + \frac{x}{2}) - h_1 \rho g x$
 $- h_2 \rho g (y_2 - \frac{x}{2} + \Delta y) - h_3 \rho g (y_1 - \Delta y) = P_B$

$$P_A - P_B = -h_3 \rho g \Delta y + h_2 \rho g \Delta y - h_2 \rho g \frac{x}{2} - h_2 \rho g \frac{x}{2} + h_2 \rho g \Delta y - h_3 \rho g \Delta y$$

$$P_A - P_B = -2 h_3 \rho g \Delta y + 2 h_2 \rho g \Delta y - h_2 \rho g x$$

$$P_A - P_B = \rho g \left[(h_1 - h_2) + (h_2 - h_3) \frac{a}{A} \right]$$

neglected

$$P_A - P_B = (h_1 - h_2) \rho g x$$

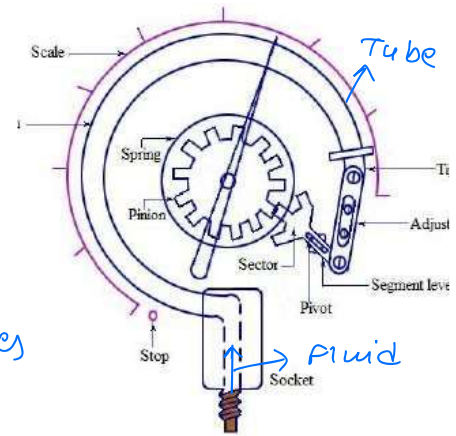
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(c) MECHANICAL GAUGES :-

- Four pressure measurement following types of mechanical gauges are available.

(i) BOURDON GAUGE :-

- when the tube is open to the atmosphere, the tube is undeflected and needle on dial is calibrated to read zero.
- when fluid in the tube is pressurized, the tube stretches and moves the needle in proportion to the applied pressure.



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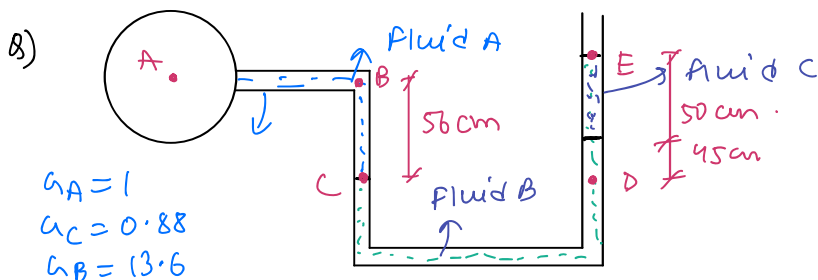
(ii) Strain gauge transducer :-

- A strain gauge is calibrated to give the measure of pressure.

(iii) Piezometric Transducer :-

- principle of piezoelectricity is used.
- piezometric material are those which produces current when pressure is applied
- The current is further calibrated by the pressure

eg :- Quartz, Roschelle salt, Topaz.



Find pressure at pt A in terms of head of water.

$$\text{SOL} \quad P_A + \gamma_A \times 0.56 - \gamma_B \times 0.45 - \gamma_C \times 0.50 = 0$$

$$\frac{P_A}{\gamma_w} = 13.6 \times 45 + 0.88 \times 50 - 1 \times 56$$

$$h_w = \frac{P_A}{\gamma_w} = 600 \text{ cm} = 6 \text{ m.}$$

- 8) A cylindrical container whose wt is 65 N is inverted and pressed into water as shown. Determine the differential height (h) of the manometer and the force (F) needed to hold the container in position.

Sol

$$P_A + 2.1 \rho_w h = P_D = P_C$$

$$P_C = 2.1 \rho_w h$$

$$\text{Now, } P_H = P_E = P_C \text{ (Air)}$$

$$\rho_w \times 20 = 2.1 \rho_w h$$

$$h = 9.52 \text{ m}$$

$$\sum F_y = 0$$

$$F + 65 \text{ N} = \rho_w \times 20 \times A$$

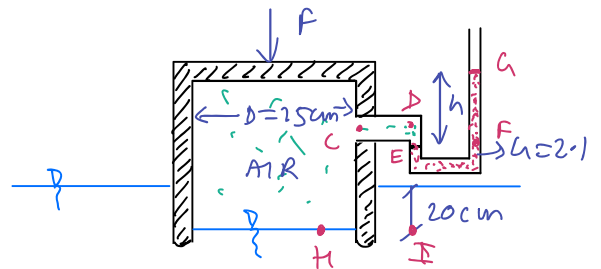
$$F + 65 = 1000 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 0.20 \text{ m} \times \frac{\pi}{4} \times 0.25^2$$

$$F = 96.30 - 65$$

$$F = \underline{\underline{31.30 \text{ N}}}$$

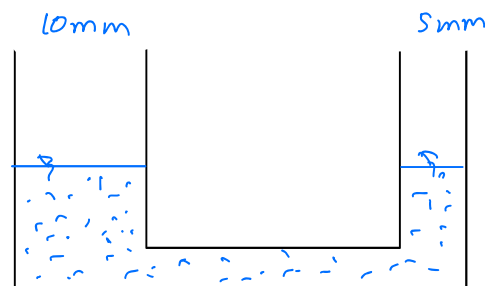
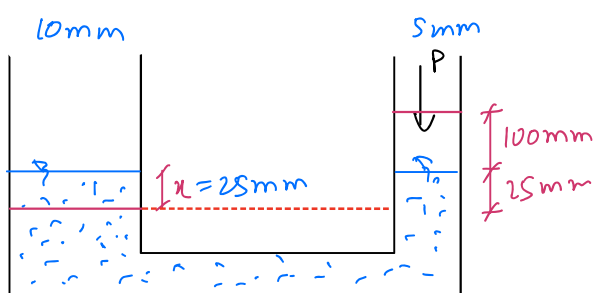
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$$\rho_w = 9810 \text{ N/m}^3$$



- 8) when a pressure, P is applied at 5mm dia meter limb, water rises in it by 100 mm from initial level. The other limb is open to atmosphere (P_a) find $P - P_a$

Sol



By vol. conservation,

$$A x = a \times 100$$

$$10^2 x = 5^2 \times 100$$

$$x = 25 \text{ mm}$$

$$P + \rho_w 125 = P_a$$

$$P - P_a = -125 \rho_w$$

$$P - P_a = -9810 \times 0.125$$

$$= -1226.25 \text{ N/m}^2$$

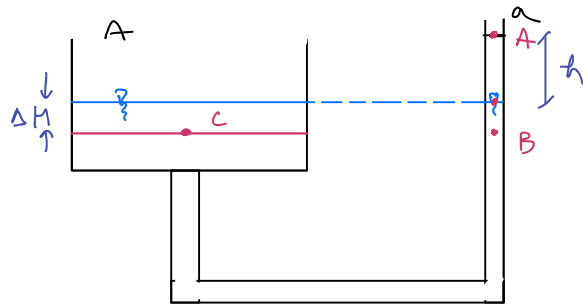
$$= -1.22 \text{ (kN/m}^2\text{)}$$

Q) If $\frac{A}{a} = 500$, Pressure diff b/w two limbs can be measured by taking measurement on one limb only, then find the error induced in measurement.

Sol) $\frac{A}{a} = 500$

$$\Delta H \times A = h \times a$$

$$\frac{h}{\Delta H} = 500$$



$$p_A + \rho_w (h + \Delta H) = p_c$$

$$p_c - p_a = \rho_w (h + \Delta H)$$

Now, if pressure at one end is recorded then

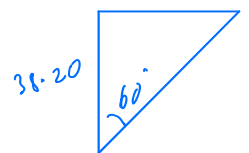
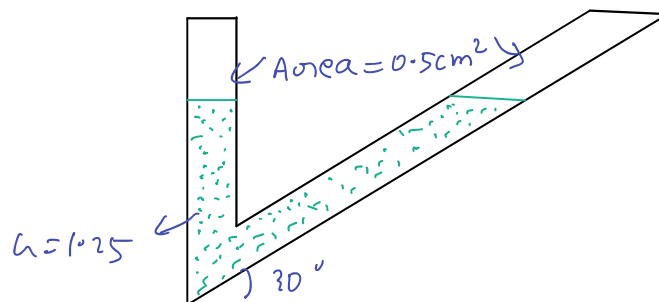
$$p_c - p_a = \rho_w h$$

$$\% \text{ Error} = \frac{\rho_w (h + \Delta H) - \rho_w h}{\rho_w h} \times 100$$

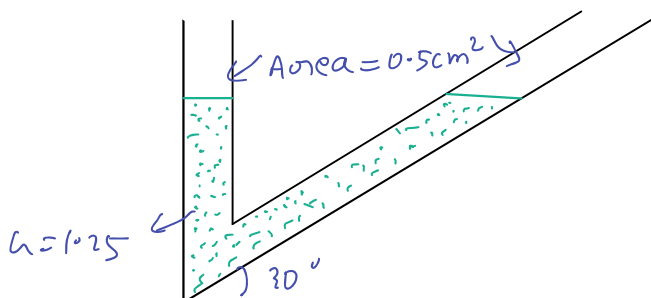
$$= \frac{\Delta H}{h} \times 100 = \frac{100}{500} = 0.2 \%$$

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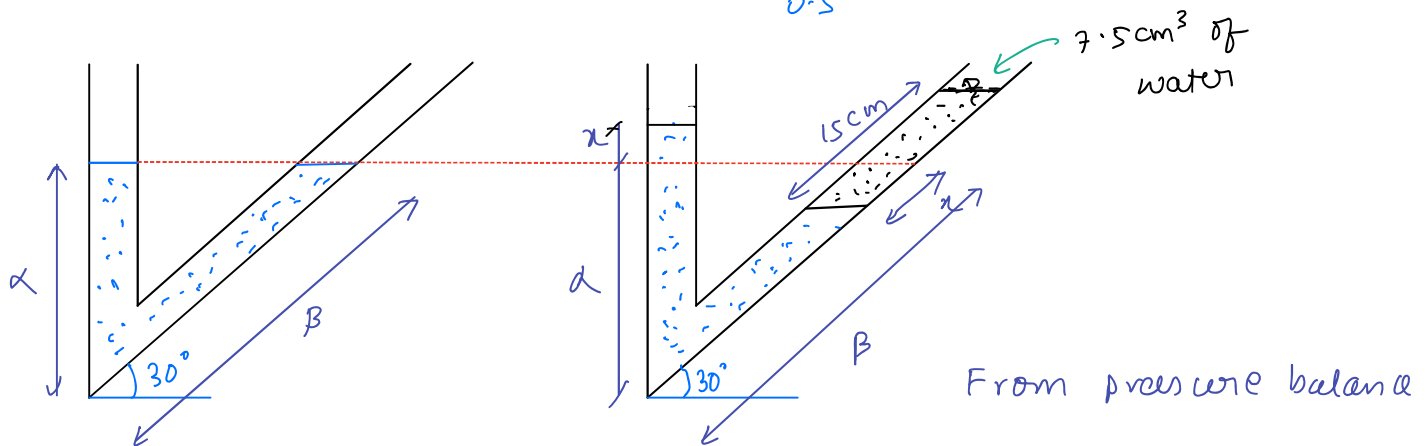
Q) Find the rise in the meniscus in vertical tube if additional 7.5 cm^3 of water is added to the inclined limb.



Sol)



Height of water @ $7.5 \text{ cm}^3 = \frac{7.5}{0.5} = 15 \text{ cm}$.



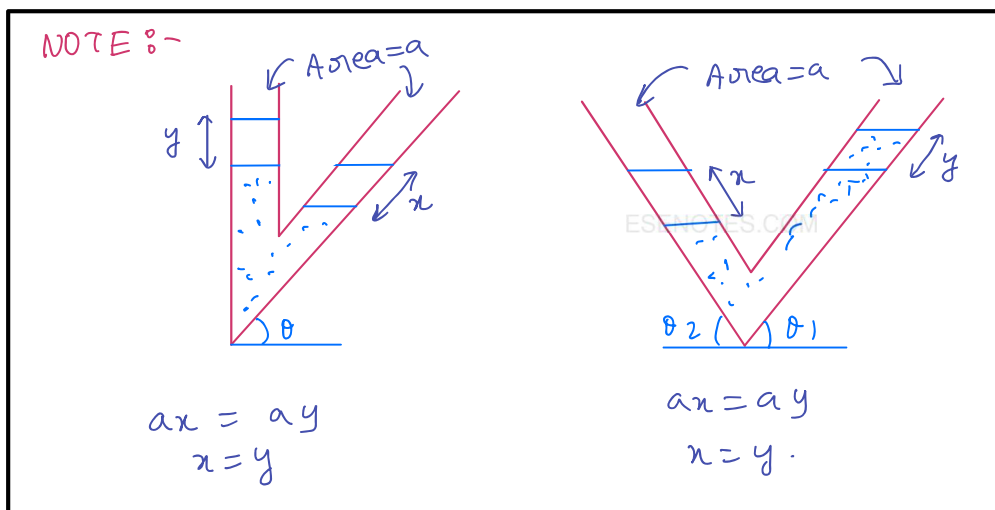
From initial pressure balance: $\rho_1 (\alpha + \alpha) \rho_w = \rho_2 \rho_w (15 \sin 30^\circ + \rho_2 \rho_w (\beta - \alpha) \sin 30^\circ$

$\rho_1 \rho_w \alpha = \rho_2 \rho_w \beta \sin 30^\circ$

$\beta = 2\alpha$

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$\alpha = 4 \text{ cm}$



OR (HOLDEN CONCEPT)

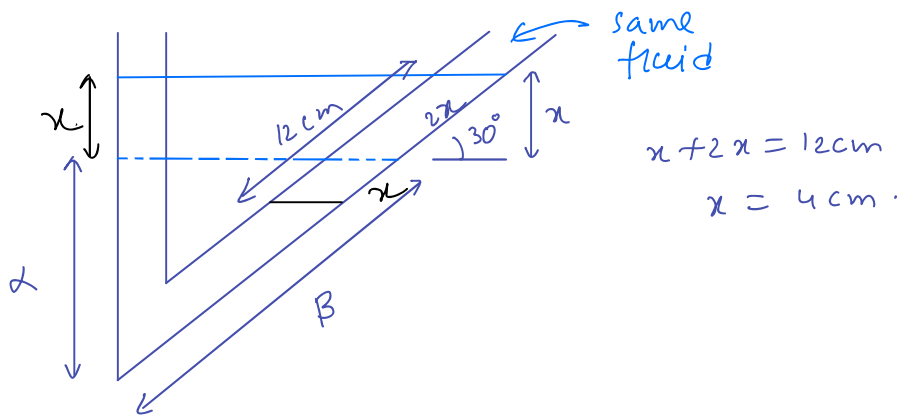
converting water into the same fluid (ρ_1).

now pressure exerted by water and the other fluid at any pt. must be same.

$\rho_1 \rho_w h = \rho_2 \rho_w y$

$1.25 (h) = 1 \cdot y$

Hence when $y = 15 \text{ cm}$, $h = 12 \text{ cm}$.

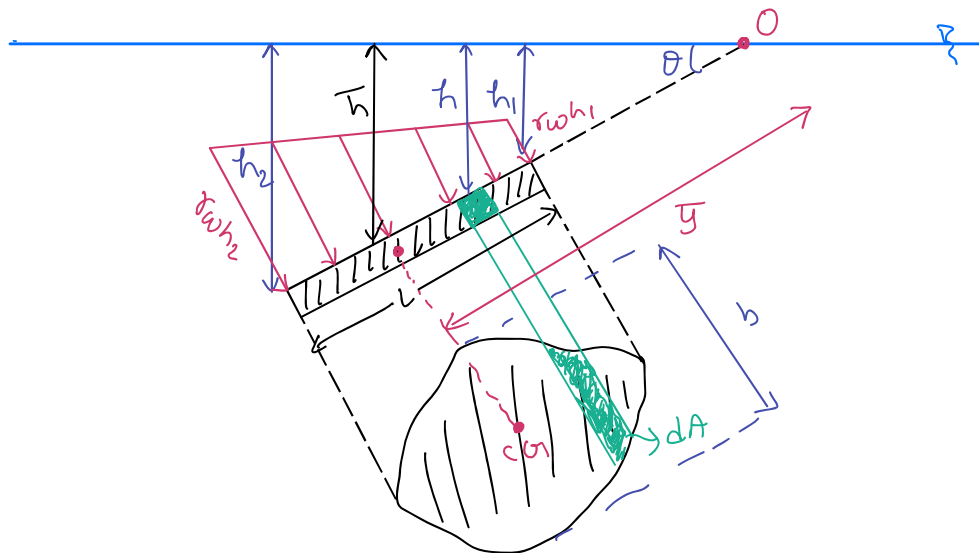


HYDROSTATIC FORCES ON PLANE & CURVED SURFACES

- pressure is the normal force applied by the surface per unit area.
- This pressure leads to the development of the force over the surface that can be computed as follows

(a) PLAIN SURFACE :-

- let us consider a plain surface inclined at an angle θ with the horizontal.



Pressure on element- at any point on the plate is

$$P = \rho h$$

$$\therefore P = \rho y \sin \theta \quad \text{Now, } h = y \sin \theta$$

$$\text{hydrostatic force, } dF = (\rho y \sin \theta) dA$$

$$\text{Total force, } F = \int \rho y \sin \theta dA$$

$$F = \rho \sin \theta \int y dA \quad [\rho = \text{const (assume)}]$$

Now, $\int y dA = A \bar{y}$

$\therefore F = \rho \sin \theta A \bar{y}$

$\bar{y}$ = distance of C.G. of plate from point O

$F = \rho \bar{h} A$

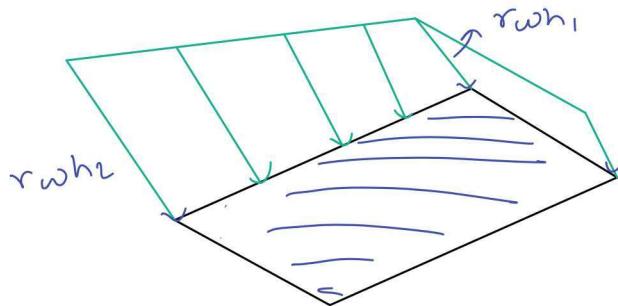
$F = \bar{P} A$ where, $\bar{P} = \rho \bar{h}$

Thus, the magnitude of the resultant hydrostatic force acting on a plane surface of a completely submerged plate in a const. density fluid (homogeneous fluid) is equal to product of "PRESSURE at centroid of the SURFACE ($\bar{P}$) and the AREA OF the SURFACE"

OR PRESSURE PRISM CONCEPT :-

- If the width "B" of the plate is constant then the magnitude of pressure force on it is equal to volume of pressure prism.

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Hydrostatic force = vol. of pressure prism.

= Ar. of pressure distr. diagram $\times$ width inside plate

$F = \left[\frac{1}{2} (rwh_2 + rwh_1) \times L \right] \times b$

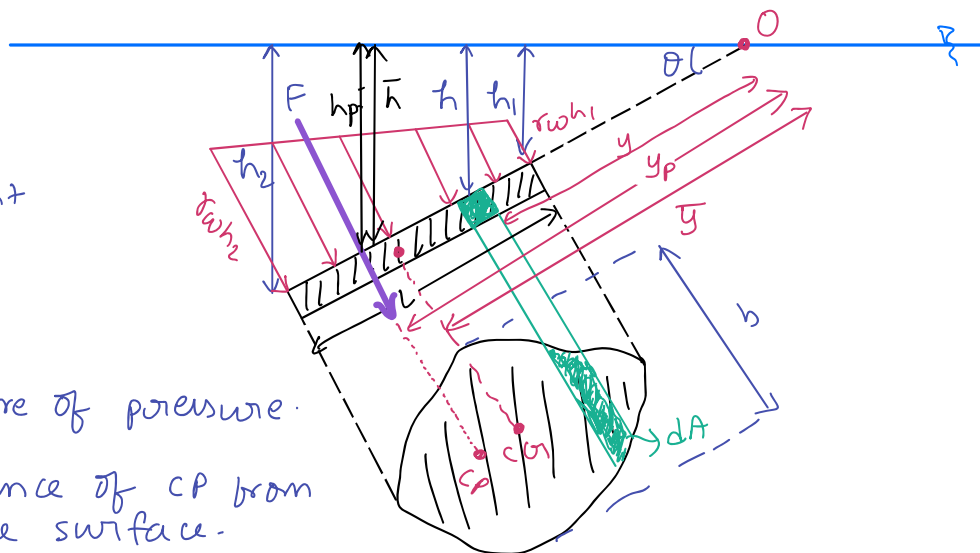
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F = resultant force.

$$F = \bar{P} \cdot A$$

CP = centre of pressure.

h_p = distance of CP from free surface.



To find the location of the point of application of resultant hydrostatic force i.e. centre of pressure. Take the moment of all the small forces or infinitesimal forces about "O".

$$F \times y_p = \int P \cdot dA \cdot y.$$

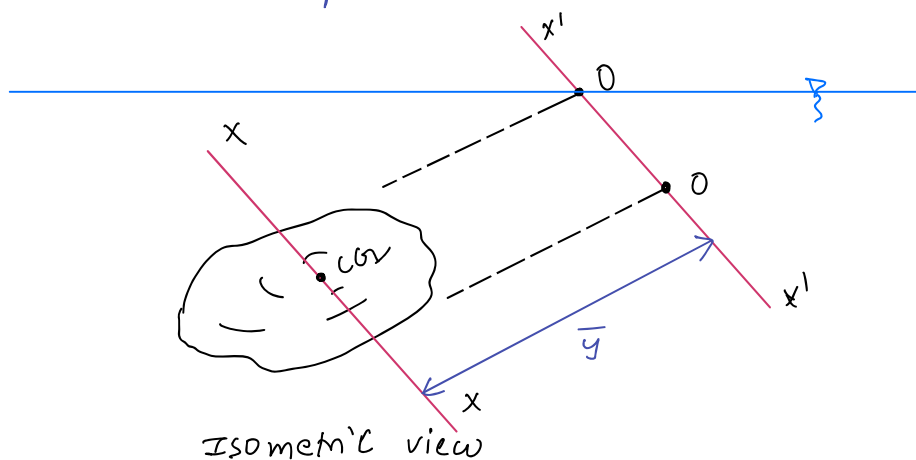
$$\text{Now, } P = \rho y \sin \theta$$

$$F \cdot y_p = \int \rho y \sin \theta \cdot y \cdot dA$$

$$F \cdot y_p = \rho \sin \theta \int y^2 dA$$

Now, $\int y^2 dA$ = MOI of the surface about the plane passing from "O" perpendicular to the surface.

$$F \cdot y_p = \rho \sin \theta I_0$$



$$I_{xx'} = I_{xx} + A \bar{y}^2$$

$$\therefore F y_p = \rho \sin \theta [I_{xx} + A \bar{y}^2]$$

$$y_p = \frac{\rho \sin \theta [I_{xx} + A \bar{y}^2]}{F_p}$$

$$y_p = \frac{r \sin \theta [I_{xx} + A y^2]}{r \sin \theta y \cdot A}$$

$$y_p = \frac{I_{xx}}{A y} + y \Rightarrow \text{along the plane of the surface}$$

Now, $y_p = \frac{h_p}{\sin \theta}$ $y = \frac{h}{\sin \theta}$

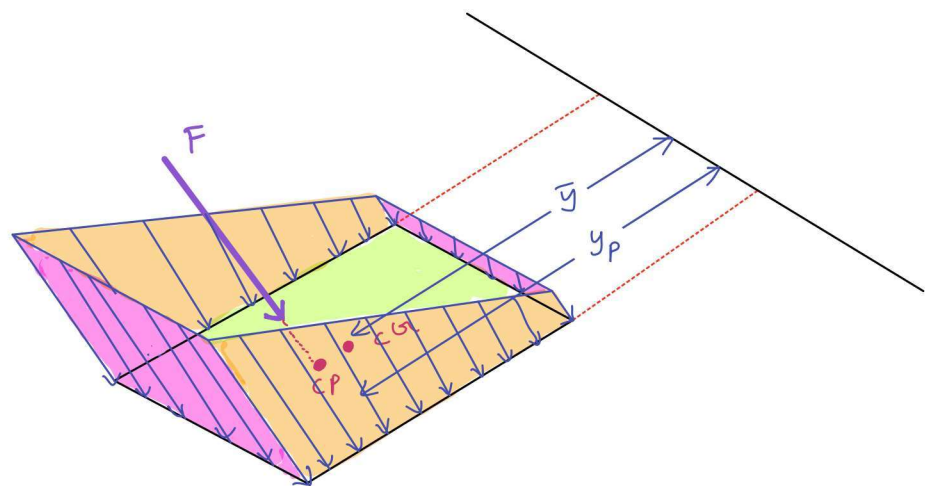
$$\frac{h_p}{\sin \theta} = \frac{I_{xx}}{A h} \sin \theta + \frac{h}{\sin \theta}$$

$$h_p = \frac{I_{xx}}{A h} \sin^2 \theta + h \Rightarrow \text{along the vertical plane.}$$

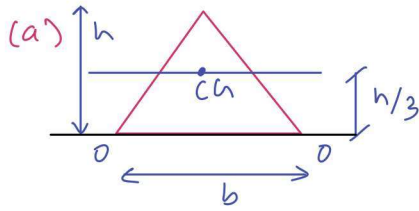
Thus, CP will ^{Souvik 2022} always be below CG .

NOTE:- ① If we go deeper difference between CG and CP will go on reducing as h or y will go on increasing, hence the additional term with h and y will decrease.

② The pressure force passes through CG of the pressure prism and its projection of CG of the pressure prism on the surface of the plate is called centre of pressure.



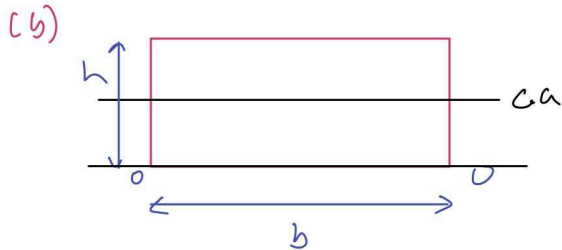
NOTE:- CG and MOI of some significant geometries.



$$A = \frac{bh}{2}$$

$$I = \frac{bh^3}{36}$$

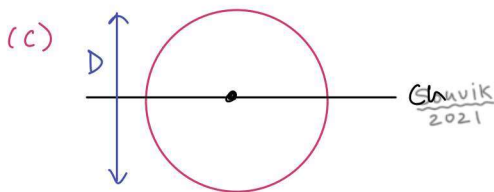
$$I_0 = \frac{bh^3}{12}$$



$$A = bh$$

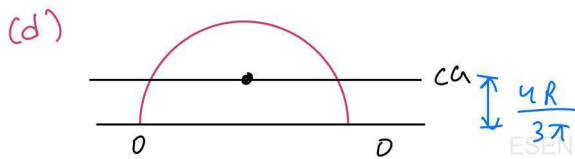
$$I = \frac{bh^3}{12}$$

$$I_0 = \frac{bh^3}{3}$$



$$A = \frac{\pi D^2}{4}$$

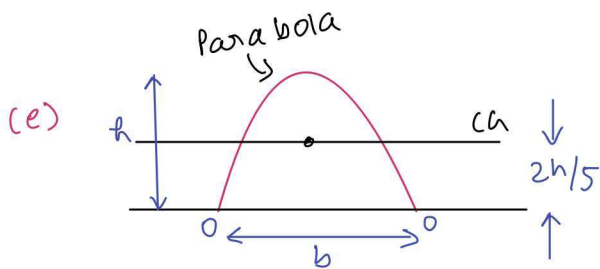
$$I = \frac{\pi D^4}{64}$$



$$A = \frac{\pi}{8} D^2$$

$$I = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) \frac{D^4}{16}$$

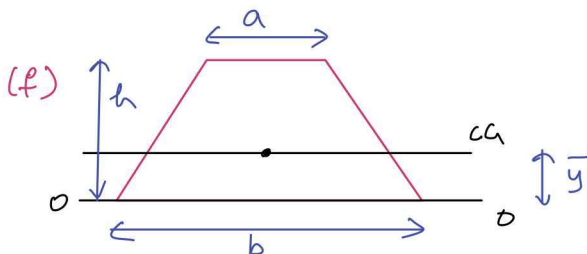
$$I_0 = \frac{\pi D^4}{128}$$



$$A = \frac{2}{3} bh$$

$$I = \frac{8}{175} bh^3$$

$$I_0 = \frac{16}{105} bh^3$$



$$A = \frac{a+b}{2} \times h$$

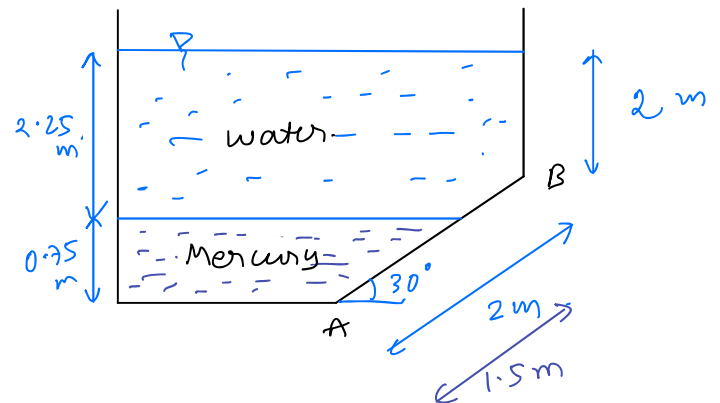
$$\bar{y} = \left(\frac{2a+b}{a+b} \right) \frac{h}{3}$$

$$I = \left[\frac{a^2 + b^2 + 4ab}{36(a+b)} \right]^3 \cdot h$$

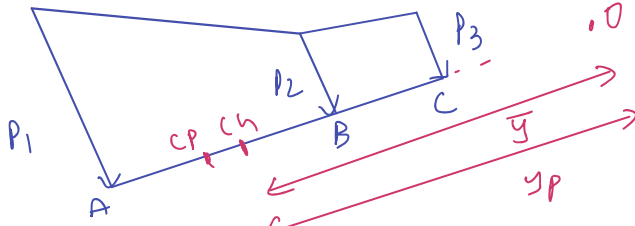
- Q) Find the magnitude of force acting on AB and its point of application. water in tank is upto the depth of 2.25m and below that there is Mercury, width of plate is 3m.

$$\rho_{\text{water}} = 1$$

$$\rho_{\text{Hg}} = 13.6$$



Sol :-



$$P_1 = 2.25 \times \rho_w + 13.6 \times \rho_w \times 0.75$$

$$= 122.13 \text{ kN/m}^2$$

$$P_2 = 2.25 \times \rho_w = 22.07 \text{ kN/m}^2$$

$$P_3 = 2 \times 9.81 = 19.62 \text{ kN/m}^2$$

$$\text{Total force} = F_{AB} + F_{BC}$$

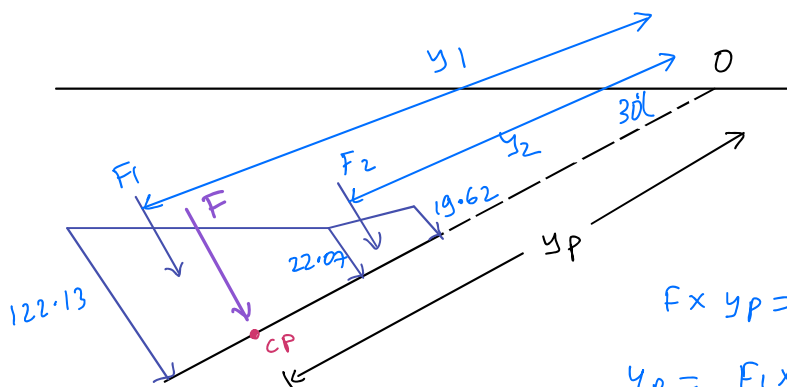
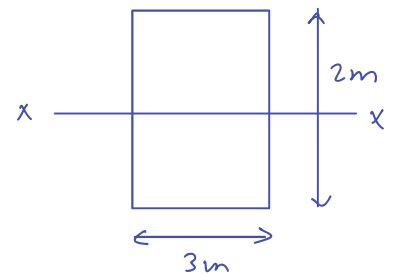
$$= \frac{1}{2} (P_1 + P_2) \times 1.5 \times 3 + \frac{1}{2} \times (P_2 + P_3) \times 0.5 \times 3$$

$$= 324.45 + 31.26$$

$$= 355.71 \text{ (kN)}$$

$$y_p = \frac{I_{xx}}{A \bar{y}} + \bar{y}$$

$$I_{xx} = \frac{3 \times 2^3}{12} = 2 \text{ m}^4$$



$$F \times y_p = F_1 \times y_1 + F_2 \times y_2$$

$$y_p = \frac{F_1 \times y_1 + F_2 \times y_2}{F}$$

$$F_1 = 324.45 \text{ kN} \quad F_2 = 31.26 \text{ kN}$$

$$y_1 = \frac{2}{\sin 30} + 0.5 + \left[1.5 - \frac{2 \times 22.07 + 122.13}{22.07 + 122.13} \right] \times \frac{1.5}{3}$$

$$= 4.67 \text{ m}$$

$$y_2 = \frac{2}{\sin 30} + \left(\frac{2 \times 22.07 + 19.62}{22.07 + 19.62} \right) \times \frac{0.5}{3} = 4.25 \text{ m}$$

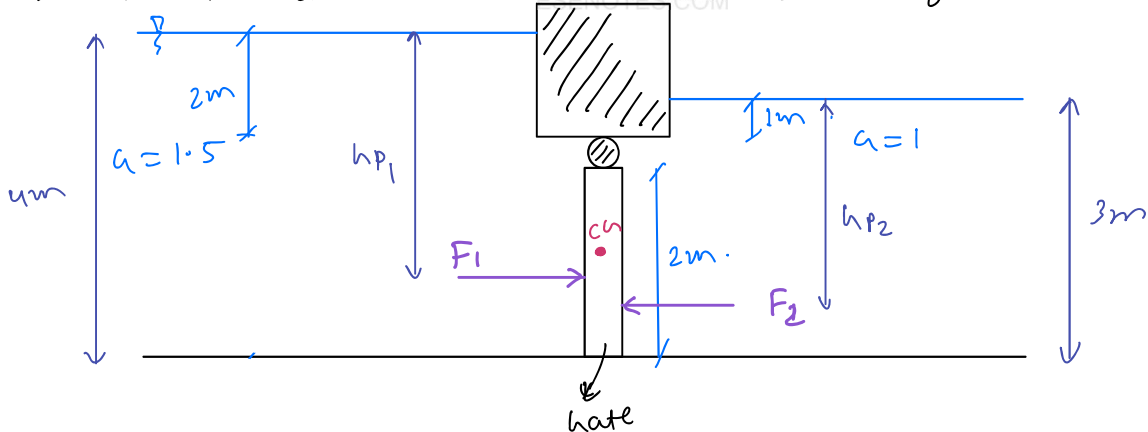
$$y_p = \frac{324.45 \times 4.67 + 31.26 \times 4.25}{324.45 + 31.26}$$

$$y_p = 4.63 \text{ m from point "O"}$$

July 17

- Q) A sluice gate 4m wide and 2m deep is hinged at the top. Liquid of sp gravity 1.5 stands to a height of 2m above the top of the gate on upstream side. On the downstream side water stands to a height of 1m above the top of the gate. compute

- (a) Resultant force acting on the gate.
(b) Point at which resultant force act.



Sol

$$F_1 = \text{Pressure at c.g.} \times \text{Area}$$

$$= \bar{P}_1 \times A$$

$$\bar{P}_1 = \gamma \bar{h}$$

$$F_1 = \gamma_1 \bar{h}_1 A$$

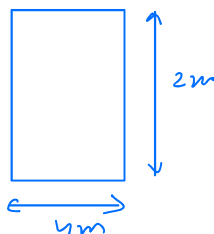
$$= \gamma_1 \gamma_w \bar{h}_1 A$$

$$= 1.5 \gamma_w (2+1) \times (2 \times 4)$$

$$= 36 \gamma_w = 36 \times 10 = 360 \text{ kN}$$

$$h_{p1} = \bar{h}_1 + \frac{I_{xx} \sin^2 \theta}{A \bar{h}_1} \quad I_{xx} = \frac{4 \times 2^3}{12}$$

$$= 3 + \frac{4 \times 2^3}{12 \times 4 \times 2 \times 3} = 3.11 \text{ m}$$



$$F_2 = \bar{P}_2 A = \gamma_2 \bar{h}_2 A = 1 \times \gamma_w (1+1) \times 8$$

$$= 16 \gamma_w = 160 \text{ kN}$$

($\gamma_w = 10 \text{ kN/m}^3$)
Assume.

$$h_{p2} = \bar{h}_2 + \frac{I_{xx} \sin^2 \theta}{\bar{h}_2 A}$$

$$= 2 + \frac{4 \times 2^3}{12 \times 2 \times 2 \times 4} = 2.16 \text{ m}$$

Thus, F_2 is 0.16 m below CG and A is 0.11 m below CG.

hence the force direction and position is correct

Resultant, $F_R = F_1 - F_2 = 360 - 160$
 $= 200 \text{ (kN)}$

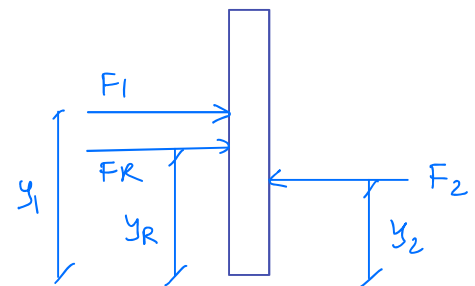
point of application of F_R about base

$$F_R \times h = F_1 (4 - h_{p1}) - F_2 (3 - h_{p2})$$

$$h = \frac{360 \times 0.89 - 160 \times 0.84}{200}$$

$$h = 0.93 \text{ m}$$

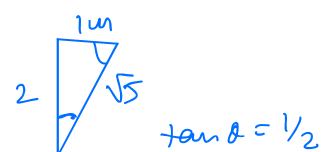
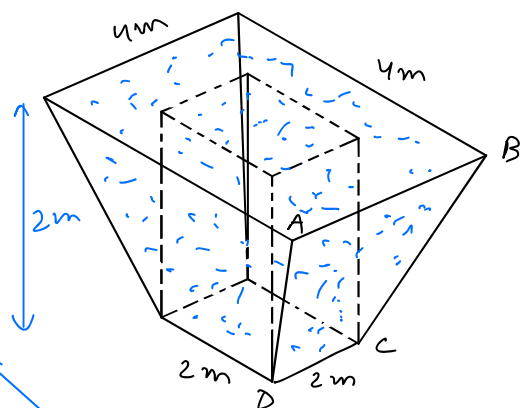
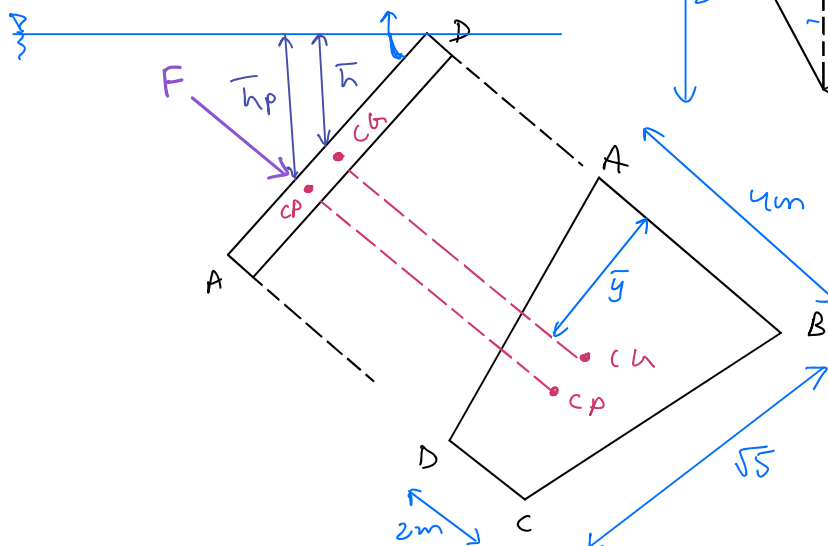
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Now, $y_1 = 0.89 \text{ m}$
 $h_1 = 0.93 \text{ m}$
 $y_2 = 0.84 \text{ m}$ } hence F_R is above F_1 .

8) Find the hydrostatic force on any face.

sol let us find force on face ABCD.



$$\bar{y} = \left(\frac{2 \times 2 + 4}{2 + 4} \right) \times \frac{\sqrt{5}}{3} = \frac{4\sqrt{5}}{9}$$

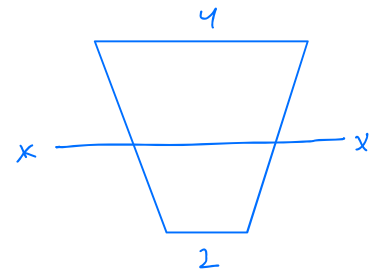
$$\frac{2}{\sqrt{5}} = \frac{\bar{h}}{9} \frac{g}{4\sqrt{5}}$$

$$\bar{h} = 8/g$$

$$F = \rho \bar{h} \cdot A$$

$$= 9.81 \times \frac{8}{9} \times 6.70$$

$$F = 58.42 \text{ (kN)}$$



$$A = \frac{1}{2} \times (4+2) \times \sqrt{5}$$

$$= 6.70 \text{ m}$$

- 8) Find the depth of water "h" for the flat board as given when the board is about to tip. Also compute the compressive force on the board when it is about to tip. Assume the width of Board as 1m.

sol concept :- The board will be about to tip when Hydrostatic pressure will totally pass thru hinge

i.e around $R_{xx} = 0$.

$h' = h - d \sin \theta$ = Centre of pressure.

$$\text{Also, } h' = \bar{h} + \frac{I_{xx} \sin^2 \theta}{A \bar{h}}$$

$$A = \frac{h}{\sin \theta} \times 1$$

$$\bar{h} = \frac{h}{2 \sin \theta} \quad \sin \theta = \frac{h}{2}$$

$$I_{xx} = \frac{1 \times \left(\frac{h}{\sin \theta} \right)^3}{12}$$

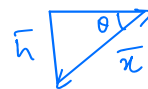
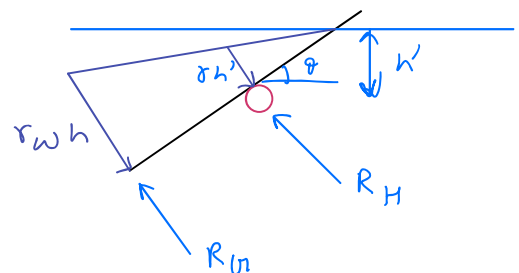
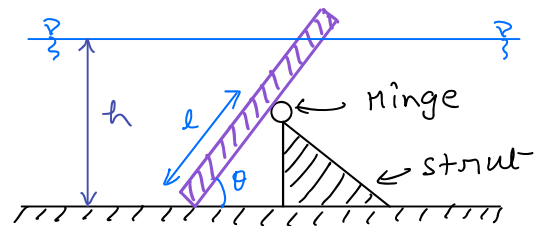
$$h' = \frac{h}{2} + \frac{\frac{h^3}{12 \sin^3 \theta} \sin^2 \theta}{\frac{h}{\sin \theta} \times \frac{h}{2}} = \frac{h}{2} + \frac{h^3}{12 \sin^3 \theta} \times \frac{2 \sin \theta}{h^2} \sin^2 \theta$$

$$= \frac{h}{2} + \frac{h}{6} = \frac{2h}{3}$$

Now,

$$h - L \sin \theta = \frac{2h}{3}$$

$$h = 3L \sin \theta$$



comp. force, $R = F = P \cdot A$

$$= \gamma_w \bar{h} \cdot A$$

$$= \gamma_w \times \frac{h}{2} \times \frac{h}{\sin \theta}$$

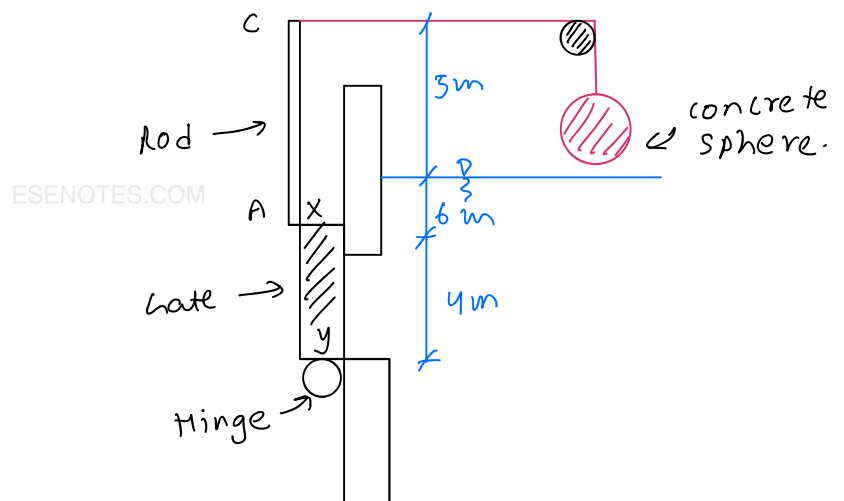
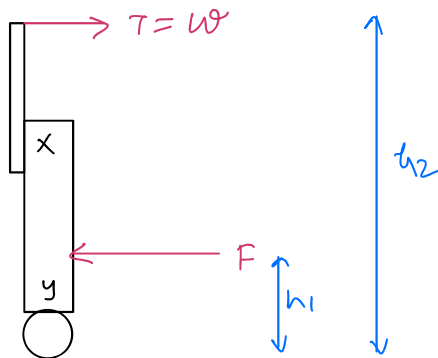
$$= \frac{\gamma_w h^2}{2 \sin \theta} \quad \text{Ans, } \gamma_w = 10 \text{ kN/m}^3$$

$$= \frac{\gamma_w (3.2 \sin \theta)^2}{2 \sin \theta} = 4.5 h^2 \sin \theta$$

July 20

- Q) Gate xy shown is 3m wide. It is hinged at its bottom and is held in position against water pressure with the help of rod AC, Rope and concrete sphere. Find the diameter of the sphere if the specific gravity of concrete is 2.4

Sol) FBD of the gate.



$$T = W = \gamma \times V = \gamma \times \frac{\pi D^3}{2}$$

$$\begin{aligned} F &= P A \\ &= \gamma_w \bar{h} A \\ &= \gamma_w \times 8 \times 12 \\ F &= 96 \gamma_w \text{ (kN)} \end{aligned}$$

$$h_2 = 15 \text{ m}$$

$$\therefore h_1 = 10 - 8.166 \text{ m} = 1.834 \text{ m}$$

$$\Sigma M_y = 0 \therefore F \times h_1 = T \times h_2$$

$$96 \gamma_w \times 1.834 = \gamma \frac{\pi D^3}{2} \times 15 \quad ; \quad D = 1.95 \frac{\gamma_w}{\gamma} = \frac{1.95}{2.4}$$

$$D = 814 \text{ mm (2.1 m)}$$

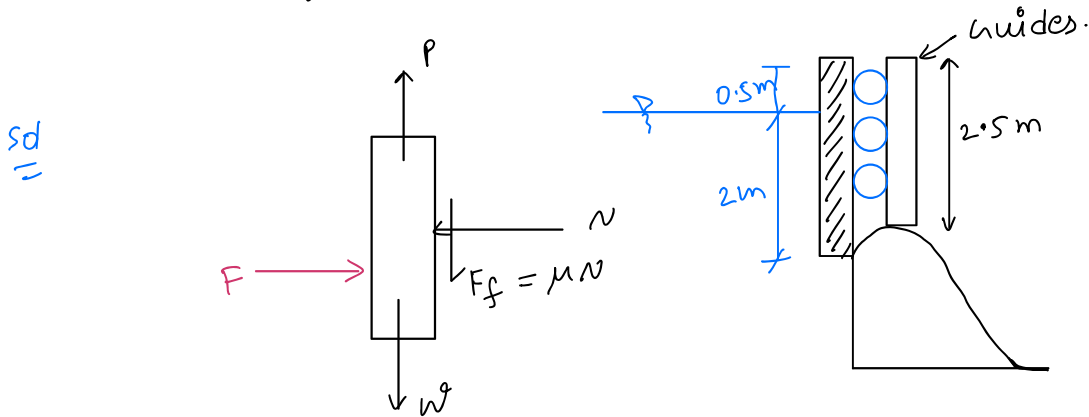
$$\begin{aligned} \bar{h} &= 6 + 2 = 8 \text{ m} \\ A &= 3 \times 4 = 12 \text{ m}^2 \end{aligned}$$

$$h_p = \bar{h} + \frac{I_{xx}}{A \bar{h}} \sin^2 \theta$$

$$\begin{aligned} h_p &= 8 + \frac{3 \times 4^3}{12 \times 3 \times 4 \times 8} \\ &= 8 + \frac{1}{6} \\ &= 8.166 \text{ m} \end{aligned}$$

$$\begin{aligned} &4 \pi \left(\frac{D}{2} \right)^3 \\ &4 \pi \frac{D^3}{8} \\ &\frac{\pi D^3}{2} \end{aligned}$$

- 8) A vertical gate $5\text{m} \times 2.5\text{m}$ in size weighing 0.5 tonnes slides along guides (coeff of friction is 0.25), fitted on the side wall of an overflow spillway and its crest. what force will have to be exerted at the hoisting mechanism to lift the gate when the head of water over the crest is 2m .



$$\Sigma F_y = 0 :- \quad P = W + F_f \quad \Sigma F_x = 0 :- \quad N = F \quad \text{--- (i)}$$

$$P = W + \mu N \quad \text{--- (ii)}$$

Now,

$$F = P \cdot A = \rho \omega h \cdot A$$

$$A = 2 \times 5 = 10$$

$$h = \frac{2}{2} = 1\text{m}$$

$$F = 10 \times 1 \times 10 = 100 \text{ (kN)}$$

$$P = 0.5 \times 10^3 \times \frac{10 \times 10^{-3}}{g} + 0.25 \times 100$$

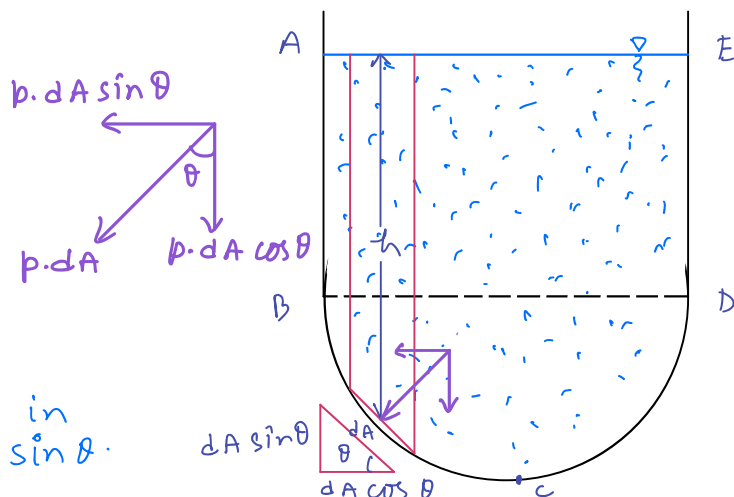
$$P = 30 \text{ (kN)}$$

HYDROSTATIC FORCE ON CURVED SURFACE :-

- Let ABCDE be the container with BCD as curved surface.

- Pressure force on area $dA = p \cdot dA = \rho \omega h dA$

- Projection of area dA in vertical plane $= dA \sin \theta$.



- Projection of area dA in horizontal plane $= dA \cos \theta$.

- Horizontal force on curved surface $= (p dA) \sin \theta$

$$= p (dA \sin \theta) \\ = p \times \text{vertical projection of curved surface}$$

Horizontal force on CS. = Force on vertical projection of curved surface

vertical force on curved surface $= (p dA) \cos \theta = r_w h dA \cos \theta$

= weight of liquid contained in the zone bound by two verticals drawn from two ends of the CS, the CS and the free surface which is applying the pressure on curved surface.

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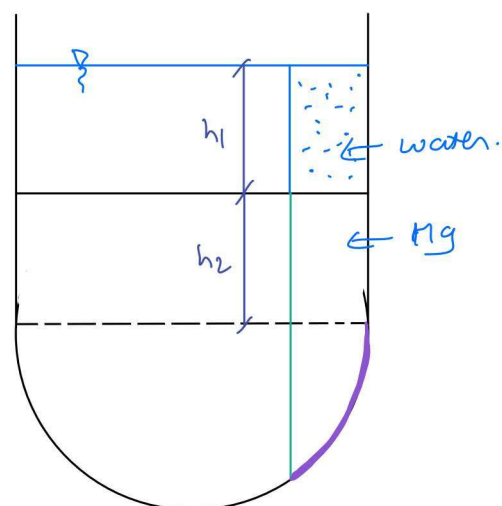
- This vertical force acts through the CG of the volume of the liquid contained above the curved surface.

- Resultant force on CS $F_R = \sqrt{F_h^2 + F_v^2}$

- If there are more than one liquid present, then we have to take the weight of all such liquids to obtain the vertical components of the force on the CS surface.

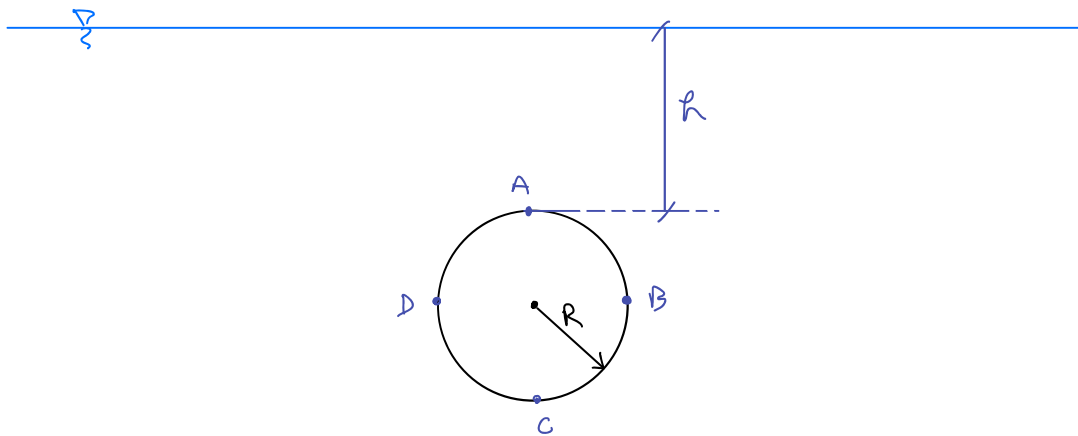
$$p = [h_1 r_w + h_2 r_w \mu_2]$$

$$F_v = p \cdot dA \cos \theta \\ = [h_1 r_w + h_2 r_w \mu_2] dA \cos \theta \\ = (h_1 dA \cos \theta) r_w + (h_2 dA \cos \theta) r_w \mu_2$$



July 21

Eg:- compute the resultant hydrostatic force on the given cylinder of unit length.



Sol considering curved surface ADC and ABC

$$F_{HADC} = \frac{1}{2} \times (\rho_w h + \rho_w (h+2R)) \times 2R \times 1$$

$$F_{HADC} = F_{HABC}$$

$$F_{HADC} = \frac{1}{2} \times 2 \rho_w (h+R) \times 2R$$

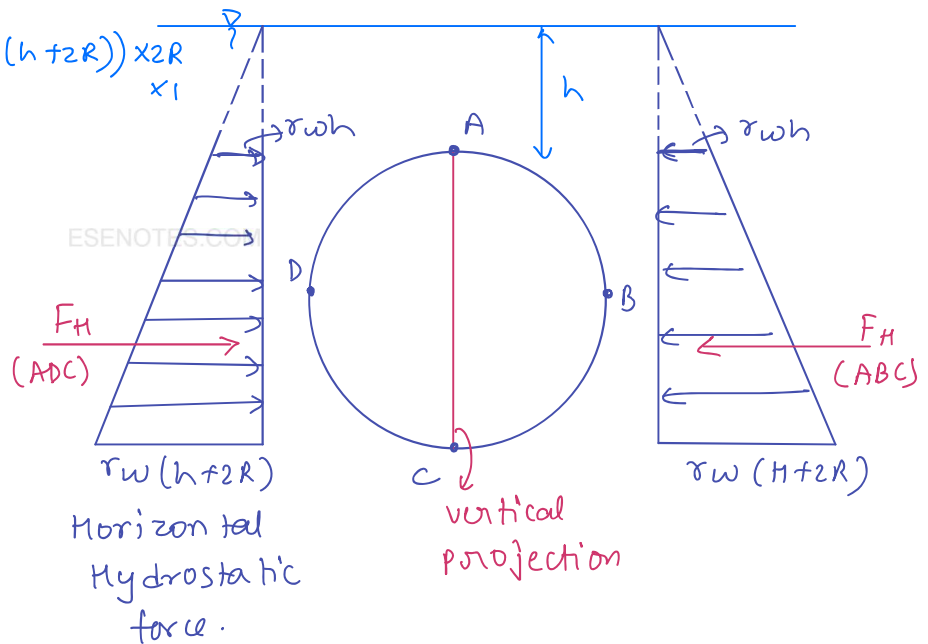
$$= 2 \rho_w R (h+R)$$

OR

$$F = \bar{P} \cdot A$$

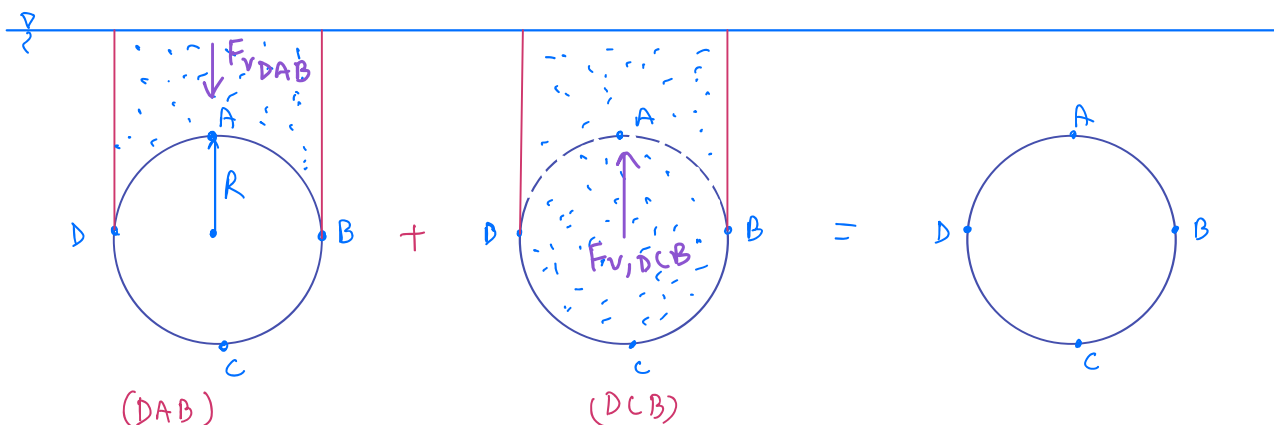
$$= \rho_w (h+R) \cdot (2R \times 1)$$

$$= 2 \rho_w R (h+R)$$



Net horizontal force, $F_H = 0$. (Both will cancel each other)

Four vertical force



$$F_{V,DAB} = \left[(h+R)2R - \frac{\pi R^2}{2} \right] \times 1 \times r_w$$

$$F_{V,DCB} = \left[(h+R)2R + \frac{\pi R^2}{2} \right] \times 1 \times r_w \quad \{\text{upward}\}$$

$$F_{V,cy} = F_{V,DCB} + F_{V,DAB}$$

$$= \left[(h+R)2R + \frac{\pi R^2}{2} \right] r_w - \left[(h+R)2R - \frac{\pi R^2}{2} \right] r_w$$

$$= \pi R^2 r_w$$

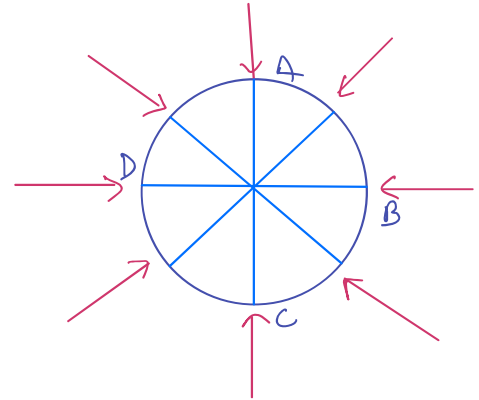
OR

$$F_v = \text{Area} \times \text{length} \times r_w$$

$$= \pi R^2 \times 1 \times r_w$$

$$= \pi R^2 r_w$$

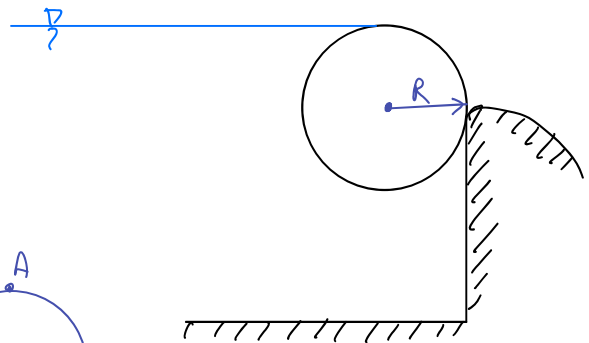
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2021



[This is why in DCB pressure was upward]

- 8) A cylinder of radius 0.3m is located in water as shown. The cylinder and the wall are smooth. For 1.5m length of cylinder. compute

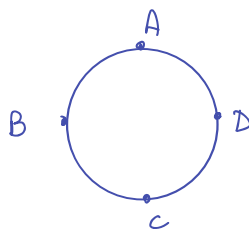
- wt. of cylinder
- Resultant force exerted by wall on cylinder.
- The resultant moment around the centre of the cylinder due to the water force on cylinder.



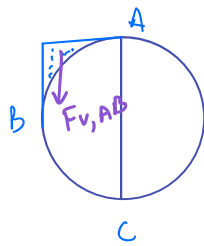
sol (a) wt of the cylinder

= vertical force acting on the cylinder.

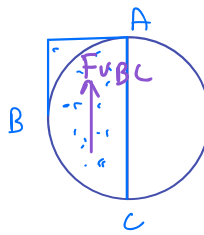
$$F_{V,cy} = F_{V,ABC} + F_{V,CD}$$



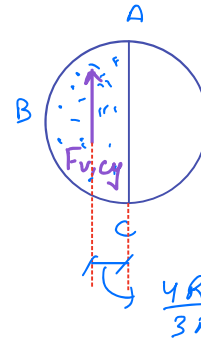
For $F_{V, ABC}$



+



=



$$F_{V, ABC} = \left(\frac{\pi R^2}{2} \times L \right) \times \gamma_w$$

$$= \left(\frac{\pi \times 0.3^2}{2} \times 1.5 \right) \times 10 = 2.12 \text{ (kN)}$$

$$\frac{4R}{3\pi} = 0.127 \text{ m}$$

For $F_{V, CD}$

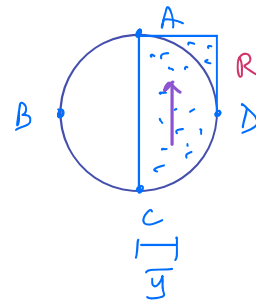
How water is above pt. D?

→ Acc. to pascals law, pressure at same level for same fluid is same.

$$\text{i.e. } P_B = P_D$$

$$\gamma_w R = \gamma_w h$$

So, At pt. R, height of water = R



$$\text{Now, } F_{V, CD} = \left[R^2 + \frac{\pi R^2}{4} \right] \times \text{Length} \times \gamma_w$$

$$= \left[0.3^2 + \frac{\pi \times 0.3^2}{4} \right] \times 1.5 \times 9.81$$

$$= 2.41 \text{ (kN)}$$

$$F_{V, cy} = F_{V, ABC} + F_{V, CD}$$

$$= 2.12 + 2.41$$

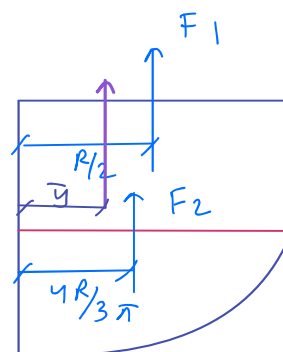
$$= 4.53 \text{ (kN)}$$

Finding $\bar{y}$

$$F_{V, CD} \times \bar{y} = F_1 \times R/2 + F_2 \times$$

$$2.41 \times \bar{y} = 1.35 \times \frac{0.3}{2} + 1.06 \times \frac{4 \times 0.3}{3\pi}$$

$$\bar{y} = 0.14 \text{ m}$$



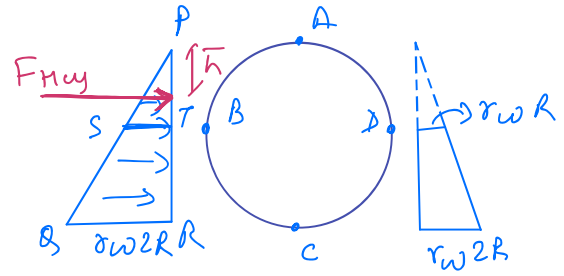
(b) Net horizontal force = PST

$$= \frac{1}{2} \times r_w R \times R \times L$$

$$= 0.5 \times 10 \times 0.3^2 \times 1.5$$

$$= 0.675 \text{ kN}$$

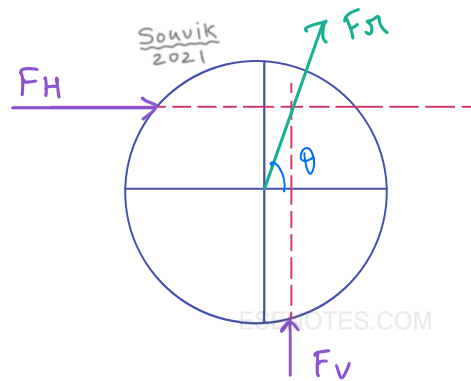
$$\bar{h} = \frac{2R}{3} = 0.2$$



Resultant force exerted by wall on the cylinder.

$$F_R = \sqrt{F_V^2 + F_H^2}$$

$$= \sqrt{4.53^2 + 0.675^2} = 4.58 \text{ kN.}$$



$$\theta = \tan^{-1} \left(\frac{0.675}{4.53} \right)$$

$$\theta = 8.52^\circ$$

NOTE:- If the weight of the cylinder is also considered.

$$w = F_{V, cy}$$

Hence force exerted by wall on cylinder

$F_R = F_{H, cy}$ as $F_{V, cy}$ and w will cancel each other.

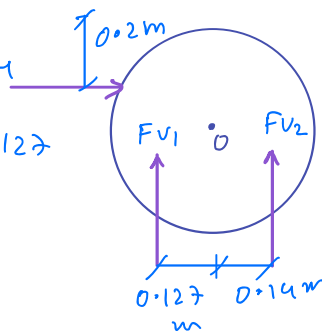
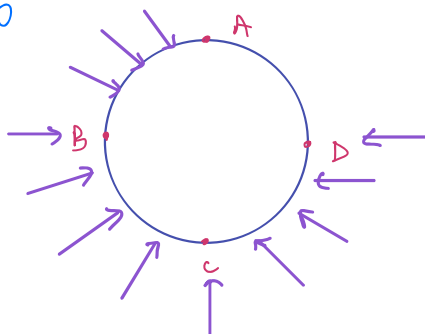
(c)

$$M_O = F_H (0.3 - 0.2) -$$

$$F_{V2} (0.14) + F_{V1} \times 0.127 - F_H$$

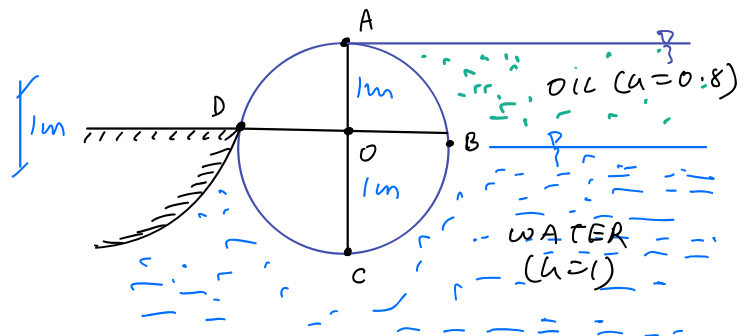
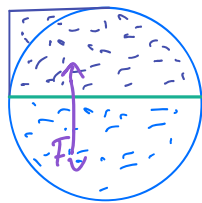
$$M_O = 0.675 \times 0.1 - 2.41 \times 0.14 + 2.12 \times 0.127$$

$$M_O = 0$$



- Q) The figure shows a cylinder of 2m dia in equilibrium calculate the vertical component of force exerted by the fluid per m length of the cylinder, and also compute horizontal pressure force.

Sol Vertical pressure force :-



$$F_{v, cy} = 0.8 \times 10 \times \left[1 \times 1 + \frac{\pi \times 1^2}{4} \right] + 10 \left[\frac{\pi \times 1^2}{2} \right]$$

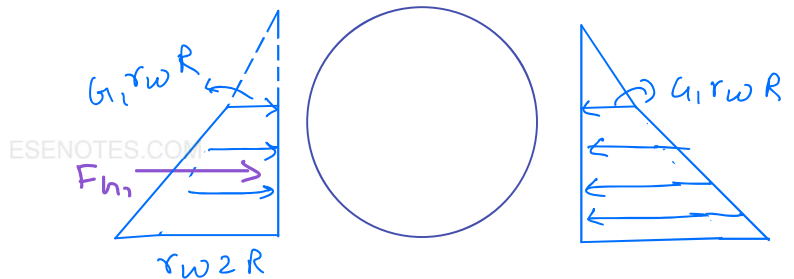
$$= 8 \times \left[1 + \frac{\pi}{4} \right] + 5\pi = 30 \text{ (kN)}$$

horizontal pressure

$$F_{H, cy} = \frac{1}{2} \times G_1 \times w \times R \times R \times 1$$

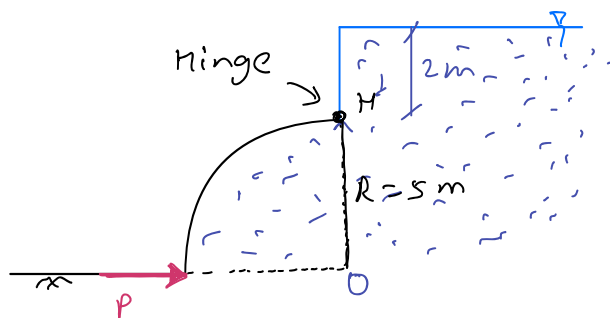
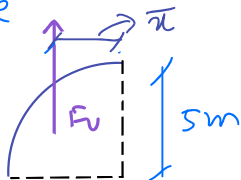
$$= \frac{1}{2} \times 0.8 \times 10 \times 1$$

$$= 4 \text{ kN (}\leftarrow\text{)}$$



- Q) A cylindrical gate as shown is to be kept closed against water pressure. Determine the force P per m width of the gate to keep it closed.

Sol vertical force on the gate



$$F_{v, cy} = 10 \times \left[2 \times 5 + \left(5 \times 5 - \frac{\pi \times 5^2}{4} \right) \right]$$

$$= 153.65 \text{ (kN)}$$

$$\pi \times F_v = \left[7 \times 5 \times 2.5 \right] \times 10 - \frac{\pi \times 5^2}{4} \times \frac{4 \times 5}{3\pi} \times 10$$

$$\pi = 2.98$$

horizontal force on the gate

$$F_H = \frac{1}{2} \times [10 \times 2 + 10 \times 7] \times 5 \times 1$$

$$= 225 \text{ (kN)}$$

$$h_p = \bar{h} + \frac{I \sin^2 \theta}{A \bar{h}}$$

$$= 4.5 + \frac{\frac{5^3}{12} \times 1}{5 \times 1 \times 4.5} = 4.96 \text{ m} \quad I = \frac{1 \times 5^3}{12}$$

from free surface

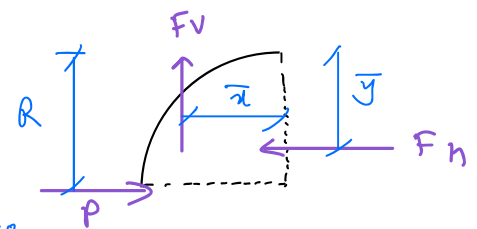
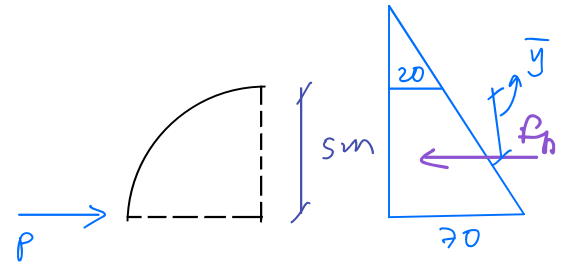
$$\therefore \bar{y} = 4.96 - 2 = 2.96 \text{ m}$$

$$\sum F_H = 0 :-$$

$$P(R) - F_H(\bar{y}) \overset{\text{Solving}}{=} F_V(\bar{x}) = 0$$

$$P \times 5 = 225 \times 2.96 - 153.65 \times 2.98$$

$$P = 41.62 \text{ (kN)}$$



July 23

- Q) A stream is spanned by a bridge, which is a single masonry arch in the form of a parabola. the crown of which is 2.5m above the springings which are 10m above measured in the direction of stream, the width of bridge is 6.5m. during the flood, the stream rises to a level of 2.1m above springings. calculate the force tending to lift the bridge from its foundation, if arch remain water tight.

sol = Force tending to lift the bridge.

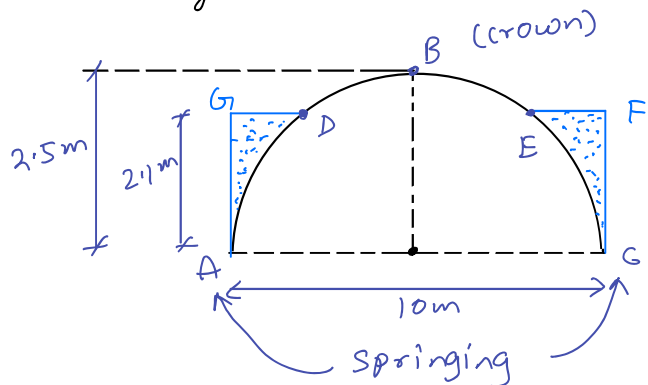
= F_v of the hydrostatic force

$$F_v = \rho \times \text{volume}$$

$$\text{Area} = A_{AGFC} - A_{ADEC}$$

$$A_{ADEC} = A_{ABC} - A_{DBE}$$

$$\text{equation of parabola, } y = kx^2 \quad [\text{origin is B}]$$



$$@ x = 5 \text{ m}, y = 2.5 \text{ m}$$

$$2.5 = k \times 5^2$$

$$k = 0.1$$

$$\text{i.e. } y = 0.1 x^2$$

$$@ y = 0.4 \text{ m}, x = \sqrt{\frac{0.4}{0.1}} = 2 \text{ m}$$

$$DE = 4 \text{ m}$$

$$A_{ADEC} = \frac{2}{3} \times 10 \times 2.5 - \frac{2}{3} \times 4 \times 0.4$$

$$A_{ADEC} = 15.6 \text{ m}^2$$

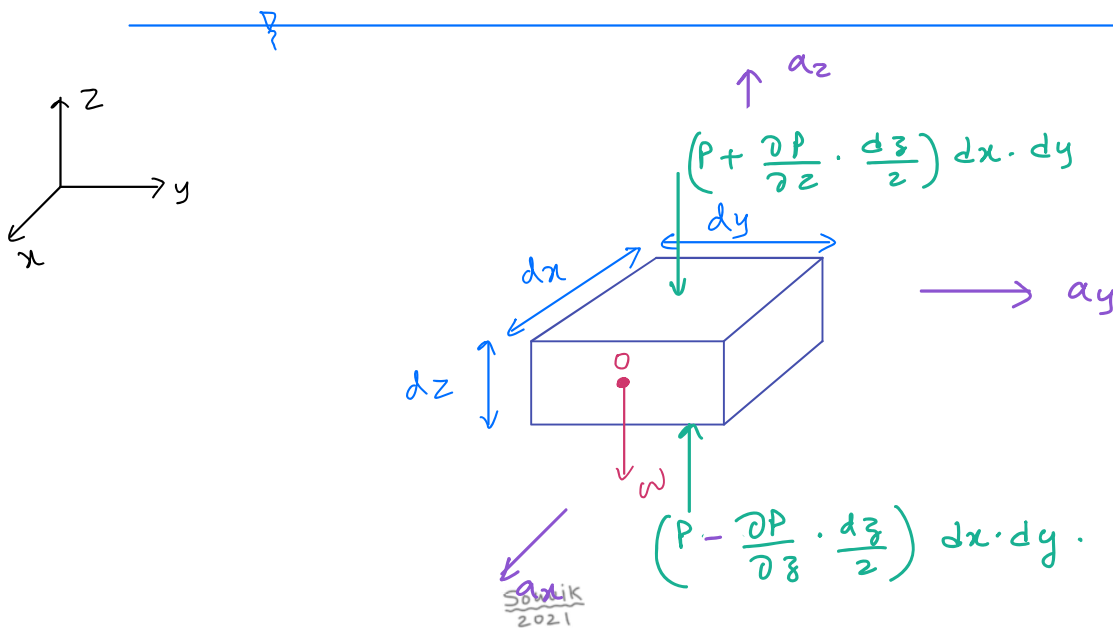
$$\text{Now, } A = \frac{21 - 15.6}{5.4} \text{ m}^2 \rightarrow \text{Total water area (ADA + ECF)}$$

$$F_v = 5.4 \times 6.25 \times 10$$

$$= 357 \text{ (kW)}$$

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LIQUIDS IN RELATIVE MOTION



- when a liquid is present in a container and the container moves, the liquid also moves along with the container as a rigid body.
- Hence there is no relative motion between different layers of the liquid and it may be considered as liquid in rest.
- If the container starts accelerating, the liquid will take different shape and geometry and for this pressure can be calculated as in previous case i.e static condition.

Now, since liquid is moving like rigid body from Newton's second law.

$$F = ma \quad \begin{matrix} \text{unit wt} \\ \uparrow \\ (P - \frac{dp}{dz} \cdot \frac{dz}{2}) dx \cdot dy - (P + \frac{dp}{dz} \cdot \frac{dz}{2}) dx \cdot dy - \rho(dx \cdot dy \cdot dz) \end{matrix} = \begin{matrix} \text{density} \\ \uparrow \\ \rho(dx \cdot dy \cdot dz) \end{matrix} a_z$$

$$- \frac{\partial P}{\partial z} \cdot (dz \cdot dx \cdot dy) - \rho(dx \cdot dy \cdot dz) = \rho(dx \cdot dy \cdot dz) a_z$$

$$- \frac{\partial P}{\partial z} + \rho - \rho \cdot a_z = 0$$

$$- \frac{\partial P}{\partial z} = \rho + \rho a_z = \rho g + \rho a_z = \rho(g + a_z)$$

$$-\frac{\partial P}{\partial z} = \rho (g + a_z)$$

NOTE:- Here -ve sign signifies that with increase in depth z from bottom pressure decreases.

- If acceleration is in downward direction, then

$$-\frac{\partial P}{\partial z} = \rho (g - a_z)$$

- Here $(g \pm a_z)$ may be termed as effective gravity

$$\therefore -\frac{\partial P}{\partial z} = \rho g_{\text{eff}} \quad \text{---(i)}$$

Similarly for x and y direction. (weight will be eliminated)

i.e. $\frac{\partial P}{\partial x} = 0$ and $a_z = a_x$ or a_y .

$$-\frac{\partial P}{\partial z} = \rho (0 + a_x)$$

$$-\frac{\partial P}{\partial x} = \rho a_x \quad \text{---(ii)}$$

and

$$-\frac{\partial P}{\partial y} = \rho a_y \quad \text{---(iii)}$$

} Euler's equation

SPECIAL CASE :-

case (i) :- Fluid / container at rest :-

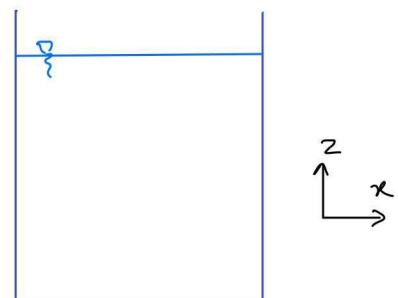
$$\begin{aligned} -\frac{\partial P}{\partial x} = 0, \quad -\frac{\partial P}{\partial y} = 0 & \left\{ \begin{array}{l} \text{pressure remains} \\ \text{const. in horizontal} \\ \text{plane.} \end{array} \right. \\ -\frac{\partial P}{\partial z} = \rho g & \left\{ \begin{array}{l} \text{pressure} \\ \text{varies linearly. with depth} \end{array} \right. \end{aligned}$$

case (ii) :- Fluid / container with constant acceleration
 a_x and a_z

$$dP = \frac{\partial P}{\partial x} \cdot dx + \frac{\partial P}{\partial y} \cdot dy + \frac{\partial P}{\partial z} \cdot dz$$

$$\text{Since } a_y = 0, \quad -\frac{\partial P}{\partial y} = 0.$$

$$dP = -\rho a_x dx + 0 - \rho (g + a_z) dz$$



At free surface, pressure at each and every point is zero. hence $dP = 0$.

$$0 = -P a_x \cdot dx - P(g + a_z) dz$$

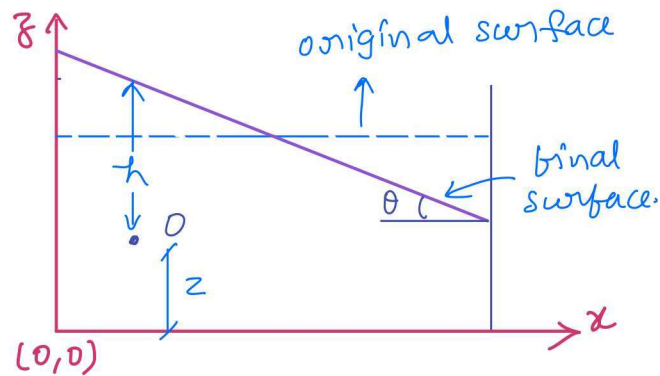
$$a_x \cdot dx = - (g + a_z) dz$$

$$\frac{dz}{dx} = - \frac{a_x}{g + a_z} = - \frac{a_x}{g_{eff}}$$

$$\frac{dz}{dx} = - \frac{a_x}{g_{eff}}$$

$$-\frac{a_x}{g_{eff}} = \tan \theta$$

slope is negative.



NOTE:- Here (-)ve sign indicates

(i) with increase in x , in direction of acceleration, z decreases.

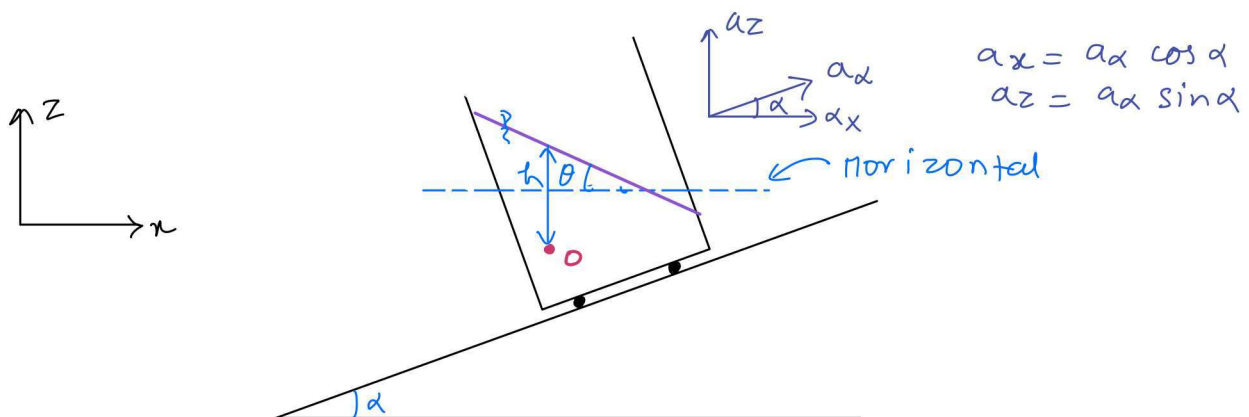
(ii) OR, it indicates water surface will slope down in direction of x .

pressure at O = $\gamma \cdot h$

$\gamma = \rho g_{eff}$

$h = \text{vertical distance from new surface of liquid}$

case (iii) fluid/container accelerating up the slope



$$\frac{dz}{dx} = - \frac{a_x}{g + a_z} = - \frac{a_x \cos \alpha}{g + a_x \sin \alpha} = \tan \theta$$

$$\frac{dz}{dx} = - \frac{a_x \cos \alpha}{g + a_x \sin \alpha} = - \frac{a_x \cos \alpha}{g_{eff}} \quad \text{where } g_{eff} = g + a_x \sin \alpha$$

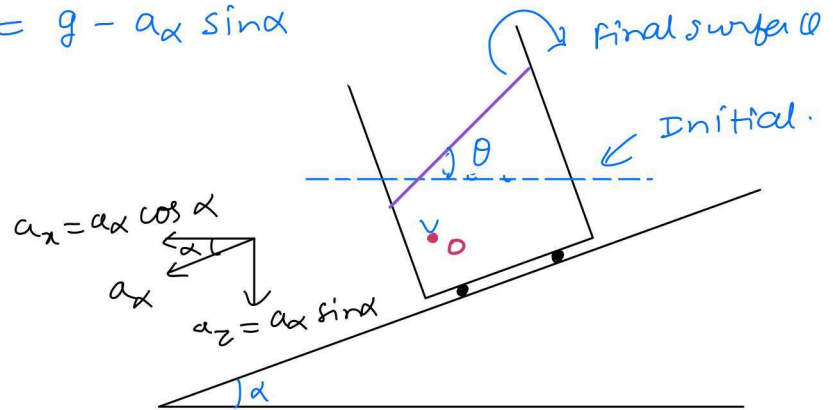
Also, pressure at O, $P_0 = \gamma h$

$\gamma = \rho g_{eff}$

case (iv) Fluid/ container accelerating down the slope

$$\frac{dz}{dx} = -\frac{a_x}{g - a_z} = -\frac{a_x \cos \alpha}{g - a_x \sin \alpha} = -\frac{a_x \cos \alpha}{g_{\text{eff}}} = \tan \theta$$

$$g_{\text{eff}} = g - a_x \sin \alpha$$



July 24

Souvik
2021

NOTE:- For free surface (new) to be parallel to inclined plane i.e. $\theta = \alpha$.

$$\tan \theta = \frac{a_x \cos \alpha}{g - a_x \sin \alpha} = \tan \alpha$$

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$$\frac{a_x \cos^2 \alpha}{g - a_x \sin \alpha} = \sin \alpha$$

$$a_x \cos^2 \alpha = g \sin \alpha - a_x \sin^2 \alpha$$

$$a_x = g \sin \alpha$$

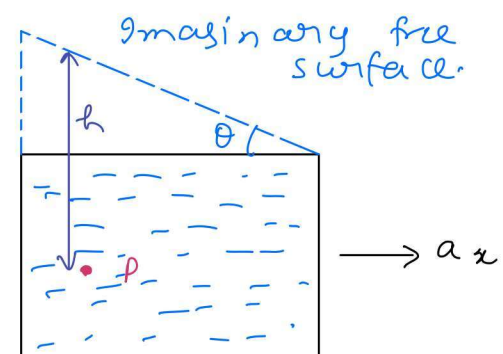
i.e. liquid is moving under the effect of its own weight.

conclusion:- If the container containing liquid rolls down an inclined plane due to its own weight, the liquid surface will become parallel to the inclined plane.

NOTE:- ① CLOSED & FILLED CONTAINER:-

$$\tan \theta = \frac{a_x}{g_{\text{eff}}} = \frac{a_x}{g} \rightarrow (a_z = 0)$$

$$\text{Pressure at } O = \gamma h = \rho g_{\text{eff}} h$$



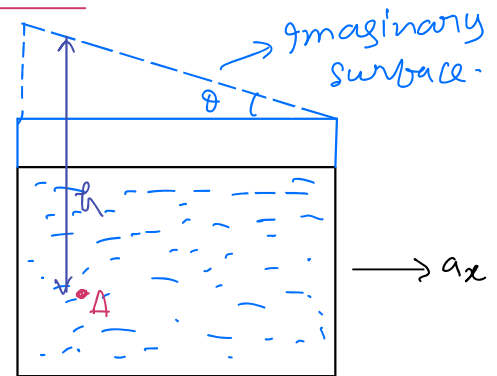
② CLOSED, FILLED and pressurized fluid :-

pressure = p_0 .

h_0 = height of liquid corresponds to the pressure

$$h_0 = \frac{p_0}{\rho}$$

$$\tan \theta = \frac{a_x}{g_{eff}} = \frac{a_x}{g} \quad (a_z = 0).$$



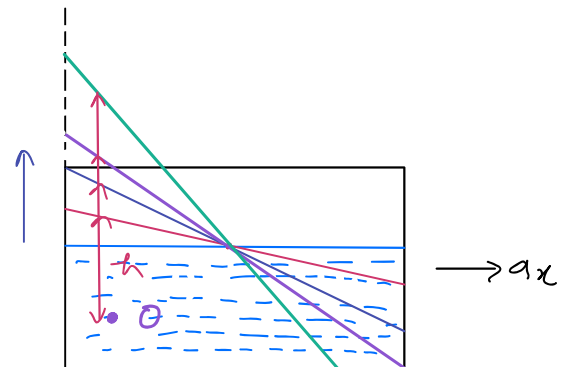
pressure at any point A = $\rho h = \rho g_{eff} h$.

③ Partially filled & closed container :-

$$\tan \theta = \frac{a_x}{g_{eff}} = \frac{a_x}{g} \quad \text{Sowvik 2021}$$

pressure at O = $\rho \times$ corresponding height.

$$= \rho g_{eff} h.$$



- Q) A 60cm high and 40cm dia cylinder containing water is translated on a level road, the highest acceleration is 4 m/s^2 . Determine the allowable initial water height in tank, if no water is to spill out during acceleration.

sol $a_x = 4 \text{ m/s}^2$

Let height be h .

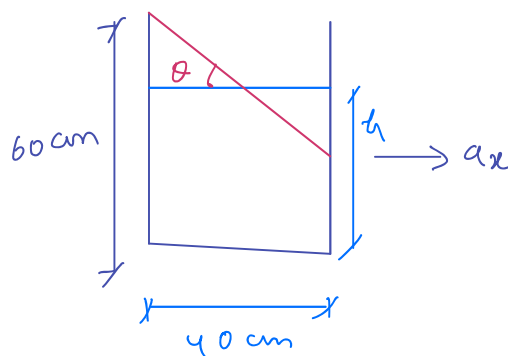
For no spill out

$$\tan \theta = \frac{60 - h}{20}$$

Also, $\tan \theta = \frac{a_x}{g_{eff}} = \frac{4}{10}$

$$\tan \theta = 0.4$$

$$0.4 = \frac{60 - h}{20} \quad h = 52 \text{ cm.}$$

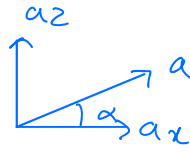


Q) For the given system find the slope of liquid surface and pressure at point "B".

$$h = 0.8$$

So

$$\tan \theta = \frac{a_x}{g_{\text{eff}}}$$



$$a_x = a \cos \alpha = 5 \cos 30 = 4.33 \text{ m/s}^2$$

$$g_{\text{eff}} = g + a_z = g + a \sin \alpha$$

$$g_{\text{eff}} = 10 + 5 \sin 30 = 12.5 \text{ m/s}^2$$

$$\tan \theta = \frac{4.33}{12.5}$$

$$\theta = \frac{19.11}{20.1}$$

now pressure at B.

$$P_B = \rho g_{\text{eff}} \times h$$

$$\tan \theta = \frac{2-h}{1.5}$$

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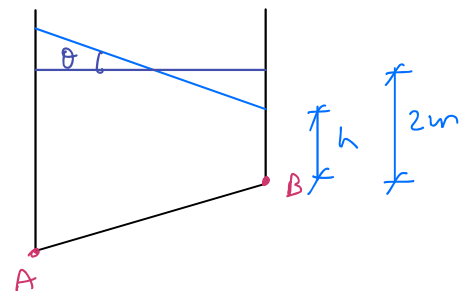
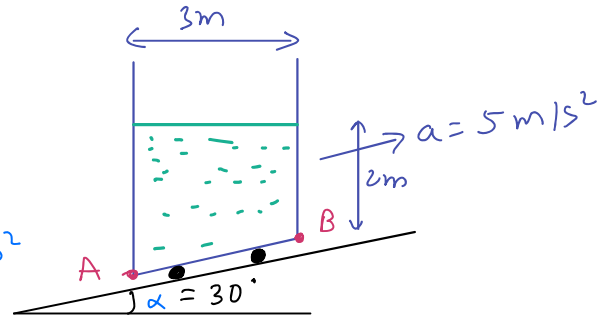
$$2-h = 0.519$$

$$h = 1.48 \text{ m}$$

$$\therefore P_B = 0.8 \times 1000 \frac{\text{kg}}{\text{m}^3} \times 12.5 \frac{\text{m}}{\text{s}^2} \times 1.48 \text{ m}$$

$$P_B = 14800 \text{ N/m}^2$$

$$= 14.8 \text{ kN/m}^2$$



Q) A closed tank 2m deep, 1.8m wide and 3.5m long has been filled with oil of sp. gravity 0.8 up to a depth of 1.6m. calculate the acceleration which may be imparted to the tank in the direction of its length so that bottom front end of the tank is just exposed. Also compute the net horizontal force acting on the tank sides and show that this is equal to force necessary to accelerate the liquid mass.

$$\text{Sol} \quad \tan \theta = \frac{ax}{g_{\text{eff}}} = \frac{ax}{g}$$

Also,

$$\tan \theta = \frac{1.6}{1.75}$$

$$\theta = 42.43^\circ$$

$$\frac{1.6}{1.75} = \frac{ax}{10}$$

$$ax = 9.14 \text{ m/s}^2$$

Pressure at B

$$= 0.8 \times 1000 \times 10 \times 3.2 \text{ N/m}^2$$

$$= 25.6 \text{ KN/m}^2$$

Pressure at A

$$= 0.8 \times 1000 \times 10 \times 1.6$$

$$= 12.8 \text{ KN/m}^2$$

$$F = \frac{1}{2} (25.6 + 12.8) \times 2 \times 1.8 = 69.12 \text{ KN}$$

Now, Force req to accelerate the oil container

$$F = m \times a$$

$$F = \gamma \times \text{vol.} \times a$$

$$F = 44.1 \text{ (not coming)}$$

IInd approach

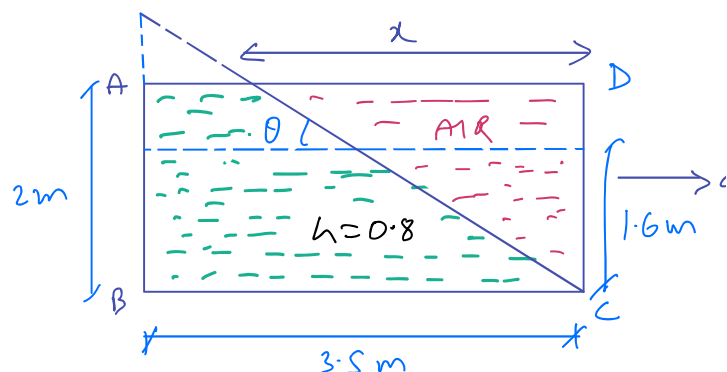
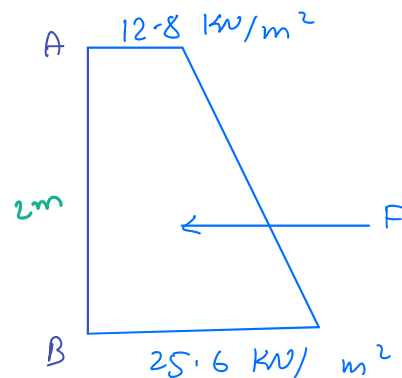
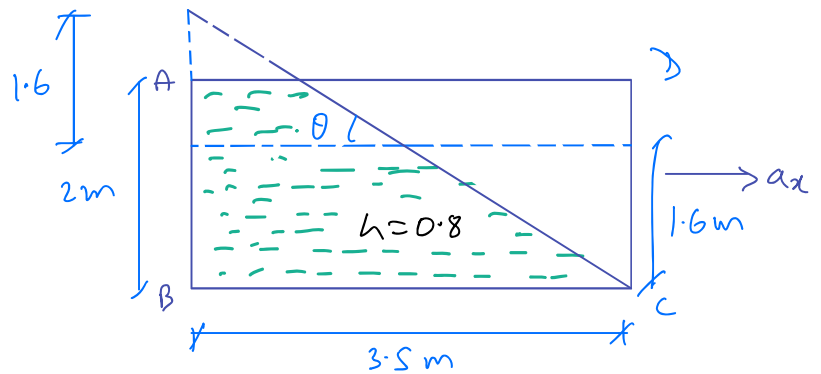
vol. of air before accel_n = vol. of air after accel_n

$$3.5 \times 1.8 \times 0.4 = \left(x \times 2 \times \frac{1}{2} \right) \times 1.8$$

$$x = 1.4 \text{ m}$$

$$\tan \theta = \frac{2}{1.4}$$

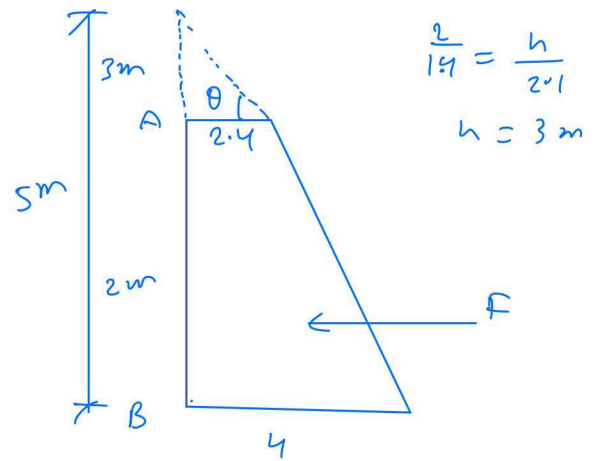
$$\text{Also, } \tan \theta = \frac{ax}{g_{\text{eff}}} = \frac{2}{1.4}$$



$$a_x = 14.28 \text{ m/sec}^2$$

Now, Force on wall.

$$\begin{aligned} F &= \bar{P} \cdot A \\ &= \gamma h \cdot A \\ &= 0.8 \times 10 \times (2 \times 1.8) \times (4) \\ &= 115.2 \text{ (kN)} \end{aligned}$$



Force necessary to accelerate.

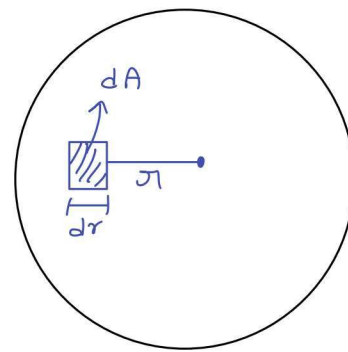
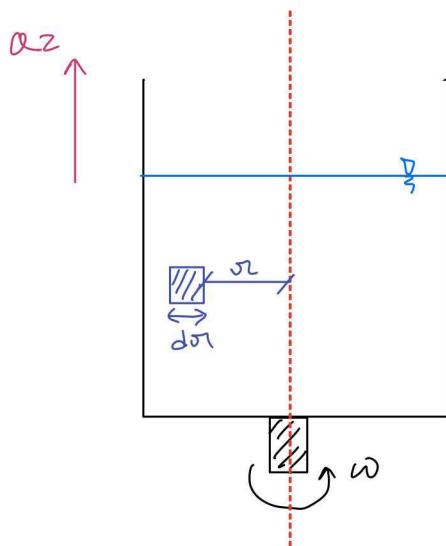
$$\begin{aligned} F &= m \times a = F_{CD} - F_{AB} \\ &= \gamma \times \text{Vol.} \times a \\ &= 0.8 \times 1000 \times (1.6 \times 3.5 \times 1.8) \times 14.28 \\ &= 115.2 \text{ (kN)} \end{aligned}$$

(same proved)

$$\ominus F_{AB} = 115.153$$

force is in opp. direct to accel

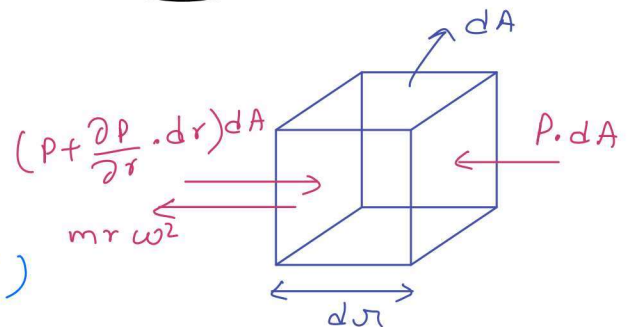
ROTATION IN CYLINDRICAL CONTAINER



$$(P + \frac{\partial P}{\partial r} \cdot dr) dA - P \cdot dA = \frac{m v^2}{r}$$

$$\frac{\partial P}{\partial r} \cdot dr \cdot dA = P \cdot dA \cdot dr \left(\frac{v^2}{r} \right)$$

$$\boxed{\frac{\partial P}{\partial r} = P \frac{v^2}{r}}$$



$$dP = \frac{\partial P}{\partial r} \cdot dr + \frac{\partial P}{\partial z} \cdot dz \rightarrow g_{\text{eff}} = g \pm a_z$$

$$dP = \frac{P v^2}{r} dr + (-P g_{\text{eff}}) dz$$

here $g_{eff} = g + a_z$

$$dp = \frac{\rho v^2}{r} dr - \rho (g + a_z) dz \rightarrow$$

This equation is general equation of vortex motion i.e. when fluid is moving in circular motion.

July 25

- vortex motion are of two types

(a) Free vortex motion

(b) Forced vortex motion.

(a) FREE VORTEX MOTION :-

- If the fluid is rotating without the application of external torque, ^{Souvik} the motion is called free vortex motion. Eg- water draining from wash basin.

- For free vortex motion angular momentum remains conserved as external torque is zero.

$$mvr = \text{const.}$$

since $m = \text{const.}$, $vr = \text{const.}$

$$v \propto \frac{1}{r} \rightarrow \text{for free vortex motion.}$$

(b) FORCED VORTEX MOTION :-

- If the fluid is rotated with constant angular velocity " ω " by application of external torque it is termed as forced vortex motion.

Eg:- motion of water in washing machine.

Here, $v = \omega r$

i.e. $v \propto r \rightarrow \text{for forced vortex motion.}$

- In our case the cylinder is being rotated by application of external torque, hence it is forced vortex motion.

$$dp = \frac{\rho v^2}{r} dr - \rho (g + a_z) dz$$

Since, $v = \omega r$

$$\therefore dP = \frac{\rho(\omega r)^2}{r} dr - \rho(g + a_z) dz$$

$$dP = \rho \omega^2 r dr - \rho(g + a_z) dz$$

$$P = \int dP = \int \rho \omega^2 r dr - \rho(g + a_z) dz$$

$$P = \rho \omega^2 \frac{r^2}{2} - \rho(g + a_z) z + C$$

At $P=A$, $r=0$, $z=z_0$ i.e. at free surface

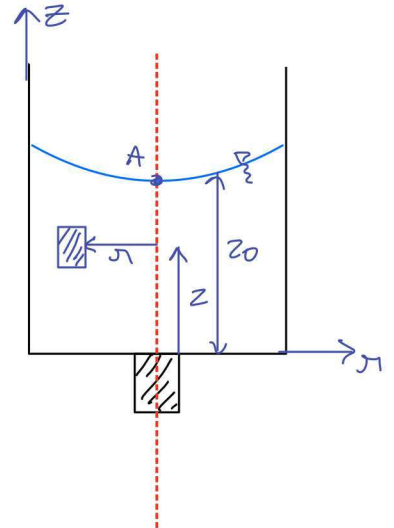
$$P=0$$

$$\rho \omega^2 \frac{r^2}{2} - \rho(g + a_z) z + C = 0$$

$$C = \rho(g + a_z) z_0$$

$$\therefore P = \rho \omega^2 \frac{r^2}{2} - \rho(g + a_z) z + \rho(g + a_z) z_0$$

$$P = \rho \omega^2 \frac{r^2}{2} - \rho g_{\text{eff}} (z - z_0)$$



Now, at free surface, $P=0$.

$$0 = \rho \omega^2 \frac{r^2}{2} - \rho(a + g_z)(z - z_0)$$

$$(z - z_0) = \frac{\omega^2 r^2}{2(g + a_z)} = \frac{\omega^2 r^2}{2g_{\text{eff}}}$$

$$(z - z_0) = \frac{\omega^2 r^2}{2g_{\text{eff}}}$$

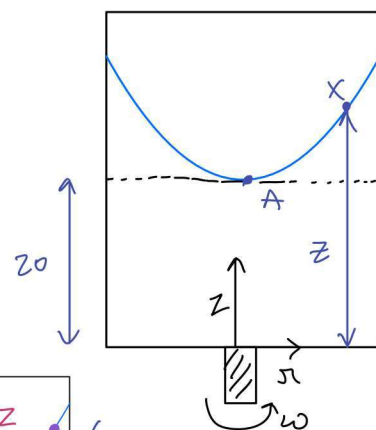
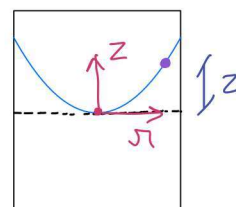
$\rightarrow z$ - measured from bottom
 $\rightarrow a_z$ is taken to be upward

water surface
 i.e. $(z - z_0) \rightarrow$ elevation from lowest point of the free surface level. (A)

NOTE:- If distances are measured from the lowest level "A" of the water surface then, $z_0 = 0$.

i.e.

$$z = \frac{\omega^2 r^2}{2g_{\text{eff}}}$$



Also, $P_0 = \rho h = \rho g_{eff} \cdot h$

$$g_{eff} = g \pm a_z$$

The above expression is valid for all shapes of container.

Eg:- compute the difference in level in the two limbs.

(a)

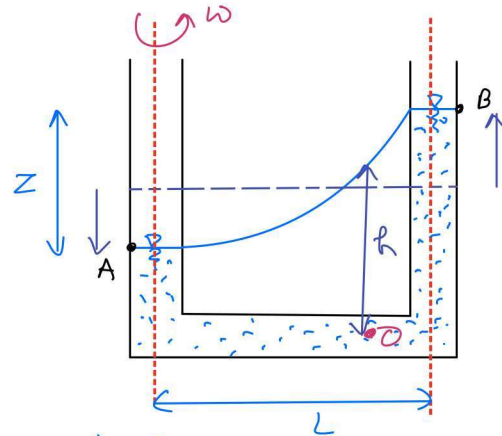
sol: A is the lowest point.

$$z = \frac{\omega^2 r^2}{2g_{eff}}$$

$$z = \frac{\omega^2 L^2}{2g}$$

pressure at O = ρh

$$= \rho g_{eff} \cdot h \quad \text{Just find "h".}$$



(b)

$$z_1 = \frac{\omega^2 r^2}{2g_{eff}}$$

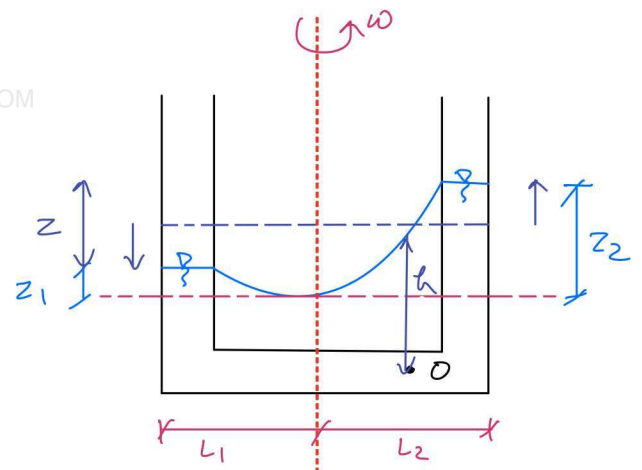
$$z_1 = \frac{\omega^2 L_1^2}{2g}$$

$$z_2 = \frac{\omega^2 L_2^2}{2g}$$

$$z = z_2 - z_1 = \frac{\omega^2}{2g} (L_2^2 - L_1^2)$$

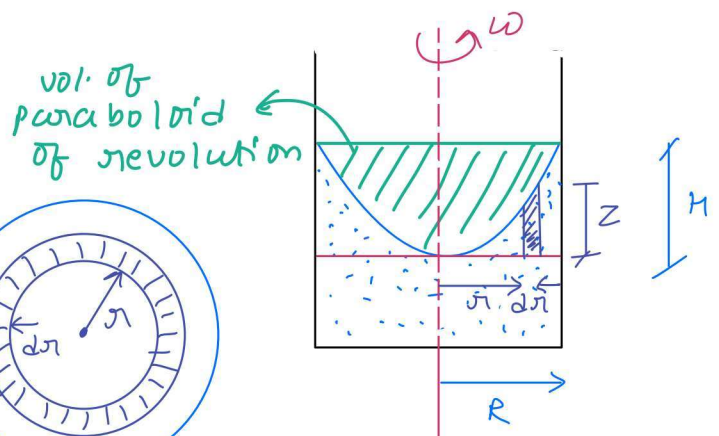
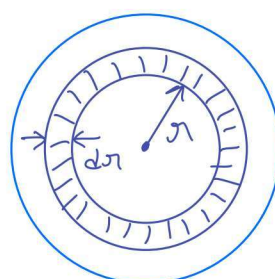
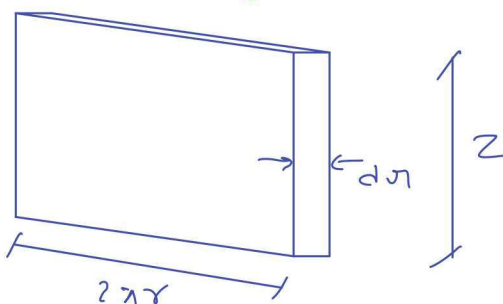
pressure, at O = ρh

$$= \rho g_{eff} h$$



NOTE:- VOLUME OF PARABOLOID OF REVOLUTION :-

$$z = \frac{\omega^2 r^2}{2g}$$



$$\therefore \text{volume, } dv = 2\pi r \cdot z \cdot dr$$

$$= 2\pi r \cdot \frac{\omega^2 r^2}{2g} dr$$

$$V_1 = \int dv = \int_0^R \frac{\omega^2 r^3}{g} \pi dr$$

$$V_1 = \frac{\pi \omega^2}{g} \left[\frac{r^4}{4} \right]_0^R$$

$$V_1 = \frac{\pi \omega^2}{g} \frac{R^4}{4}$$

$$\text{Now, } r = \frac{\omega^2 R^2}{2g} \quad \therefore V_1 = \frac{\pi H R^2}{2}$$

$$\text{vol. of paraboloid of revolution, } V = \pi R^2 H - V_1$$

$$V = \pi R^2 H - \frac{\pi H R^2}{2}$$

$$V = \frac{\pi R^2 H}{2}$$

i.e

$$V = V_1 = \frac{\pi R^2 H}{2}$$

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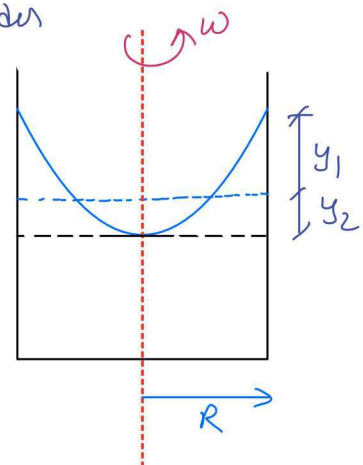
volume of paraboloid of revolution = $\frac{\text{vol. of enclosing cylinder}}{2}$

NOTE:- ① In case of cylindrical container, rise above the original level will be equal to the fall below the original level, if the liquid is free to rise and fall.

$$V_1 = \pi R^2 y_2 = \pi R^2 \left(\frac{y_1 + y_2}{2} \right) \quad \text{half of cylinder vol.}$$

$$2y_2 = y_1 + y_2$$

$$y_1 = y_2$$



July 26

- ② A cylinder is filled initially upto top and rotated such that half of the water spills out, then pressure at axis of rotation is $p_0 = 0$ (atm pressure).

③

$$\omega_0 < \omega_1 < \omega_2 < \omega_3 < \omega_5$$

For $\omega = \omega_1$ and ω_2 , rise and fall will be equal, thereafter two points will be fixed at top and its height will keep on increase in angular speed.

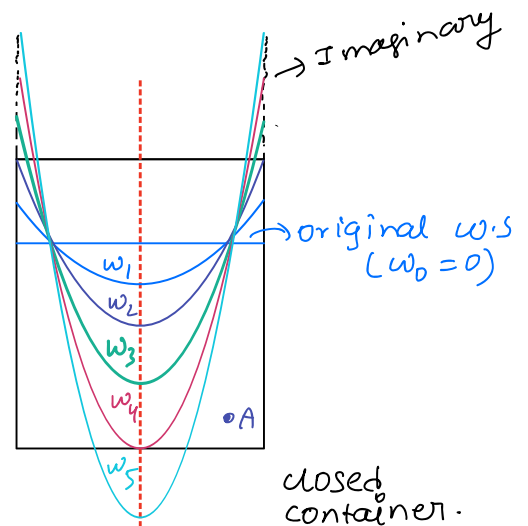
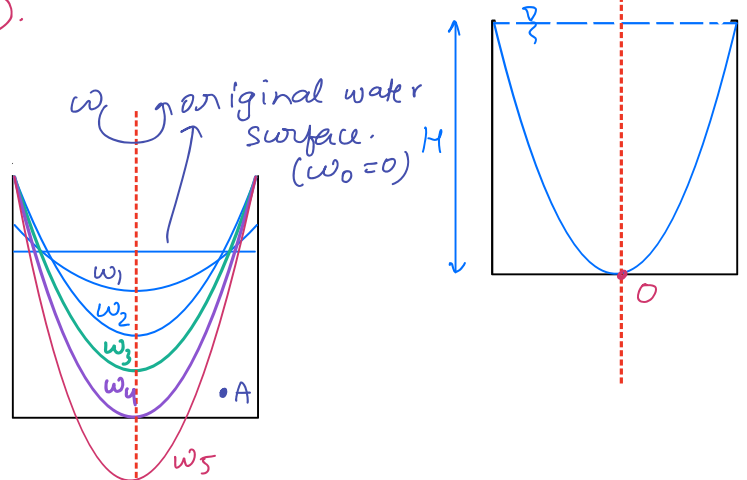
$$P_A = \gamma h \quad (h \text{ from free surface})$$

$$= \rho g_{\text{eff}} h$$

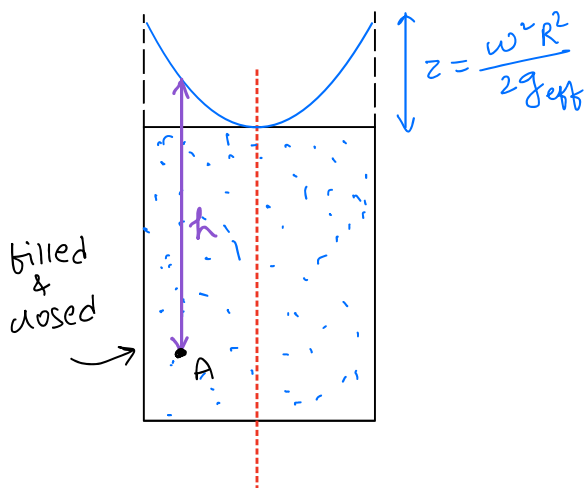
- ④ closed container & partially tilted

$$P_A = \gamma h \quad (h \text{ from free surface})$$

$$P_A = \rho g_{\text{eff}} \cdot h$$



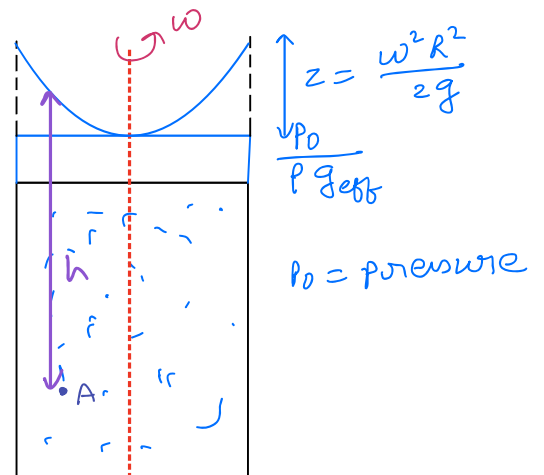
- ⑤ closed & fully filled.



$$P_A = \gamma h$$

$$= \rho g_{\text{eff}} \cdot h$$

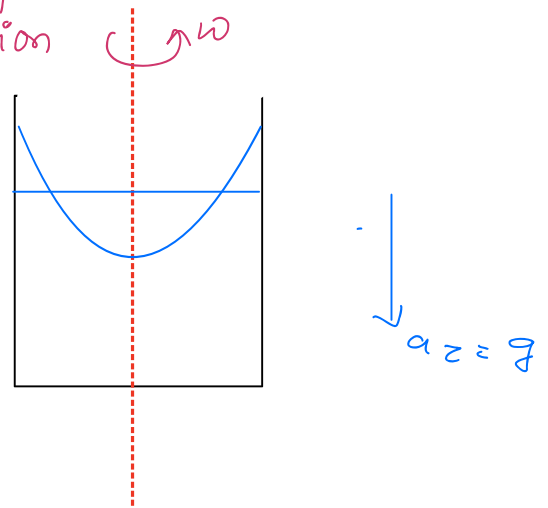
- ⑥ Filled under pressure & closed



$$P_A = \rho g_{\text{eff}} \times h$$

- ⑦ In case of free fall pressure will not vary in vertical direction and will only vary in radial direction.

$$\begin{aligned} dP &= \frac{\partial P}{\partial r} \cdot dr + \frac{\partial P}{\partial z} \cdot dz \\ &= \frac{\rho v^2}{r} \cdot dr - \rho(g - a_z) dz \\ &= \frac{\rho v^2}{r} dr \end{aligned}$$



- ⑧ For the given case, find the volume of water spilled out.

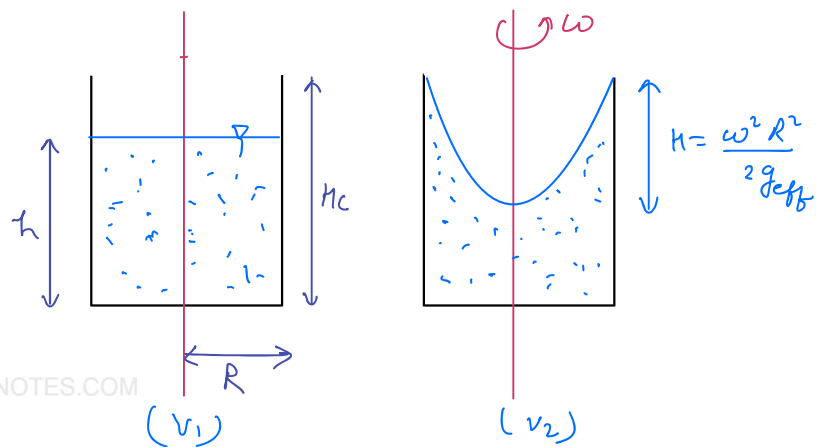
Sol. vol. spilled out = $V_1 - V_2$ Souvik 2021

$$V_1 = \pi R^2 \cdot h$$

$$V_2 = \pi R^2 H_c - \frac{1}{2} \times \pi R^2 H$$

$$V_2 = \pi R^2 H_c - \frac{\pi R^4 \omega^2}{4g}$$

$$\therefore \text{Spilled out} = V_1 - V_2$$



(9)

$$H - H_c = \frac{\omega^2 r_0^2}{2g}$$

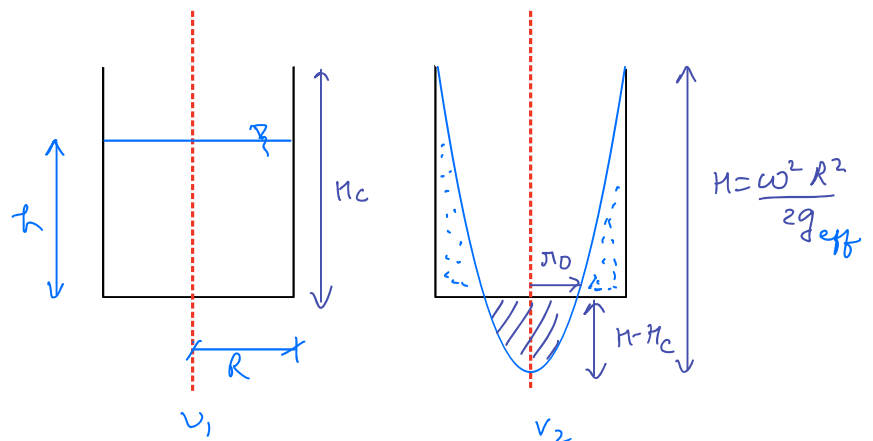
water spilled out is = $V_1 - V_2$

$$V_1 = \pi R^2 h$$

$$V_2 = \text{Total vol.} - \text{vol. of whole paraboloid} + \text{vol. of small paraboloid}$$

$$V_2 = \pi R^2 H_c - \frac{1}{2} \times (\pi R^2 H) + \frac{1}{2} (\pi r_0^2 (H - H_c))$$

$$\text{los of water} = \pi R^2 h - \left[\pi R^2 H_c - \frac{\pi R^2 H}{2} + \frac{\pi r_0^2 (H - H_c)}{2} \right]$$



- Q) A closed cylinder 0.4m in diameter and 0.4m in height is filled with oil of sp. gravity 0.8. If the cylinder is rotated about its vertical axis at a speed of 200 rpm. Calculate the thrust of oil on top and bottom of the cylinder.

Sol

$$N = 200 \text{ rpm}$$

$$h = \frac{\omega^2 r^2}{2g_{\text{eff}}} = \frac{\omega^2 r^2}{2g}$$

$$P = \rho_{\text{oil}} g_{\text{eff}} \cdot h$$

$$f = P \cdot dA = \rho_{\text{oil}} g \frac{\omega^2 r^2}{2g} \cdot (2\pi r \cdot dr)$$

Total force,

$$F = \int f = \rho_0 \pi \omega^2 \int_0^R r^3 \cdot dr$$

Top element $F_t = \rho_0 \pi \omega^2 \frac{1}{4} R^4$

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$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s.}$$

$$h = \frac{\rho_0}{\rho_{\text{oil}}}$$

$$F_t = 0.8 \times 1000 \times \pi \times (20.94)^2 \times \frac{0.2^4}{4}$$

$$F_t = 440.81 \text{ N}$$

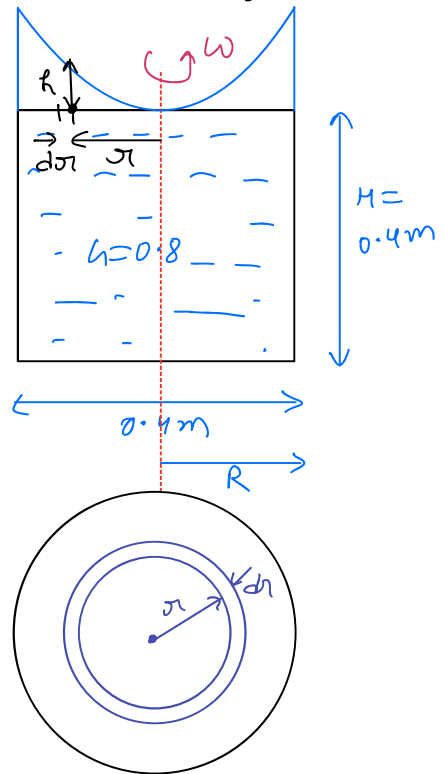
Now, at the bottom.

The radial pressure distribution will remain constant for both top and bottom. only an additional thrust due to weight will come.

$$F_b = F_t + \rho_0 g H \times \text{Area}$$

$$= 440.81 + 0.8 \times 1000 \times 9.81 \times 0.4 \times (\pi \times 0.2^2)$$

$$F_b = 835.29 \text{ N} \quad \checkmark$$



Jul 27

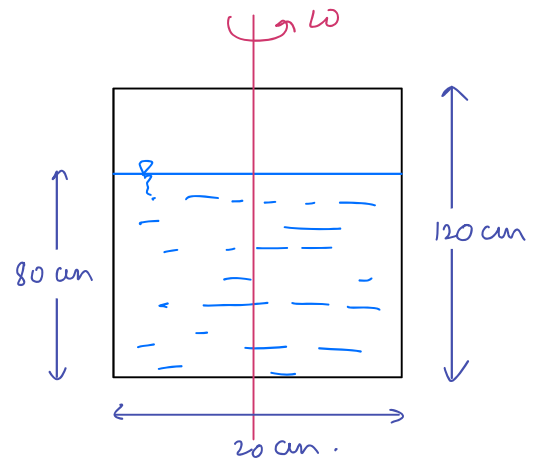
Q) For the given system, find the height of paraboloid in the tank. $N = 400 \text{ rpm}$

Sol =
$$h = \frac{\omega^2 r^2}{2g_{\text{eff}}} \quad \omega = \frac{2\pi \times 400}{60}$$

$$h = \frac{41.88^2 \times 10^2}{2 \times 981 \text{ cm/s}^2} = 41.88 \text{ rad}$$

$$h = 89.4 \text{ cm.}$$

space available = $2 \times (120 - 80) = 80 \text{ cm.}$



Hence it will keep on increasing down ward.

Let the height be z .

Total vol. of air before rotation = Total vol. of air after rotation.

$$\pi \times R^2 \times 40 = \frac{1}{2} \times (\pi \times r^2 \times z)$$

$$z = \frac{2R^2 \times 40}{r^2} = 2 \times \frac{100 \times 40}{r^2}$$

$$z = \frac{8000}{r^2} \quad \text{--- (i)}$$

Also,

$$z = \frac{\omega^2 r^2}{2g} = \frac{41.88^2}{2 \times 981} \times r^2$$

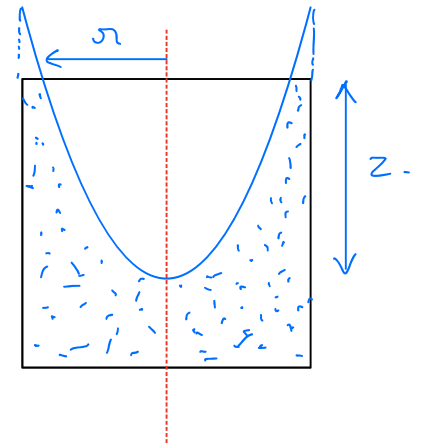
$$z = 0.89 r^2 \quad \text{--- (ii)}$$

from (i) & (ii)

$$0.89 r^2 = \frac{8000}{r^2}$$

$$r = 9.73 \text{ cm}$$

$$z = 84.50 \text{ cm.}$$



- Q) A cylindrical vessel 12cm dia and 30cm deep is filled with water up to top. Find the quantity of water left $N = 600 \text{ rpm}$.

$$\text{Sol} \quad z = \frac{\omega^2 R^2}{2g}, \quad \omega = \frac{2\pi \times 600}{60} = 20\pi$$

$$z = \frac{(20\pi)^2 \times 6^2}{2 \times 981} = 72.43 \text{ cm.}$$

which is more than height of the cylinder.

Now, Vol. of water present

$$= \pi \times R^2 \times H - \left[\frac{1}{2} \times \pi R^2 z \right] + \left[\frac{1}{2} \pi r^2 \cdot (z-H) \right]$$

$$\text{Also, } z-H = \frac{\omega^2 r^2}{2g}$$

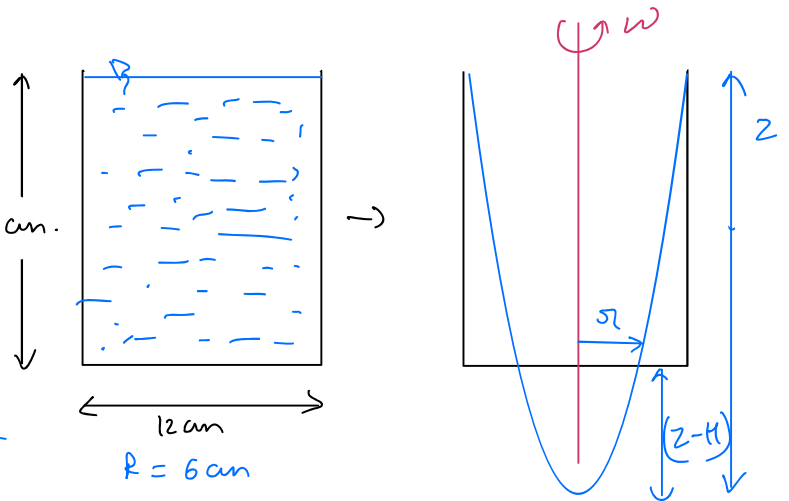
$$r = \sqrt{\frac{2g(z-H)}{\omega^2}} = \sqrt{\frac{2 \times 981 \times (72.43 - 30)}{(20\pi)^2}}$$

$$r = 4.59 \text{ cm.}$$

$$\begin{aligned} \text{Vol.} &= \pi \times 6^2 \times 30 - \left(\frac{\pi \times 6^2 \times 72.43}{2} \right) + \left(\frac{1}{2} \times \pi \times 4.59^2 \times (72.43) \right) \\ &= 707.4 \text{ cm}^3 \end{aligned}$$

- Q) A closed cylindrical tank 1m in diameter and 2m high, contains water up to 1.5m; when at rest. It is rotated about its longitudinal axis. Find

- Angular speed when water just touches the top.
- Depth of water at the centre if the tank is rotated at 127 rpm .
- Angular speed when axial depth is zero.
- Area of base exposed when the tank is rotated at 190 rpm .



sol
 $R = 0.5 \text{ m}$
 $H = 2 \text{ m}$

(a) when water will just touch the top

$$z = 2 \times (2 - 1.5) = 1 \text{ m}$$

$$z = \frac{\omega^2 R^2}{2g}$$

$$\omega^2 = \frac{2 \times 9.81 \times 1}{0.5^2}$$

$$\omega = 8.85 \text{ rad/s.}$$

$$\omega = \frac{2\pi \times 127}{60} = 13.3 \text{ rad/s}$$

(b) $z = \frac{13.3^2 \times 0.5^2}{2 \times 9.81} = 2.25 \text{ m}$

Space available = 1m.

Hence top surface will come down ward.

$$h = \frac{\omega^2 r^2}{2g}$$

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$$h = \frac{(13.3)^2 \times r^2}{2 \times 9.81} = g r^2 \quad \text{--- (i)}$$

now, As the container is closed, Hence.

Vol. of air before rotation = Vol. of air after rotation

$$\pi R^2 \cdot 0.5 = \frac{1}{2} \pi r^2 \cdot h$$

$$h = \frac{2 \times 0.5 \times 0.5^2}{r^2}$$

$$h = \frac{0.25}{r^2} \quad \text{--- (ii)}$$

putting value in eqn (i)

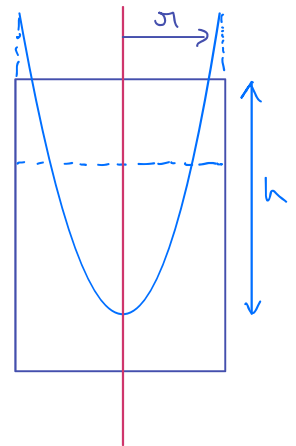
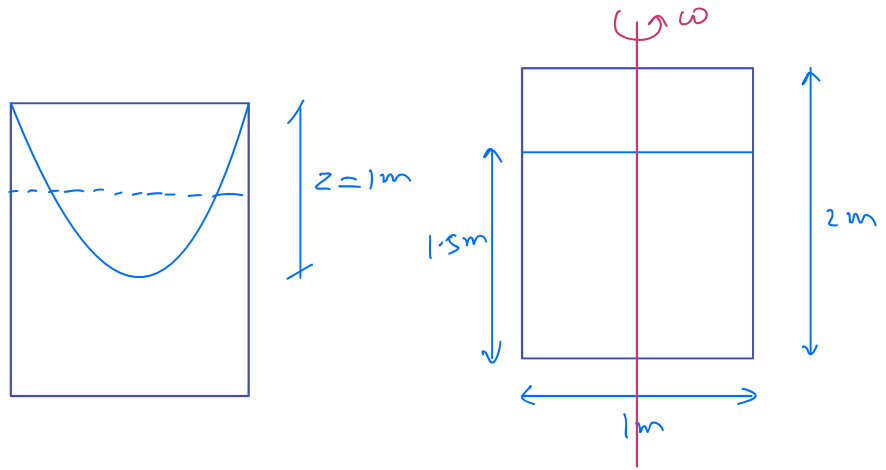
$$\frac{0.25}{r^2} = g r^2$$

$$r^4 = \frac{0.25}{g}$$

$$r = 0.408 \text{ m}$$

$$\therefore h = \frac{13.3^2 \times 0.408^2}{2 \times 9.81} = 1.5 \text{ m}$$

Hence water depth at centre = $2 - 1.5 = 0.5 \text{ m}$



(c) Angular speed when axial depth = 0

$$h = \frac{\omega^2 r^2}{2g}$$

$$2 = \frac{\omega^2 r^2}{2g}$$

$$\omega^2 r^2 = 4g \quad \text{--- (i)}$$

vol. of air before = vol. of air after rotation.

$$\pi R^2 \times 0.5 = \frac{1}{2} \times \pi r^2 \times 2$$

$$r^2 = R^2 \times 0.5$$

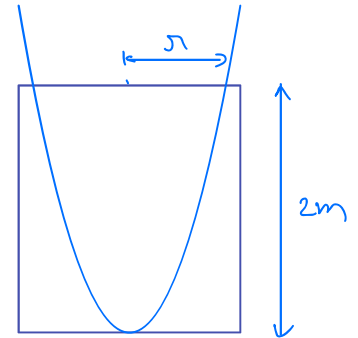
$$r^2 = 0.5^3$$

$$r = 0.353 \quad \text{--- (ii)}$$

$$\therefore \omega^2 \times (0.353)^2 = 4 \times 9.81$$

$$\omega = 17.74 \text{ rad/s}$$

$$N = \frac{17.74 \times 60}{2\pi} = 169.45 \text{ RPM.}$$



(d) Area of base exposed $N = 190 \text{ rpm}$.

$$h = \frac{\omega^2 r^2}{2g}$$

$$\omega = \frac{2\pi \times 190}{60}$$

$$= 19.9 \text{ rad/s}$$

$$h = \frac{19.9^2 \times r^2}{19.62}$$

$$h = 20.18 r^2 \quad \text{--- (i)}$$

Also,

(Va) before = (Va) after.

$$\pi R^2 H = \frac{1}{2} \times \pi r^2 h - \pi r_1^2 (h - H) \quad \text{--- (ii)}$$

Also,

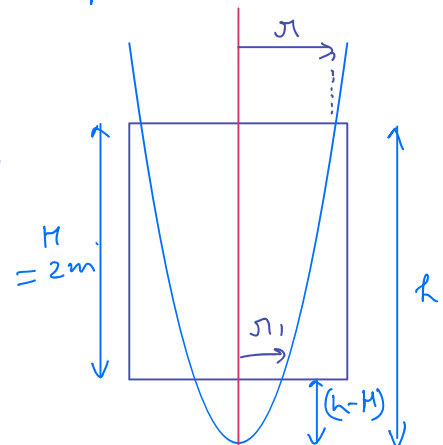
$$h - H = \frac{\omega^2 r_1^2}{2g}$$

$$\frac{2g (20.18 r^2 - H)}{\omega^2} = r_1^2 \quad \text{or} \quad r_1^2 = 0.495 (20.18 r^2 - 2)$$

putting this value in eqn (ii)

$$\pi \times 0.5^2 \times 2 = 0.5 \times \pi \times r^2 \times 20.18 r^2 - \pi \times 0.495 (20.18 r^2 - 2) (20.18 r^2 - 2)$$

$$1.57 = 36.7 r^3 - (31.38 r^2 + 3.11) (20.18 r^2 - 2)$$



$$1.57 = 36.7 r^3 - 633.24 r^4 + 62.76 r^2 + 62.75 r^2 - 6.22$$

$$633.24 r^4 - 36.7 r^3 - 125.5 r^2 + 7.79 = 0$$

on solving $r_1 = 11.3 \text{ cm}$

$$\text{Area exposed} = 3.14 \times 11.3^2$$

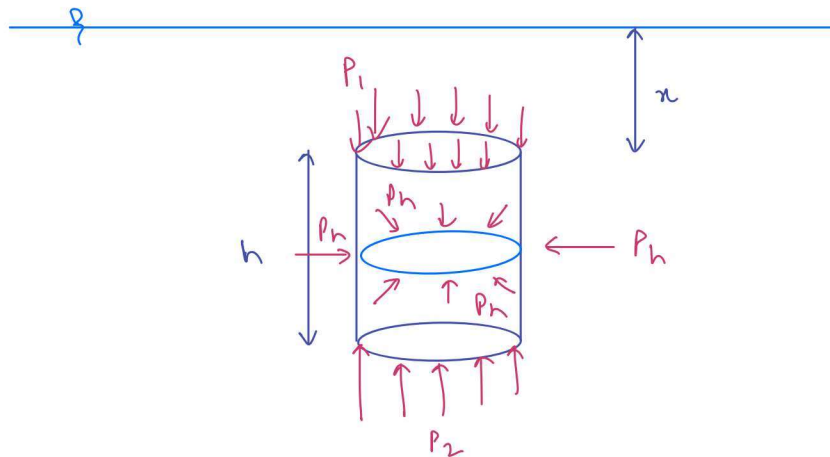
$$= 401 \text{ cm}^2$$

Souvik
2021

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Buoyancy and floatation

BUOYANCY



$$P_1 = \rho_w x$$

$$F_1 = P_1 \cdot A$$

$$P_2 = \rho_w (x+h)$$

$$F_2 = \rho_w (x+h) \times A$$

Net force in ^{Souvik} upward direction, $F_{net} = F_2 - F_1$

$$= \rho_w (h+x)A - \rho_w xA$$

$$F_{net} = \rho_w hA$$

Here $A \cdot h = \text{volume of body} = \text{vol. of liquid displaced}$.

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$\therefore F_{net} = \text{wt. of liquid displaced} \Rightarrow \text{Archimedes principle.}$

Buoyant force = Net upward force. = wt. of liquid displaced.

July 29

- For a submerged or floating body the resultant force exerted on the body by static fluid is called "buoyant force"
- Buoyant force always act in vertically upward direction and has no horizontal component.
- As per "Archimedes principle", this Buoyant force is equal to the weight of liquid displaced.
- point of application of this force at the ch of the displaced liquid and is termed as "CENTRE OF BUOYANCY".

FLOATATION :-

- A body floats in a liquid when its weight is equal to the weight of liquid displaced by the submerged portion of the body termed as buoyant force.

From static equilibrium condⁿ.

$$W - F_B = 0$$

$$W = F_B$$

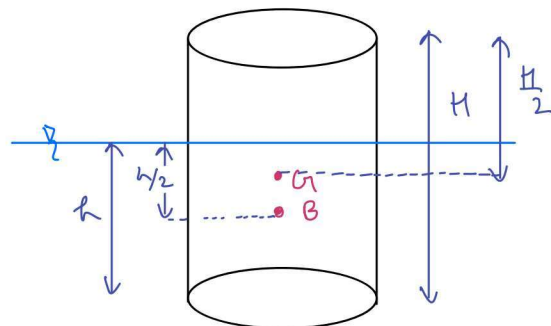
$$V \gamma_s = V_d \times \gamma_w$$

V = vol. of body

V_d = vol. of liquid displaced = vol. of submerged portion.

$$A H \gamma_s = A h \gamma_w$$

$$h = \frac{H \gamma_s}{\gamma_w}$$



NOTE:- The above expression is only valid when, $A = \text{const}$ along the height (H) .

- Q) A metallic body floats at the interface of mercury ($\gamma = 13.6$) and water such that 30% of its volume is submerged in mercury and 70% in water. Find the density of the metal.

Sol

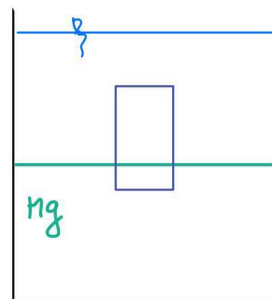
$$W = F_{B_{\text{water}}} + F_{B_{\text{Hg}}}$$

$$\gamma_s V = \gamma_w (0.7 V) + 13.6 \gamma_w (0.3 V)$$

$$\rho_s = \rho_w \times 0.7 + 13.6 \times \rho_w \times 0.3$$

$$\rho_s = 1000 \times 0.7 + 13.6 \times 1000 \times 0.3$$

$$\rho_s = 4780 \text{ kg/m}^3$$



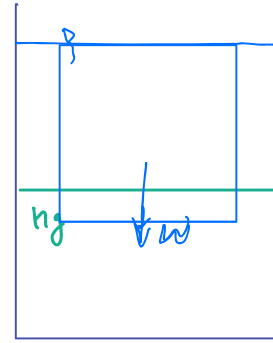
- Q) A metallic cube 30cm side and of weight 450 (N) is lowered into a tank containing two fluid layers of water and mercury. Top edge of the cube is at water surface. Determine the position of block, at water-mercury interface, when it has reached equilibrium.

sol $W - F_{Bw} - F_{Bhg} = 0.$

Let, $x\%$ of block be at water
so $(100-x)$ in the hg.

$$F_{Bw} = \gamma_w \times \left(\frac{x}{100}\right) V = 9810 \times 0.3^3 \times \frac{x}{100}$$

$$= 2.648x$$



$$F_{Bhg} = \gamma_w \times \left(\frac{100-x}{100}\right) V = 13.6 \times 9810 \times \left(\frac{100-x}{100}\right) \cdot V$$

$$= 36.022 (100-x)$$

$$450 = 2.648x + 36.022 (100-x)$$

$$450 = 2.648x + 3602.2 - 36.022x$$

$$33.37 \overset{\text{Spvik}}{\cancel{402x}} = 3152.2$$

$$x = 94.45\%$$

i.e. 94.45% of vol. is in water.

As Area is const = 0.3×0.3

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So, 94.45% height will be in water.

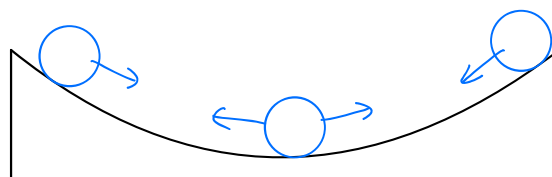
Height in water $h_w = 28.33 \text{ cm}$

Height in hg $h_{hg} = 1.66 \text{ cm}.$

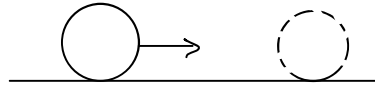
STABILITY OF SUBMERGED & FLOATING BODY

- stability of the body is classified as follows.

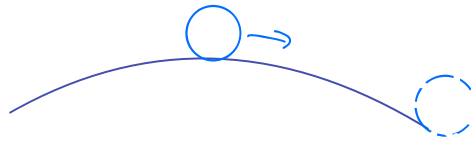
(a) Stable equilibrium - when body is displaced from its original position and it tends to come back to its original position it is said to be in STABLE EQUILIBRIUM



(b) Neutral Equilibrium :- when body is displaced from its original position, and if it remains to stay in its original position, then it is called neutral equilibrium.



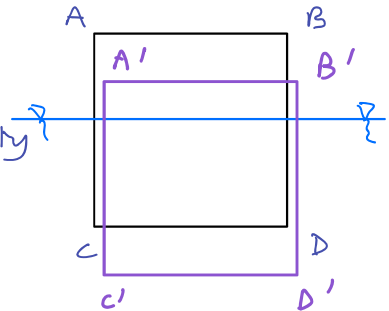
(c) UNstable Equilibrium :- when body is displaced from its original position and it continues to move away, it is said to be in unstable equilibrium.



NOTE :- STABILITY of the body can also be classified ^{Souvik} ~~2021~~

(a) LINEAR STABILITY :-

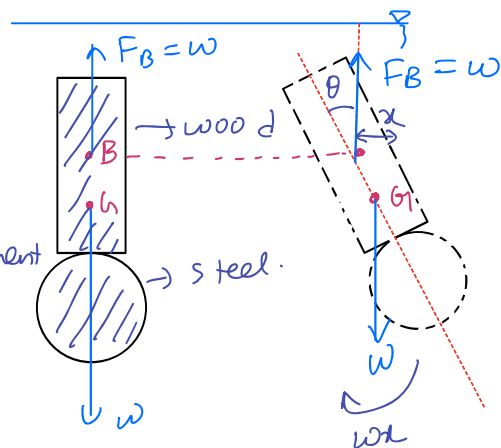
- When small linear displacement sets up restoring force, the body is said to have linear stability.



(b) Rotational Stability :-

- G is below B , this is possible when most of the weight is concentrated below.

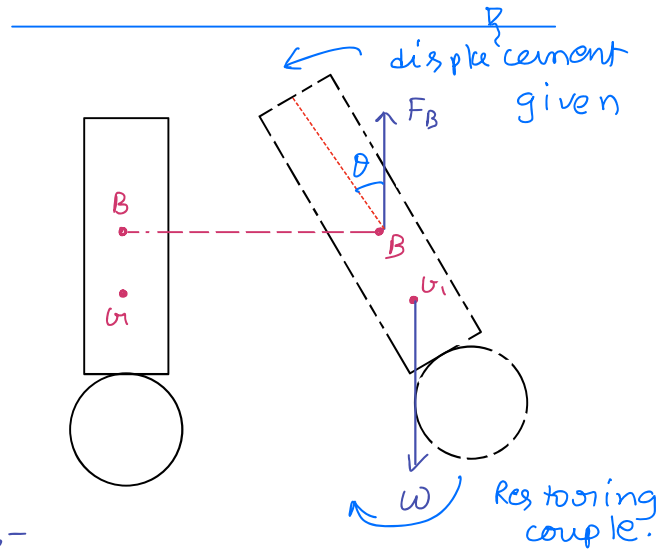
- when small angular displacement sets up restoring couple, the stability is called rotational stability.



ROTATIONAL STABILITY OF completely submerged body.

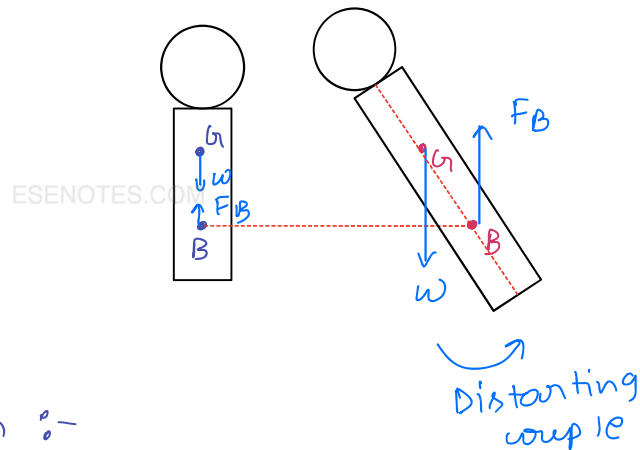
(a) Stable Equilibrium :-

For stable equilibrium G should be below B .
to get the restoring couple.



(b) Unstable Equilibrium :-

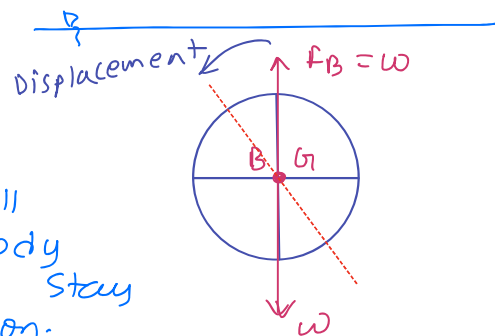
For unstable equilibrium G must be above B .



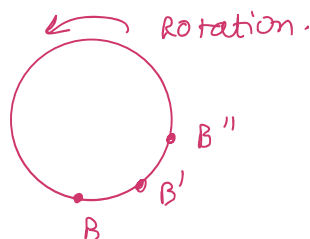
(c) Neutral Equilibrium :-

- For neutral equilibrium G must coincide with B .

- No restoring couple will generate, hence body will continue to stay in its new position.

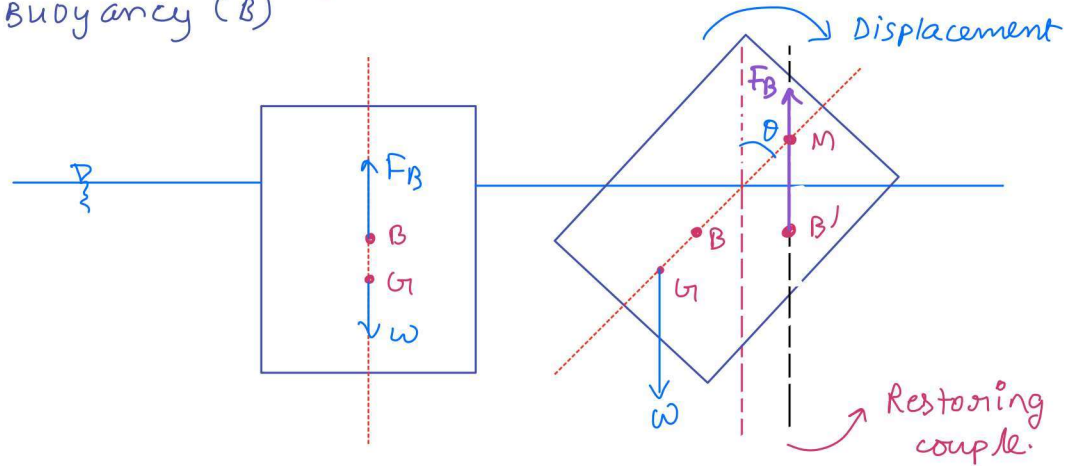


NOTE:- B and B' will not be on same line, if rotation is given.



ROTATIONAL STABILITY OF FLOATING BODY :-

- A floating body will be in stable equilibrium against rotation if centre of gravity (G) is below the centre of buoyancy (B)



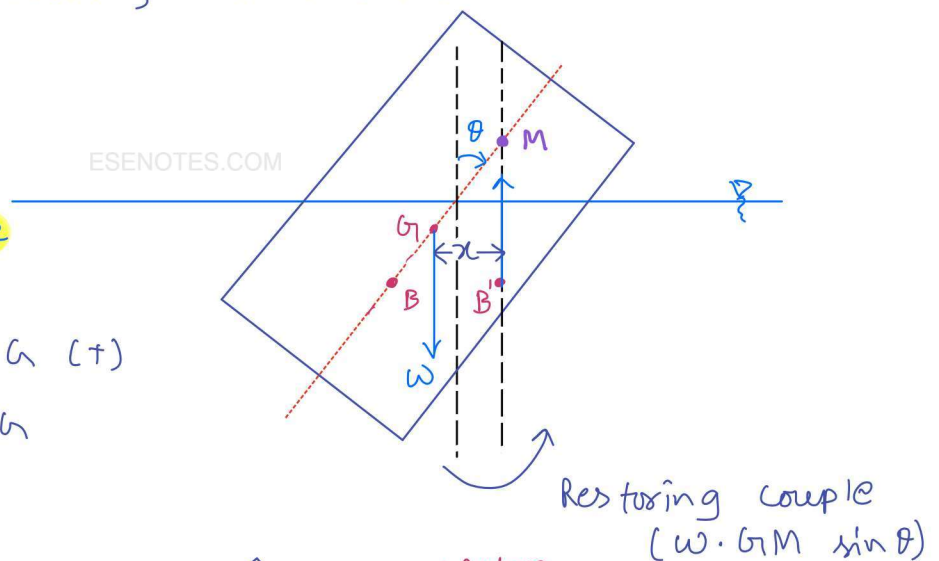
- A floating body can remain in stable equilibrium against rotation even if " B " is below " G " unlike submerged body (which becomes rotationally unstable) in following situation.

case (a) :-

If M lies above G , restoring couple develops and **stable** equilibrium exists.

$$GM = BM - BG (+)$$

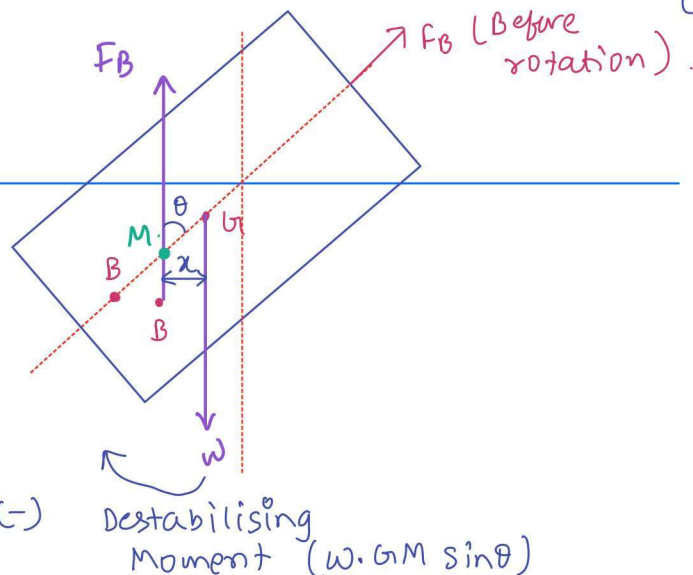
condition $\Rightarrow BM > BG$
for stable



case (b) :-

If M is below G , destabilising couple generates and **UNSTABLE** equilibrium exists.

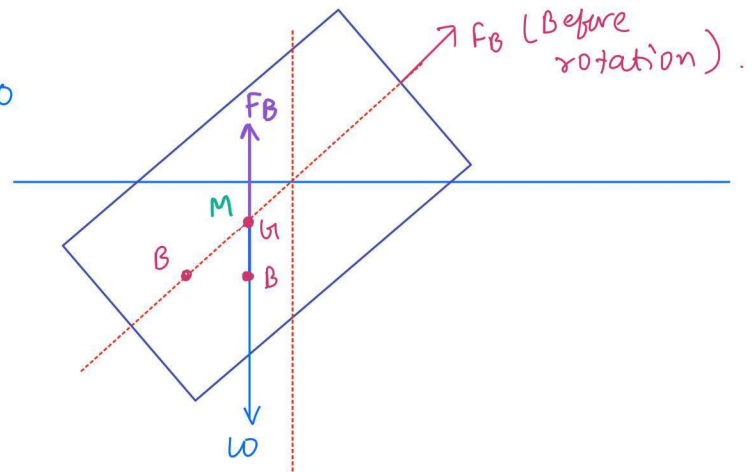
condition :- $BM < BG$
 $GM = BM - BG (-)$



case (c) :-

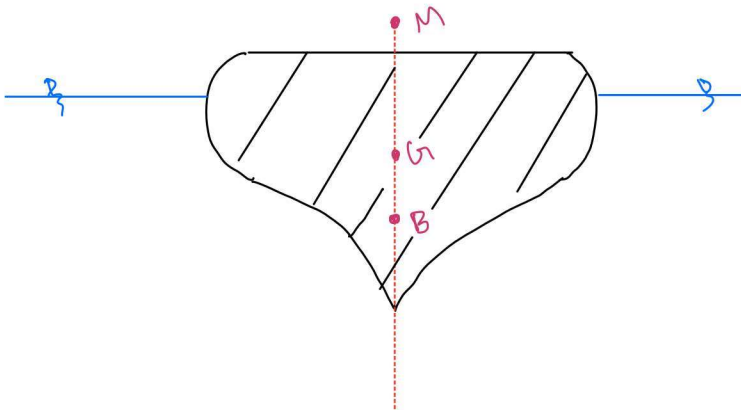
If M and G coincide, no moment will generate and develops **NEUTRAL** Equilibrium.

condition :- $\begin{cases} BM = BG \\ GM = 0 \end{cases}$



NOTE :-

- Here " M " is termed as Meta centre, which is the point of intersection of line of action of buoyant force before and after rotation.
- Height GM is called "metacentric height"
Souvik 2021
 $GM = BM - BG$.
- If body is rotated, displacement of liquid, in which it is floating on one side increases (in the direction of rotation) and on the other side decreases (in the direction opposite to rotation), due to which centre of buoyancy is displaced in the direction of rotation.
- Position of " M " changes with angular rotation (θ), however for **HULL SHAPES OF THE SHIPS**, position of M is considered to be fixed for small rotation of angle up to 20° .



- As restoring couple is $w \cdot GM \sin \theta$, larger is the meta centric height more will be the stability.

CALCULATION OF META CENTRIC HEIGHT :-

$$GM = BM - BG$$

- BG can be computed once location of centre of gravity and centre of buoyancy is known.
- BM depends on axis of rotation of floating body and is given by.

$$BM = \frac{I}{V}$$

- height of elementary area

$$= x \tan \theta \quad (\theta = \text{small})$$

$$dh = x \theta$$

- volume of element considered

$$dv = dA \times dh$$

$$= dA \cdot x \theta$$

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- Buoyancy force on this element

$$dF = (dA \cdot x \theta) \cdot \gamma$$

$$= x \theta dA \cdot \gamma$$

Moment of this force, about axis of rotation = $dF \cdot x$

$$\text{Total moment, } M = \int x \cdot dF$$

$$M = \int x \cdot x \theta \gamma dA$$

$$M = \gamma \theta \int x^2 dA$$

$$M = \gamma \theta I_{yy}$$

For rotational stability,

$$\gamma \theta I_{yy} = F_B \times BB'$$

$$\gamma \theta I_{yy} = \gamma V BM \theta$$

V = vol. of liq. displaced

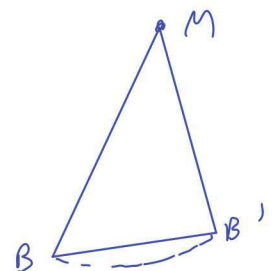
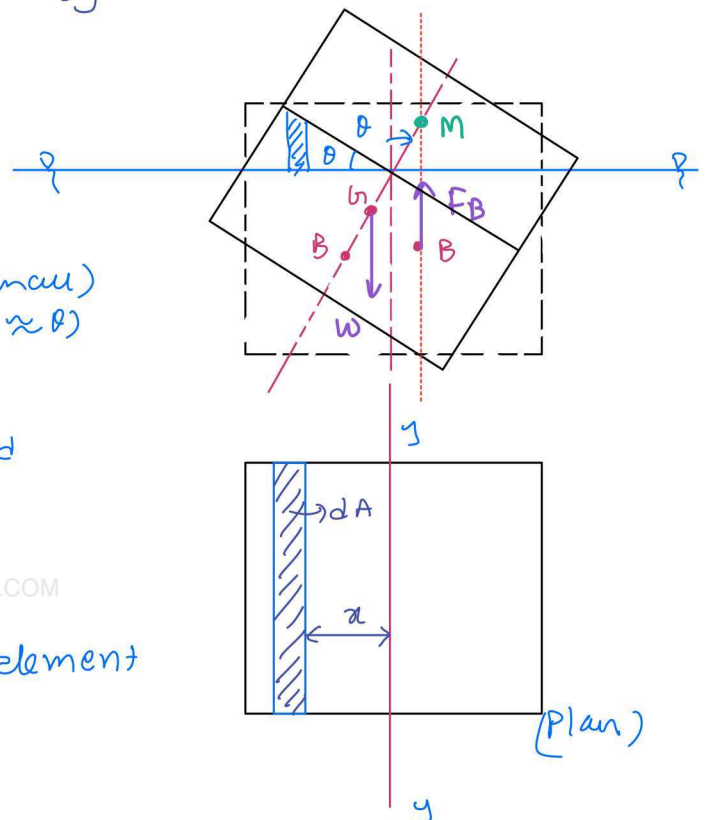
I = MOI about

rotation axis.

(Plan area) at

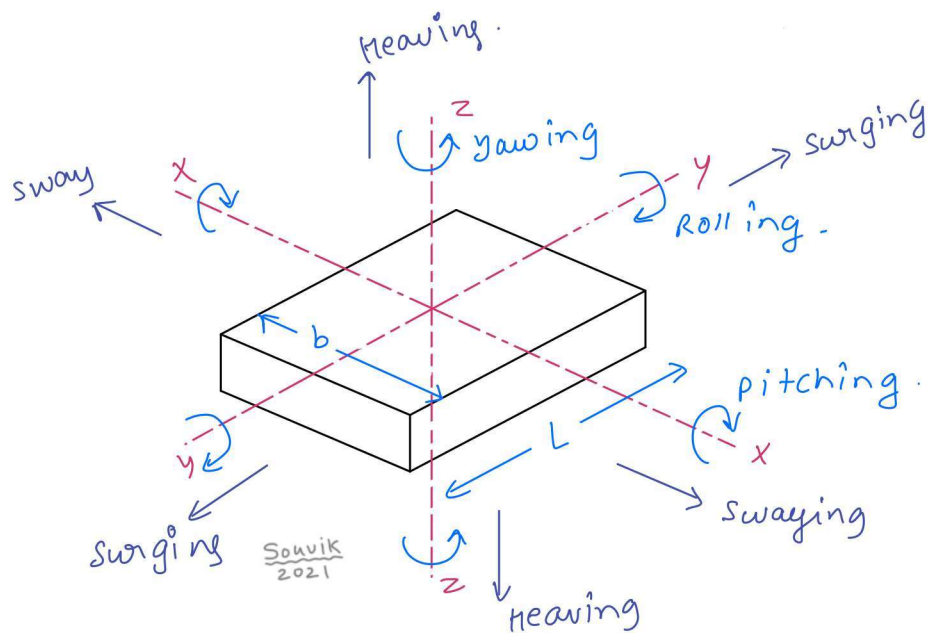
$$BM = \frac{I_{yy}}{V}$$

the level of liquid surface.



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NOTE:- Different movements or degree of freedom of ship.



Now,

$$BM = \frac{I}{V}$$

For pitching :- $BM = \frac{I_{xx}}{V} = \frac{b L^3}{12 V}$

For rolling :- $BM = \frac{I_{yy}}{V} = \frac{L b^3}{12 V}$

Also, For Rotational stability, $GM > 0$.

$$GM = BM - BG > 0.$$

Also, higher the value of GM , higher is rotational stability.

For higher GM , BM should be higher.

$$\text{Since } L > b, \quad I_{xx} > I_{yy}$$

$$(BM)_{\text{pitching}} > (BM)_{\text{rolling}}$$

$$(GM)_{\text{pitching}} > (GM)_{\text{rolling}}$$

Hence Rolling governs the rotational stability of the ship (floating body).

If ship is safe in rolling, it will be automatically safe in pitching also.

- Q) A homogenous circular cylinder of length "h", radius "r" and $h = "s"$, floats in water. It is noted that $r = \frac{2h}{3}$. Under what condition will the flotation be unstable.

Sol

$$r = \frac{2h}{3}$$

For floating

$$Wt = F_B$$

$$S r w \times (\pi r^2 h) = r w (\pi r^2 y)$$

$$S h = \frac{Sowik}{2029} - (i)$$

For unstable equilibrium (rotational).

$$GM < 0$$

$$BM - BG \leq 0 \rightarrow \text{critical condition.}$$

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$$\text{i.e. } BM \leq BG$$

$$OG = \frac{h}{2}, \quad OB = \frac{y}{2}, \quad BG = OG - OB = \frac{h-y}{2}$$

$$BM = \frac{I}{V} = \frac{\pi d^4}{64 \cdot \pi r^2 y} = \frac{\pi 16 r^4}{64 \pi r^2 y}$$

$$BM = \frac{r^2}{4y}$$

$$\therefore \frac{r^2}{4y} \leq \frac{h-y}{2}$$

$$\frac{r^2}{4 \times h s} \leq \frac{h-hs}{2}$$

$$\frac{\left(\frac{2h}{3}\right)^2}{4hs} \leq \frac{h(1-s)}{2} \Rightarrow \frac{4h^2}{9 \cdot (4hs)} \leq \frac{h(1-s)}{2}$$

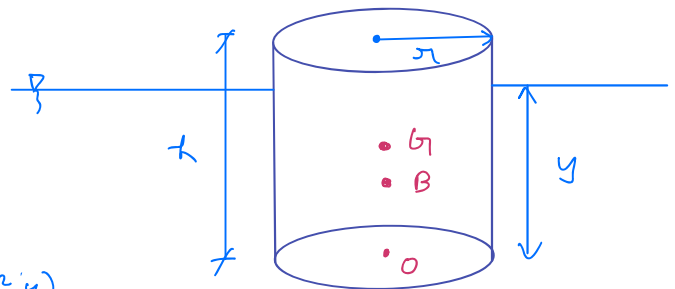
$$\Rightarrow \frac{2}{9} \leq s(1-s)$$

$$s - s^2 \geq \frac{2}{9}$$

$$9s - 9s^2 \geq 2$$

$$9s^2 - 9s + 2 \leq 0$$

$$s = \frac{1}{3} \text{ and } \frac{2}{3}$$



$\therefore s$ should be between, $\left(\frac{1}{3} \text{ to } \frac{2}{3}\right)$
For rotational instability.

Hence, for stability $s > \frac{2}{3}$ or $s < \frac{1}{3}$.

TIME PERIOD OF OSCILLATION :-

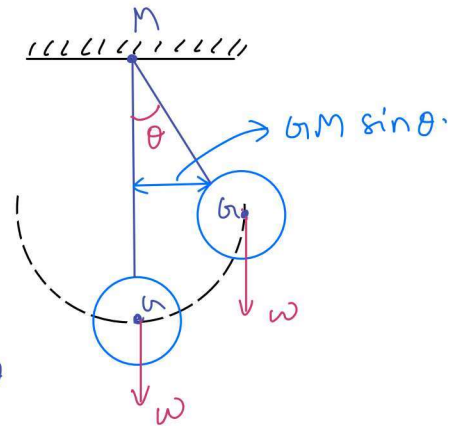
- A ship can be considered as a pendulum with centre of rotation as metacentre.

For SHM :-

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$T = \frac{2\pi}{\omega}$$

α = angular acceleration



$$\text{Restoring couple/moment} = W \cdot GM \sin\theta = I\alpha$$

$\alpha \rightarrow$ angular acc.

$$\alpha = \frac{d^2\theta}{dt^2}$$

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$$I\alpha = W GM \sin\theta$$

$$\theta \approx \sin\theta \approx \tan\theta$$

$$I \frac{d^2\theta}{dt^2} = -W(GM)\theta$$

Restoring moment is in opposite direction of angular disp. (θ - A.C.W)

$$\frac{d^2\theta}{dt^2} = -\frac{W(GM)\theta}{I}$$

$$\text{also, } \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\therefore \omega^2 = \frac{W(GM)}{I}$$

$$T = \frac{2\pi}{\sqrt{\frac{W(GM)}{I}}}$$

$$\text{or } 2\pi \sqrt{\frac{I}{W(GM)}}$$

conclusion :-

- comfort of the ship is more when time period is more.
- Hence for greater comfort for given I , GM should be less.
- But smaller GM would decrease rotational stability of ship.
- Hence without compromising rotational stability if time period is to be increased, I must be increased (by making large ship).
- Hence, for passenger ship GM is less and balance between stability and comfort is achieved.

And for cargo ^{Souvik 2021} ships GM is more, as stability is prime criteria.

- 8) A cone floating in the water with its apex down has diameter " D " and vertical height " H ", if the $b = S$, then for stable equilibrium show that

$$H^2 < \frac{D^2 S^{1/3}}{4(1-S^{1/3})}$$

Sol For floating.

$$F_B = \text{weight}$$

$$\rho_w \left(\pi \left(\frac{d}{2} \right)^2 h \right) = \rho_w \left(\pi \left(\frac{D}{2} \right)^2 \cdot H \right)$$

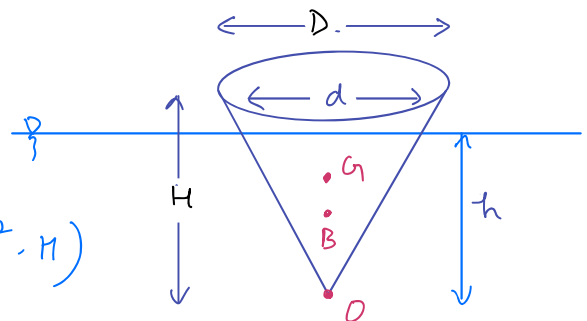
$$d^2 h = S D^2 H$$

$$\text{Also, } \frac{D}{H} = \frac{d}{h}, d = D \frac{h}{H}$$

$$\left(D \frac{h}{H} \right)^2 \cdot h = S D^2 H$$

$$h^3 = S H^3$$

$$h = S^{1/3} H$$



For rotational stability. $GM > 0$

$$BM - BG > 0.$$

$$BM = \frac{I}{V} \rightarrow \text{at the liquid surface.}$$

$$I = \frac{\pi d^4}{64}, \quad V = \frac{\pi d^2}{4} \cdot \frac{4}{3}$$

$$BM = \frac{\pi d^4 \cdot 12}{64 \cdot \pi d^2 h} = \frac{3d^2}{16h} = \frac{3}{16h} D^2 \frac{h^2}{h^2}$$

$$BM = \frac{3}{16} \cdot \frac{D^2 h}{h^2} = \frac{3}{16} \cdot \frac{D^2}{h^2} s^{1/3} h$$

$$BM = \frac{3}{16} \cdot \frac{D^2 s^{4/3}}{h}$$

For BG :-

Souvik
2021

$$BG = OG - OB = \frac{3}{4} H - \frac{3}{4} \cdot h = \frac{3}{4} (H - h)$$

$$BG = \frac{3}{4} (H - H s^{1/3})$$

$$BG = \frac{3H}{4} (1 - s^{1/3}).$$

$$\therefore BM - BG > 0$$

$$\frac{3}{16} \cdot \frac{D^2 s^{1/3}}{h} - \frac{3H}{4} (1 - s^{1/3}) > 0$$

$$\frac{D^2 s^{1/3}}{4H} > H (1 - s^{1/3})$$

$$h^2 \leq \frac{D^2 s^{1/3}}{4(1 - s^{1/3})} \quad \checkmark \text{ proved}$$

- b) A rectangular pantoon 10m long, 8m broad, 3m deep weighs 980 kN. It carries on its upper deck a boiler 6m in diameter weighing 588 kN. Longitudinal axis of the boiler is parallel to 10m side. The CG of pantoon and boiler are at CG of respective geometries and is same vertical line. Find meta centric height ($\rho_w = 10 \text{ kN/m}^3$).

Sol For Floating:-

$$F_B = W$$

$$\rho_w (8 \times y \times 10) = 980 + 588$$

$$y = 1.96 \text{ m.}$$

For rotational stability

$$GM \geq 0.$$

$$BM - BG \geq 0.$$

$$BG = OG - OB$$

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$$OG = \frac{w_1 \times OG_1 + w_2 \times OG_2}{w_1 + w_2} = \frac{588 \times 6 + 980 \times 1.5}{980 + 588}$$

$$OG = 3.1875 \text{ m}$$

$$OB = \frac{y}{2} = 0.98 \text{ m}$$

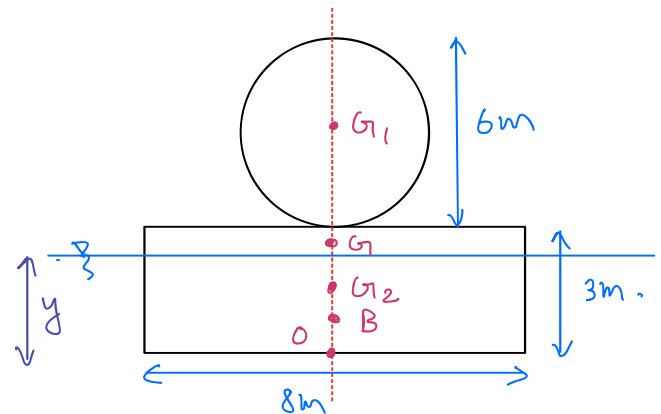
$$BG = 3.1875 - 0.98 = 2.2075 \text{ m}$$

$$BM = \frac{I_{yy}}{V} \quad I_{yy} = \frac{10 \times 8^3}{12}, \quad V = y \times 10 \times 8$$

$$BM = \frac{10 \times 8^3}{12 \times 1.96 \times 10 \times 8} = 2.42 \text{ m}$$

$$\therefore GM = 2.42 - 2.2075 = 0.5125 \text{ m.}$$

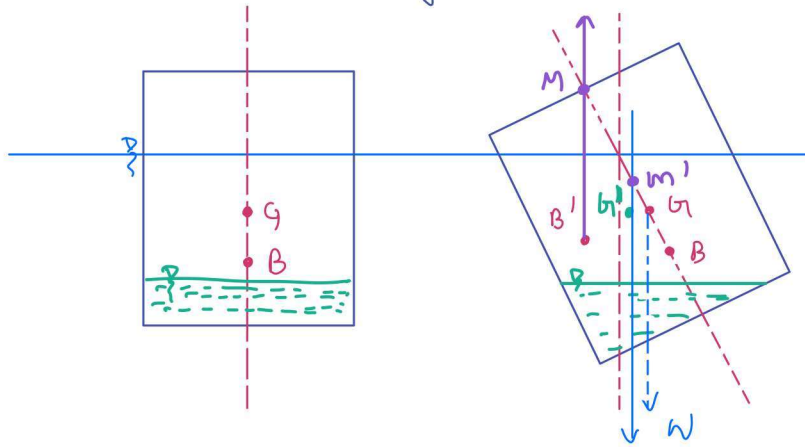
$$\therefore \text{Meta centric height} = 0.5125 \text{ m.}$$



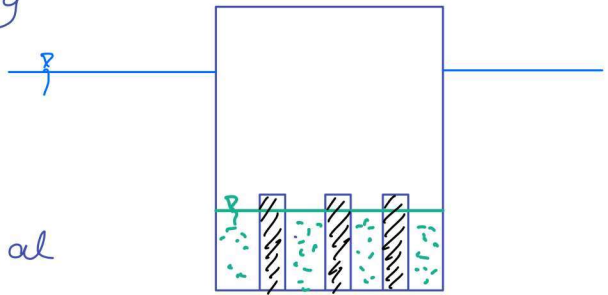
Aug 2

FLOATING BODY CONTAINING LIQUID WITH FREE SURFACE

- If a floating body contains liquid with free surface its meta centric height reduces, hence it reduces its rotational stability.



- If angular displacement is given to the floating body having liquid with free surface, the centre of gravity of the body (G) also shifts to (G') that results in decrease of meta centric height ($mm' < mh$). Thus it reduces its rotational stability.



$$mm' = hM - \frac{\gamma_l I_l}{\gamma v}$$

- γ_l = unit wt. of liquid in ship / floating body,
- I_l = MOI of free surface of liquid in floating body about longitudinal axis.
- γ = unit wt. of liquid displaced by floating body.
- v = vol. of liquid displaced by floating body.

- For lesser reduction in meta centric height, liquid is placed in separate / different compartment. In such case.

$$mm' = hM - \frac{\sum \gamma_i I_i}{\gamma v}$$

- γ_i = unit wt. of lig. in i^{th} container.
- I_i = MOI of free surface about longitudinal axis of i^{th} compartment.

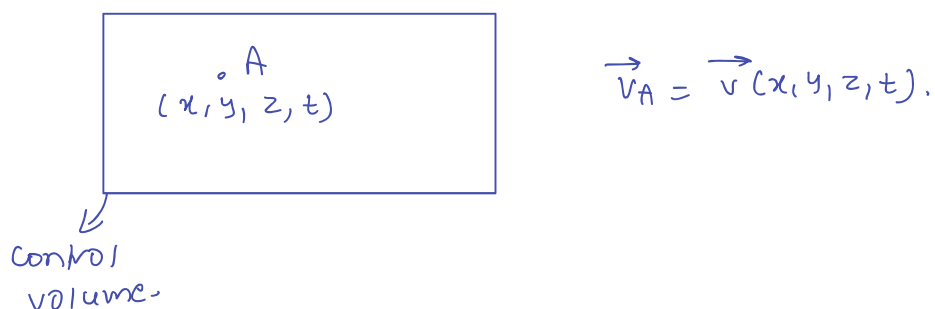
FLUID KINEMATICS

- It deals with describing the motion of the fluid without considering the forces and moments that cause the motion.
 - It deals with study of velocity acceleration and angular velocity of flow.
 - The fluid motion is described in general by two methods.
 - (a) LAGRANGIAN MTD.
 - (b) EULERIAN MTD.
- (a) In lagrangian mtd, he considered a coordinate system attached to a single fluid particle and its flow properties (velocity, acceleration) were described.

In this approach, during the fluid flow

- It is difficult to keep track of single fluid particle.
- It is difficult to describe the interaction b/w the particles.
- As the particle continues to deform and change its identity it is difficult to recognize the particle.

- (b) In Eulerian mtd., a definite volume called as "CONTROL VOLUME", through which fluid flows is considered and instead of tracking the individual fluid particle, fluid properties are described w.r.t space and time within control volume.



- In fluid mechanics, we deal with EULER'S mtd.

TYPES OF FLOW :-

(a) STEADY & UNSTEADY FLOW :-

- If fluid properties at any given location does not change wrt time, flow is termed as STEADY FLOW.

or else, if it changes with time, it is termed as unsteady flow.

Eg:- $\frac{\partial v}{\partial t} = 0$, $\frac{\partial \rho}{\partial t} = 0$, $\frac{\partial p}{\partial t} = 0$... etc. $\rightarrow$ steady flow.

(b) UNIFORM & NON-UNIFORM FLOW :-

- If fluid properties at any given instant of time does not varies with location over a space, flow it is termed as uniform flow.

- If the properties change then it is non-uniform flow.

Eg:- $\frac{\partial v}{\partial s} = 0$, $\frac{\partial \rho}{\partial s} = 0$, $\frac{\partial p}{\partial s} = 0$... $\rightarrow$ uniform flow.

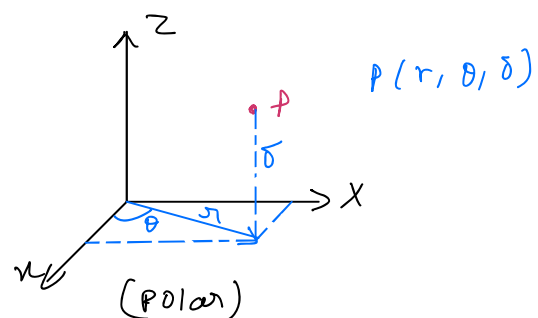
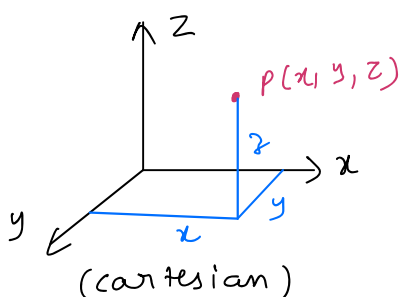
(c) ONE, TWO or THREE DIMENSIONAL FLOW :-

- If the flow properties varies in one dimension, two dim. or three dim. wrt. space only, the flow is termed as one-dime., two dim. and 3-D flow respectively.

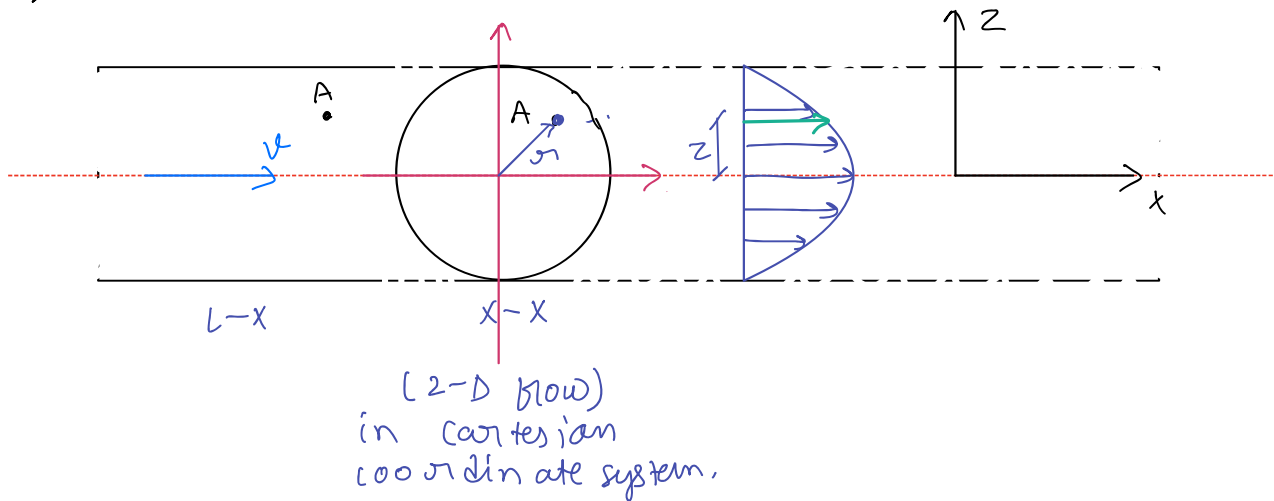
Eg:- $v = v(x, t) \rightarrow 1-D$ $v = v(x, y, t) \rightarrow 2D$.

$v = v(x, y, z, t) \rightarrow 3-D$.

NOTE:- The dimensionality of the flow depends upon choice of coordinate system and its orientation i.e. a flow may be 2D in one coordinate system and same flow may be in 1D in another coord. system.



For eg:

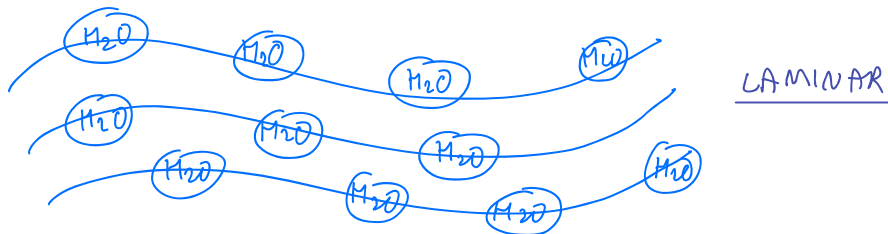


- whereas in polar coordinate system it depends only on its radius r . Hence it is one dimensional flow in cylindrical coordinate system.

Aug 3

(d) LAMINAR AND TURBULENT FLOW :-

- when fluid motion is such that one layer slides past the other layer it is termed as laminar flow.

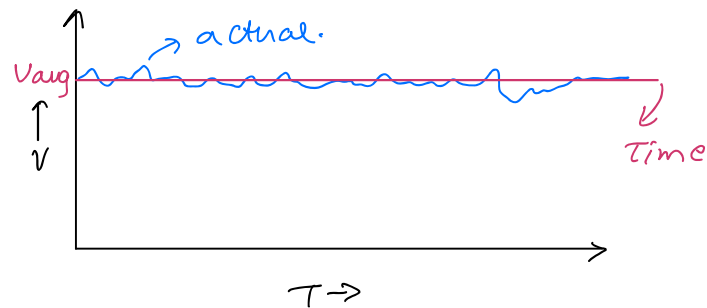
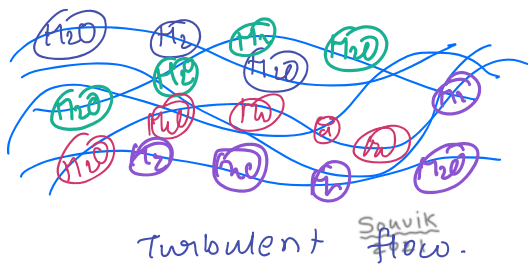


- Generally this flow occurs at low velocity
- Fluid particle of one layer does not go into another layer i.e. there is no momentum transfer.
- Flow near solid boundary is laminar.
- Flow in highly viscous liquid is laminar.

when fluid particle flows in highly random manner such that particle of one layer enters or penetrates into another layer, it is called turbulent flow.

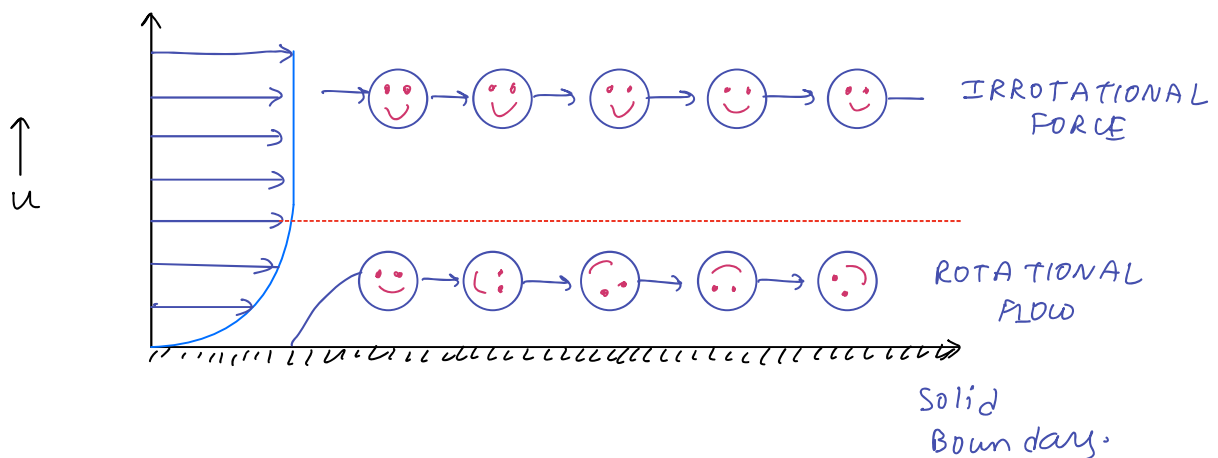
- Here momentum transfer is continuous b/w different layers.

- This flow occurs where velocity is comparatively less.
- This occurs away from solid boundary.
- Flow in less viscous fluid is turbulent flow.
- At any point in turbulent flow, vel varies with time (i.e. unsteady flow), but if vel. is averaged over a period of time (i.e. temporal mean velocity) it can be taken as steady.

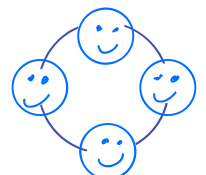
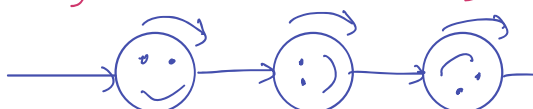


(e) ROTATIONAL & IRRATIONAL FLOW :-

- when fluid particle rotate about their CG (centre of mass) during flow it is termed as rotational flow else irrotational flow.
- Rotation of the fluid particles in a flow field is caused by viscosity, hence in viscous region flow will be rotational and where effect of viscosity is negligible, flow can be considered as irrotational.



NOTE:- Fluid moving along a straight line and fluid moving along a circle may not be rotational.



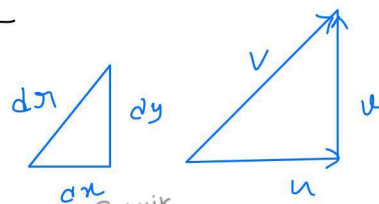
(f) compressible & incompressible flow :-

- A compressible flow is that in which density of fluid changes from point to point whereas in incompressible flow, the density of the fluid remains constant throughout the flow.

TYPES OF FLOW LINE :-

- For the visualization of the flow, certain lines are defined in flow space termed as flow line.

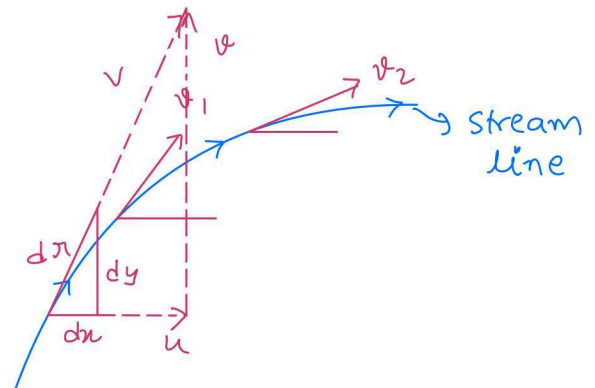
(a) STREAM LINE :-



$$\frac{dx}{u} = \frac{dy}{v} = \frac{ds}{\sqrt{u^2 + v^2}}$$

In general form :- eqn of stream line is given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

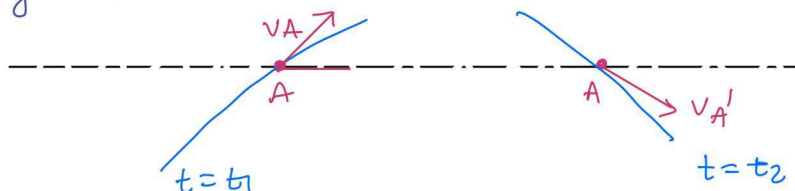


- A stream line is a curve such that tangent at any point on this curve gives the direction of instantaneous velocity.

- In 2-D :- $\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dy}{dx} = \frac{v}{u}$

- Integral of above eqn will give the eqn of stream line at any instant of time.

- Hence if flow is UNSTEADY, then stream line will vary time to time.



8) The velocity components of flow are given as

$u = -x$, $v = 2y$, $w = 5-z$, find the eqn of stream line passing through point $(2, 1, 1)$.

sol

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\int \frac{dx}{-x} = \int \frac{dy}{2y}$$

$$-\ln x = \frac{1}{2} \ln y + C$$

@ $x=2, y=1$, $-\ln 2 = \frac{1}{2} \ln 1 + C$

$$C = -\ln 2$$

$$\therefore -\ln x = \frac{\ln y}{2} - \ln 2$$

$$\ln x^2 = \ln (y/2)$$

$$x^2 = y/2$$

$$x^2 y = 2$$

$$\frac{dy}{2y} = \frac{dz}{5-z}$$

$$\frac{1}{2} \ln y = -\ln(5-z) + C_1$$

@ $y=1, z=1$,

$$\frac{1}{2} (0) = -\ln 4 + C_1$$

$$C_1 = \ln 4$$

$$\ln y^{1/2} = \ln \left(\frac{1}{5-z} \right) + \ln 4$$

$$y^{1/2} = \frac{4}{5-z}$$

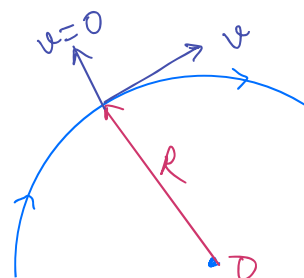
$$y = \left(\frac{4}{5-z} \right)^2$$

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NOTE:- Stream lines are set of concentric circle with origin at centres.

- There is no component of velocity $\perp$ to the streamline i.e., there will be no flow across the stream line.

- A stream line can neither intersect itself, nor any other stream line.



- The spacing b/w stream lines vary inversely with the vel. of flow.

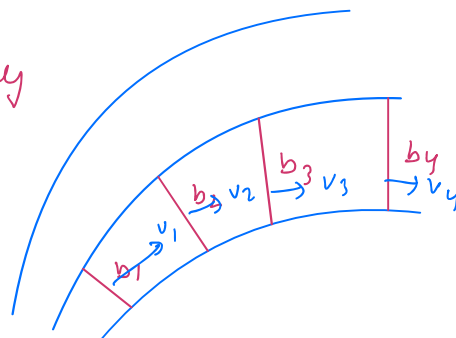
$$q = \text{const.}$$

$$q = a_1 v_1 = a_2 v_2$$

$$(b_1 \times 1) v_1 = (b_2 \times 1) v_2$$

$$v_1 = \frac{q}{b_1} \text{ and } v_2 = \frac{q}{b_2}$$

Since $b_2 > b_1$, $v_2 < v_1$



Aug 5

(b) STREAM TUBE :-

- It is a group/bundle of stream lines.

- Flow within the stream tube must remain there and cannot cross the boundary of stream tube.

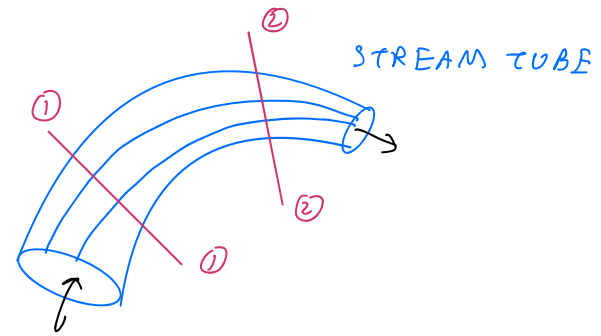
- At any instant of time, the mass flow rate passing through the section, must be constant.

i.e. $\frac{M}{t} = \text{const}$

$\frac{\rho V}{t} = \text{const}$

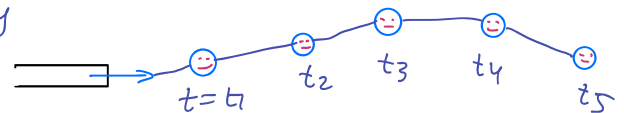
$\rho \cdot Q = \text{const}$

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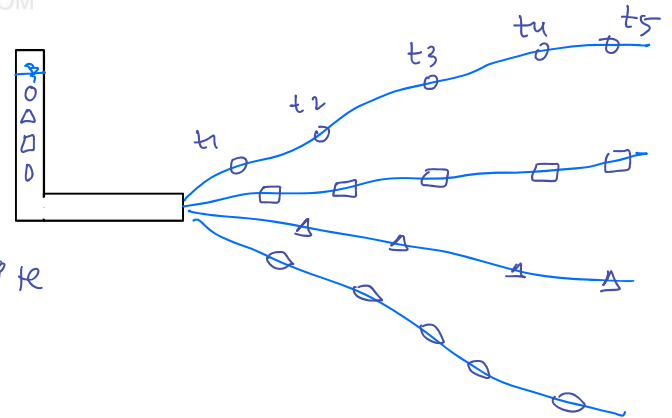
(c) PATH LINE :-

- Actual path travelled by any individual fluid particle over some period of time is called PATH LINE.



- It is a LAGRANGIAN concept.

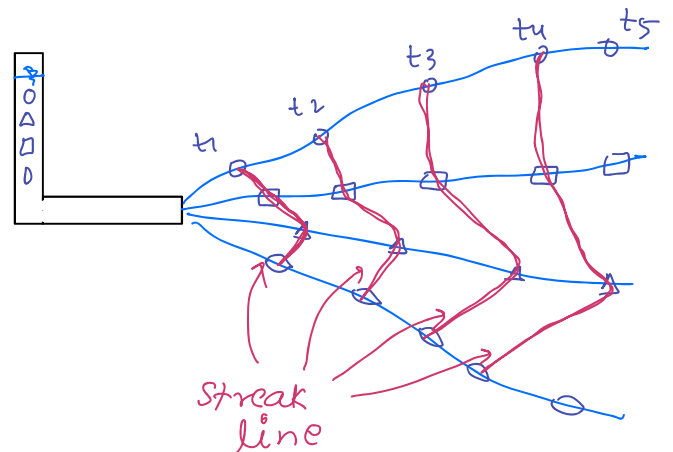
- It is not an instantaneous concept and it involves finite time period.



(d) STREAK LINE :-

- Path of the fluid particles that they have passed sequentially through a given point in flow is termed as streak line.

- It is formed by continuous introduction of dye



NOTE:-

- For steady flow, streakline, stream line and path line are same
- For unsteady flow, streakline at any instant of flow is the locus of end point of path line

CONTINUITY EQUATION :-

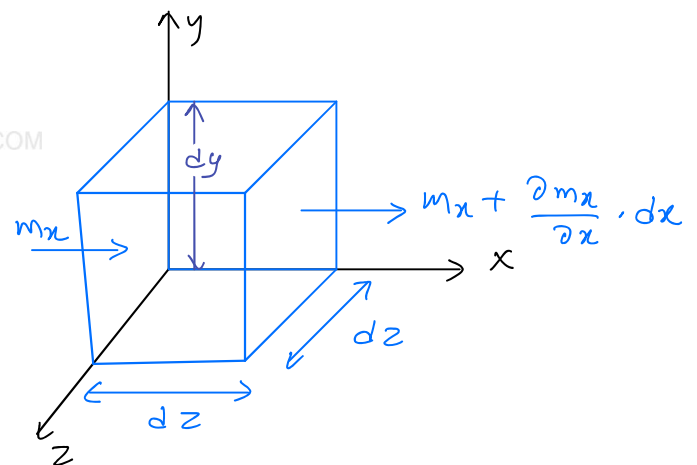
- It is based on principle of conservation of mass.
- According to this mass inflow in a fixed region = mass outflow from that region in a particular time.
- or, it can also be stated as, difference of mass inflow and mass outflow in fixed region, is equal to change in storage in that region.
- continuity eqn in cartesian coordinate system.

$$m_i - m_{out} = m_{storage}$$

In x-direction:-

$$(\dot{m}_{in} - \dot{m}_{out})_x + (\dot{m}_{in} - \dot{m}_{out})_y + (\dot{m}_{in} - \dot{m}_{out})_z = \dot{m}_{storage} \quad \text{--- (i)}$$

$$\begin{aligned} (\dot{m}_i - \dot{m}_o)_x &= \dot{m}_x - \left(\dot{m}_x + \frac{\partial \dot{m}_x}{\partial x} \cdot dx \right) \\ &= - \frac{\partial \dot{m}_x}{\partial x} \cdot dx \end{aligned}$$



mass flow rate, $\dot{m} = \rho B = \rho A v$

v = velocity.

velocity, $v = u\hat{i} + v\hat{j} + w\hat{k}$

$$\begin{aligned} \therefore &= - \frac{\partial \rho}{\partial x} dy dz \cdot u \cdot dx \\ &= - \frac{\partial \rho u}{\partial x} (dv) \end{aligned}$$

similarly, $(\dot{m}_{in} - \dot{m}_{out})_y = - \frac{\partial}{\partial y} (\rho v) dv$

$$(\dot{m}_{in} - \dot{m}_{out})_z = - \frac{\partial}{\partial z} (\rho w) dv$$

$$\text{change in storage} = \frac{\partial}{\partial t} (\rho \cdot dv)$$

from eqn (i)

$$-\frac{\partial \rho u}{\partial x} \cdot dv - \frac{\partial \rho v}{\partial y} dv - \frac{\partial (\rho w)}{\partial z} dv = \frac{\partial \rho}{\partial t} \cdot dv$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0} \quad (***)$$

↳ most generalized form of continuity eqn,

valid for all types of flow - (steady, unsteady, uniform, non-uniform, compressible, non-compressible).

For steady flow :-

$$\frac{\partial \rho}{\partial t} = 0 \quad \text{Sowvik}$$

in 3d :-

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

in 2d :-

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0$$

in 1d :-

$$(m_{in} - m_{out}) = m_{storage}$$

$$-\frac{\partial \rho (A \cdot v)}{\partial s} \cdot ds = \frac{\partial}{\partial t} (\rho A \cdot ds) \quad \left\{ s \text{ can be anything } x, y, z. \right\}$$

$$\boxed{-\frac{\partial \rho}{\partial s} A \cdot v = \frac{\partial \rho}{\partial t} \cdot A} \rightarrow \text{General eqn for 1-D.}$$

Since flow is steady. $\frac{\partial \rho}{\partial t} = 0$.

$$\frac{\partial \rho}{\partial s} A \cdot v = 0$$

Integrating.

$$\rho A v = \text{const}$$

$$\text{i.e. } \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

For steady flow, and incompressible flow $\frac{\partial \rho}{\partial s} = 0$ and $\frac{\partial \rho}{\partial t} = 0$. i.e. $\rho = \text{const}$

$$\text{in 3d :- } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{in 1d :- } A_1 v_1 = A_2 v_2$$

$$\text{in 2d :- } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

CONTINUITY EQN in CYLINDRICAL POLAR COORDINATE SYSTEM :-

In 3D :-

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{\partial (\rho v_\theta)}{r \partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

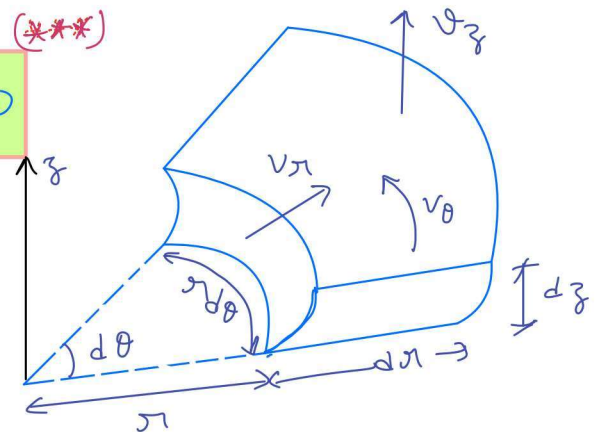
In 2D :- ($v_z = 0$) steady ($\frac{\partial \rho}{\partial t} = 0$)

In compressible flow ($\frac{\partial \rho}{\partial t} = 0$) flow.

$\rightarrow (r, \theta, z)$

$$\frac{\partial (r v_r)}{\partial r} + \frac{\partial v_\theta}{r \partial \theta} + 0 = 0$$

$$\frac{\partial r v_r}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = 0$$



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Q) If velocity components $u = \lambda xy^3 - x^2y$, $v = xy^2 - \frac{3}{4}y^4$

then value of λ for possible flow field involving steady incompressible flow would be?

Sol continuity eqn in 2D :-

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0$$

For steady flow, $\frac{\partial \rho}{\partial t} = 0$

and incompressible, hence $\rho = \text{const}$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x} (\lambda xy^3 - x^2y) + \frac{\partial}{\partial y} (xy^2 - \frac{3}{4}y^4) = 0$$

$$\lambda y^3 - 2xy + 2xy - 3y^3 = 0$$

$$\lambda = 3$$

- Q) For the flow of an incompressible the velocity component in x -direction is $u = ax^2 + by$ and the vel. comp. in z -direction $= 0$. Find the velocity comp. v in y -direction such that $v=0$ at $y=0$.

Sol vel-component in z -direction $= 0$, Hence for 2-D continuity eqn will be.

$$\frac{\partial P}{\partial t} + \frac{\partial Pu}{\partial x} + \frac{\partial Pv}{\partial y} = 0.$$

Incompressible, $P = \text{const}$

$$P \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0.$$

$$\frac{\partial}{\partial x} (ax^2 + by) + \frac{\partial v}{\partial y} = 0.$$

$$2ax + \frac{\partial v}{\partial y} = 0 \quad , \quad \frac{\partial v}{\partial y} = -2ax$$

Integrating.

$$v = -2axy + C$$

$$\text{@ } y=0, v=0. \quad \therefore C=0$$

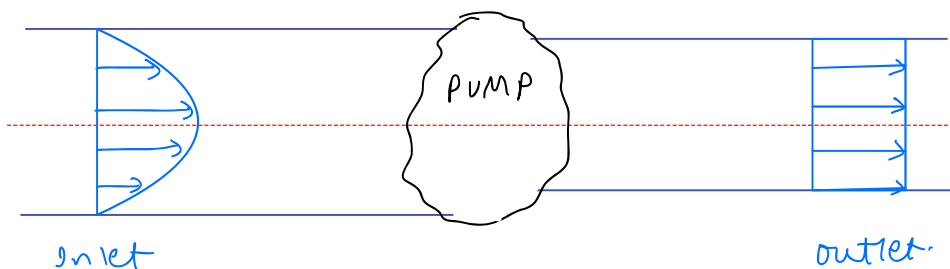
$$v = -2axy$$

- Q) water flows steadily through a pump, the velocity distribution in the inlet pipe being parabolic and that the outlet pipe being uniform. The dia of the inlet pipe is 10cm and that of the outlet pipe is 8cm. The velocity distribution in the inlet pipe is given by

$$v = 3 \left(1 - \frac{r^2}{R^2} \right) \quad \begin{array}{l} r = \text{radial distance} \\ \text{from the axis} \\ R = \text{Radius of inlet pipe} \end{array}$$

- compute the magnitude of velocity in the outlet pipe.

Sol



Since the velocity is variable in inlet pipe.

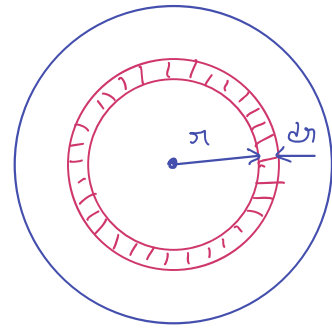
$$Q_{\text{inlet}} = \int dq = \int da \cdot v.$$

$$Q_{\text{in}} = \int 2\pi r \cdot dr \cdot 3 \left(1 - \frac{r^2}{R^2} \right)$$

$$= 6\pi \int_0^R r \left(1 - \frac{r^2}{R^2} \right) dr$$

$$= 6\pi \left(\frac{r^2}{2} - \frac{r^4}{4R^2} \right)_0^R$$

$$= 6\pi \left(\frac{R^2}{2} - \frac{R^2}{4} \right) = \frac{3\pi R^2}{2}$$



For steady flow ^{Souvik 2021} condition

$$A_1 v_1 = A_2 v_2$$

$$\frac{3\pi R^2}{2} = \frac{\pi}{4} \times (8 \times 10^{-2})^2 \times v_2$$

$$6R^2 = 64 \times 10^{-4} v_2$$

$$v_2 = 2.343 \text{ m/s.}$$

NOTE :- continuity eqn for steady incompressible flow is given by divergence of velocity = 0.
↳ change

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}, \quad \vec{v} = u\hat{i} + v\hat{j} + w\hat{k}.$$

prove :- $\vec{\nabla} \cdot \vec{v} = 0 \rightarrow$ steady incompressible flow.

$$\vec{\nabla} \cdot \vec{v} = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \cdot (u\hat{i} + v\hat{j} + w\hat{k}) = 0.$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \rightarrow \text{we need to prove this.}$$

proof :-

in general continuity eqn is given by-

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0.$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + v \frac{\partial \rho}{\partial y} + \rho \frac{\partial v}{\partial y} + w \frac{\partial \rho}{\partial z} + \rho \frac{\partial w}{\partial z} = 0$$

$$\left[\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} \right] + \rho \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right] = 0.$$

$$\rho = f(x, y, z, t)$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial \rho}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial \rho}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial \rho}{\partial t} \cdot \frac{dt}{dt}$$

$$\left(\frac{d\rho}{dt} \right) = u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t}$$

→ Total change in density wrt. time.

$$\frac{d\rho}{dt} + \rho (\vec{\nabla} \cdot \vec{v}) = 0.$$

For steady & incompressible flow, $\Rightarrow \frac{\partial \rho}{\partial t} = 0$.

$$\therefore \vec{\nabla} \cdot \vec{v} = 0$$

Q) Density at a point varies at a rate of $5 \text{ kg/m}^2/\text{sec}$ and density at that point at that instant is 1000 kg/m^3 . Find the divergence of velocity.

Sol we know,

$$\frac{\partial \rho}{\partial t} + \rho (\vec{\nabla} \cdot \vec{v}) = 0.$$

$$\vec{\nabla} \cdot \vec{v} = -\frac{1}{\rho} \cdot \frac{\partial \rho}{\partial t}$$

$$= -\frac{1}{1000 \text{ kg}} \cdot 5 \frac{\text{kg}}{\text{m}^2 \text{ sec}}$$

$$\vec{\nabla} \cdot \vec{v} = -\frac{1}{200} \text{ m/sec}$$

NOTE:- For any flow to be possible, continuity eqn must be satisfied.

ACCELERATION OF FLUID PARTICLE :-

$$\vec{v} = u(x, y, z, t) \hat{i} + v(x, y, z, t) \hat{j} + w(x, y, z, t) \hat{k}$$

$$\frac{d\vec{v}}{dt} = \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}.$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t} \cdot \frac{dt}{dt}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Similarly,

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

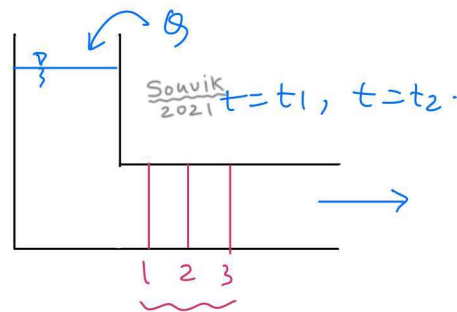
also,

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$\Downarrow$
 convective / advective acceleration
 (acceleration varying with time)

$\Downarrow$
 Local / temporal Acceleration.
 (acceleration varying with time)

For ex:-
(a)

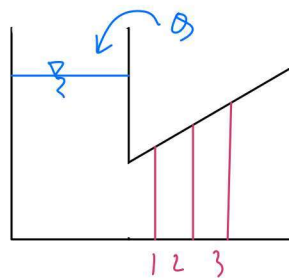


vel. is same
Hence it is not varying with space. as $A = \text{const.}$
convective accn = 0.

with respect to time also vel. is not changing as const. head is maintained. Hence local accn = 0.

STEADY & uniform flow.

(b)

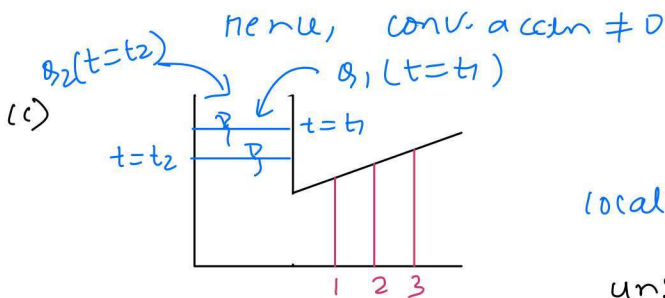


vel. is decreasing, $A \rightarrow \text{incre.}$

$t=t_1$ to $t=t_1$

with time $v = \text{const.}$
as const. head is maintained
Hence local = 0.

STEADY & NON-UNIFORM flow.



conv. accn $\neq 0$.

local accn $\neq 0$.

unsteady & non-uniform flow

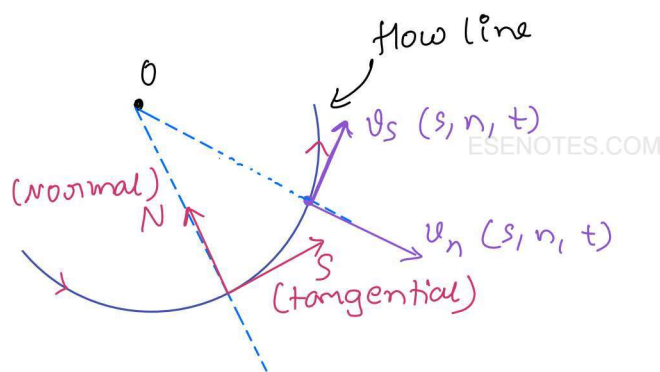
unsteady and uniform flow practically don't exist.

conclusion:-

- (a) For steady flow:- $\rightarrow$ local acc'n = 0.
Total acc = convective acc'n.
- (b) For uniform flow $\rightarrow$ convective acc'n = 0.
Total acc'n = convective acc'n
- (c) For steady uniform flow:-
Total acc'n = 0.

NORMAL & TANGENTIAL ACCELERATION :-

(a) TANGENTIAL ACCELERATION :- It is the acc'n due to change in magnitude of velocity.



tangential acceleration, $a_s = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s} \frac{ds}{dt} + \frac{\partial V_s}{\partial n} \frac{dn}{dt} + \frac{\partial V_s}{\partial t} \frac{dt}{dt}$

$$a_s = V_s \frac{\partial V_s}{\partial s} + V_n \frac{\partial V_s}{\partial n} + \frac{\partial V_s}{\partial t} \rightarrow \text{general eqn.}$$

If flow line is stream line, ($V_n = 0$)
No flow occurs across the line.

$$a_s = \boxed{V_s \frac{\partial V_s}{\partial s}} + \boxed{\frac{\partial V_s}{\partial t}}$$

$\downarrow$ convective tangential acceleration. $\rightarrow$ local tangential acceleration

Therefore, if spacing b/w streamline changes tangential acc'n exist, as vel. will change its magnitude but direction will be same.

Aug 10 (b) NORMAL ACCELERATION :- It is due to change in the direction of fluid along the curved path.

Normal acc'n $a_n = \frac{dv_n}{dt} = \frac{\partial v_n}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v_n}{\partial n} \frac{dn}{dt} + \frac{\partial v_n}{\partial t} \cdot \frac{dt}{dt}$

$$a_n = v_s \frac{\partial v_n}{\partial t} + v_n \frac{\partial v_n}{\partial t} + \frac{\partial v_n}{\partial t} \rightarrow \text{General Eqn}$$

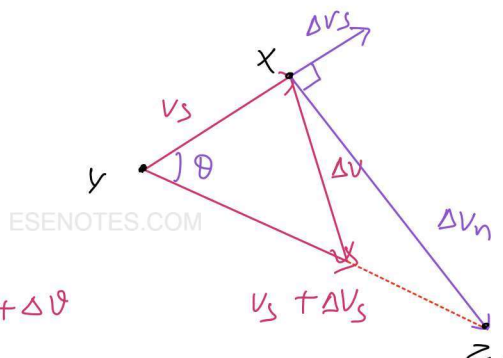
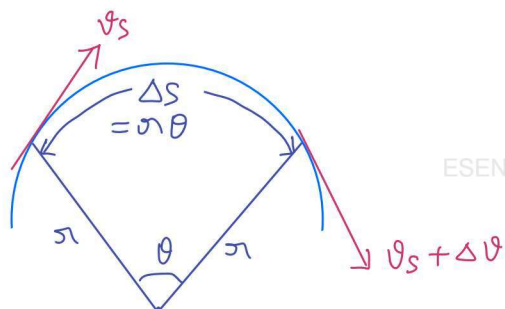
Now, if flow is taking place along stream line, $v_n = 0$.

$$a_n = v_s \frac{\partial v_n}{\partial t} + \frac{\partial v_n}{\partial t} \quad \text{--- (i)}$$

↓
convective Normal acceleration.

↓
local normal acceleration

NOTE :-



$v_s \rightarrow$ tangential acc'n

The vector triangle is drawn with Δv as the change of velocity which consist of both change in direction + change in magnitude.

Now, $\Delta v_s \rightarrow$ change in tang. velocity i.e only magn. changes, hence direction is same as that of v_s .

But, $\Delta v_n \rightarrow$ change in normal accd. i.e change in direction. Now v_n is $\perp$ to the v_s . Hence at 90° Δv_n is drawn with Δv_s , which meets the lines $v_s + \Delta v$ at point Z.

Now, in Right $\angle$ triangle YXZ :-

$$\tan \theta = \frac{xz}{xy} = \frac{\Delta v_n}{v_s}$$

if θ is small, $\tan \theta \approx \theta$.

$$\theta \approx \frac{\Delta v_n}{v_s} \quad \text{--- (i)}$$

Also, $\theta = \frac{\Delta S}{r} \quad \text{--- (ii)}$

$$\frac{\Delta v_n}{v_s} = \frac{\Delta S}{r}$$

$$\frac{\Delta v_n}{\Delta S} = \frac{v_s}{r}$$

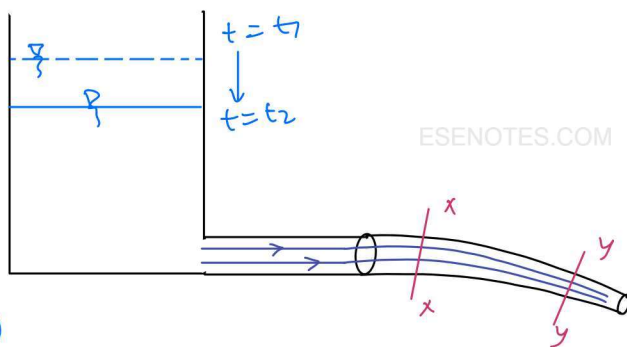
Putting this value in, $a_n = v_s \frac{\partial v_n}{\partial S} + \frac{\partial v_n}{\partial t}$

$$a_n = v_s \cdot \frac{v_s}{r} + \frac{\partial v_n}{\partial t}$$

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$$a_n = \frac{v_s^2}{r} + \frac{\partial v_n}{\partial t}$$

EX-1 :-



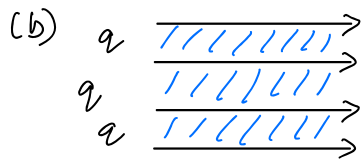
(section fixed)

- Here, local tangential accn, exist because mag. of vel. is changing w.r.t time at any given section (xx or yy) (because water level is change).

(Time fixed)

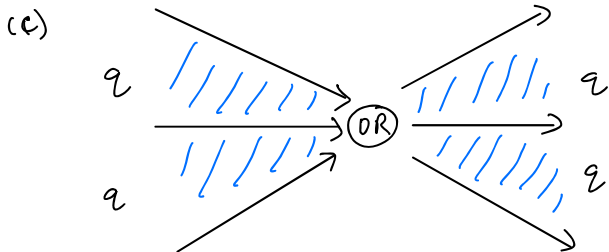
convective tangential acceleration exists because mag. of velocity is changing w.r.t section at any given time. (Because, the nozzle has tapered section, hence spacing of stream line changing hence vel. also changes).

- convective Normal accn :- exist because, direction of velocity is changing from section to section at any given time as streamlines are curved
- local Normal accn :- will not exist because flow is taking place in combined space that is not changing with time.



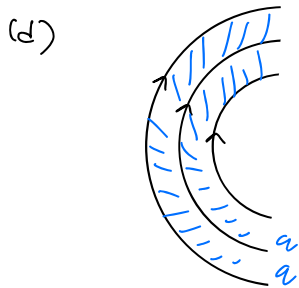
$\Rightarrow$ No acceleration.

(wrt time, $q = \text{const}$, hence no local accn
wrt space, dist. b/w stream line = same
hence no change in vel.
no conv. accn).



stream line not curved $\rightarrow$ no normal accn

mag. of vel. is changing with space.
 $\rightarrow$ conv. tang. acceleration.



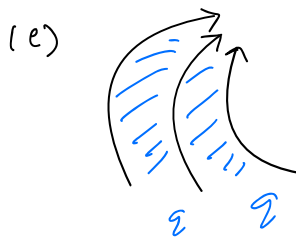
stream line curved $\rightarrow$ normal accn.

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spacing = const $\rightarrow$ no tangential accn

change in dir. wrt space

$\therefore$ convective normal accn



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curved $\rightarrow$ normal accn

spacing = not const $\rightarrow$ Tangential accn

mag. is changing wrt space.

convective Tang. accn.

dir. is changing wrt space.

conv. normal accn

- 8) A conical pipe with 10cm inlet dia, 20cm outlet dia. and 100cm length is placed horizontally. The vel. of any cross-section may be considered to be uniform.

Determine the convective and local accn at section where dia is 15cm, for following cases

ca) const inlet discharge for $0.2 \text{ m}^3/\text{sec}$.

cb) inlet discharge varying linearly from $0.2 \text{ m}^3/\text{s}$ to $0.4 \text{ m}^3/\text{s}$ over 2 sec. The time of interest is when $t=1$.

Sol :-

(a) $q_i = 0.2 \text{ m}^3/\text{s}$.

flow is in st. line.

no normal accn

mag. is changing

tangential accn exist

mag. is changing wrt space.

convective tangential accn.

with time mag. is not changing as $q = \text{const.}$

hence, local accn = 0.

At any distance ^{Souvik} ~~2x21~~,
$$Dx = 0.1 + \left(\frac{0.2 - 0.1}{1} \right) x$$
$$= 0.1 + 0.1x = 0.1(1+x)$$

$$Ax = \frac{\pi}{4} \times 0.1^2 (1+x)^2$$

velocity,
$$v_x = \frac{Q}{Ax} = \frac{Q}{7.85 \times 10^{-3} (1+x)^2}$$

$$= \frac{\partial v_x}{\partial x} = \frac{Q}{7.85 \times 10^{-3}} \left(-2(1+x)^{-3} \right)$$

$$= \frac{-2Q}{7.85 \times 10^{-3} (1+x)^3} = -254.64 Q (1+x)^{-3}$$

$$\text{convective accn} = u \cdot \frac{\partial u}{\partial x} = \frac{Q}{7.85 \times 10^{-3} (1+x)^2} \cdot \frac{(-254.64 Q)}{(1+x)^3}$$

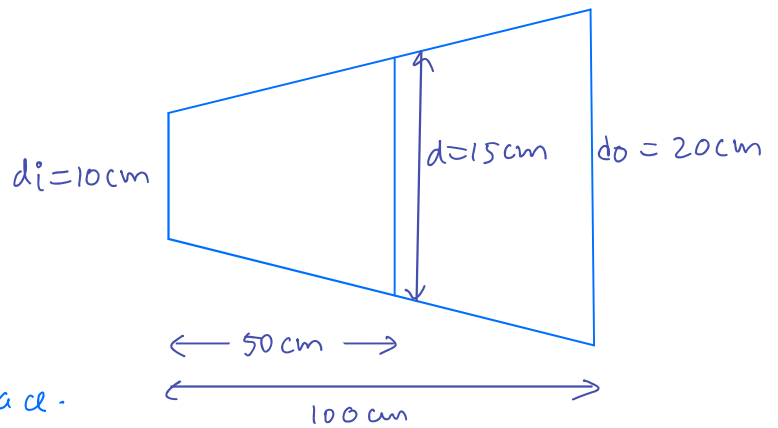
$$@ x = 0.5 \text{ m}, = \frac{0.2}{7.85 \times 10^{-3} \cdot 1.5^2} \cdot \frac{(-254.64 \times 0.2)}{(1.5)^3}$$

$$= -170.85 \text{ m/s}^2$$

(b) at $t=1$, $Q = 0.2 + \frac{(0.4 - 0.3)}{2} \times 1 = 0.3 \text{ m}^3/\text{s}^2$

Now conv. accn :
$$v_x \left[\frac{\partial v_x}{\partial x} \right]_{x=0.5} = -384.4 \text{ m/s}^2$$

local accn ;
$$\frac{\partial v_x}{\partial t} = \frac{\partial}{\partial t} v_x = \frac{\partial}{\partial t} \left[\frac{Q}{7.85 \times 10^{-3} (1+x)^2} \right]$$



$$= \frac{127.38}{(1+x)^2} \frac{\partial \theta}{\partial t} = \frac{127.38}{(1+x)^2} \times \frac{(0.3-0.2)}{1-0}$$

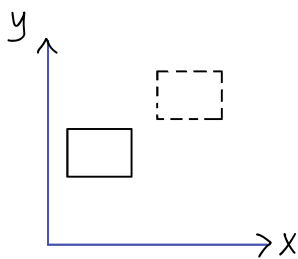
$$= \frac{127.38}{(1.5)^2} \times 0.1 = 5.66 \text{ m/sec}^2$$

hence total acc'n = $5.66 - 384.4$
 $= -378.74 \text{ m/s}^2$

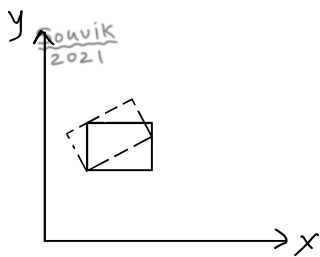
Aug 11) # ANGULAR VELOCITY :- (Ω)

NOTE:-

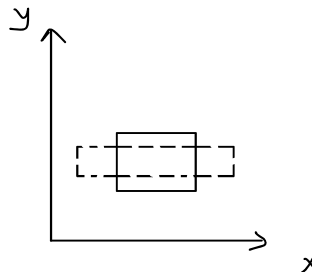
When fluid flows, it may undergo following types of motions / deformation



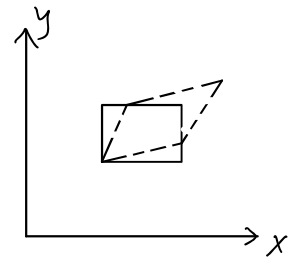
Translational motion



Rotational motion

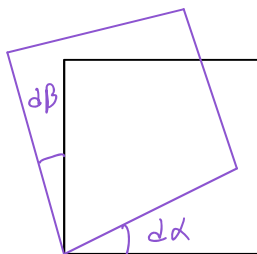


Linear Deformation



Rotational deformation

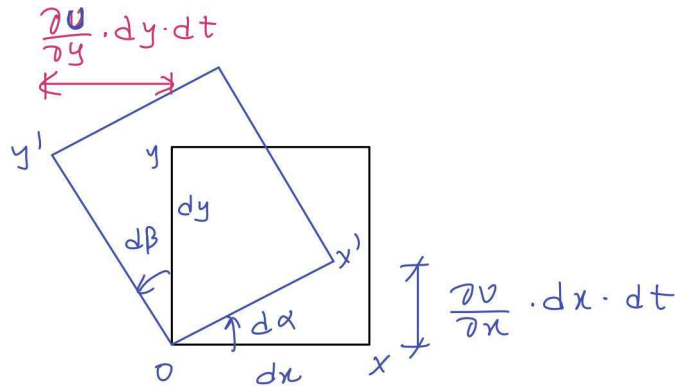
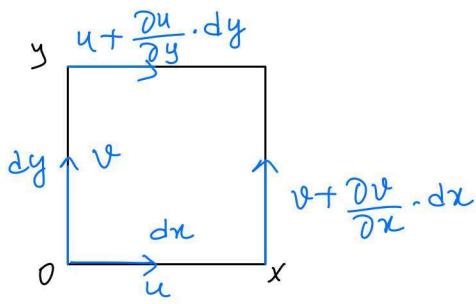
- Here rate of translational motion is termed as linear velocity.
- Also, rate of rotational motion or avg. of rotation rate of two initially perpendicular lines is termed as Angular velocity.



$$\Omega = \frac{d\theta}{dt} = \frac{1}{2} \left(\frac{d\alpha}{dt} + \frac{d\beta}{dt} \right)$$

$$\Omega = \Omega_x \hat{i} + \Omega_y \hat{j} + \Omega_z \hat{k}$$

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$



If θ is small, $\theta \approx \tan \theta$

$$\therefore d\alpha = \frac{\frac{\partial v}{\partial x} \cdot dx \cdot dt}{dx} = \frac{\partial v}{\partial x} \cdot dt \cdot \frac{dx}{dx}$$

Rate of rotation Souvik 2021 $\frac{d\alpha}{dt} = \frac{\partial v}{\partial x} \cdot \frac{dx}{dx}$

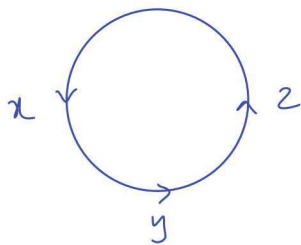
Similarly,

$$d\beta = - \frac{\frac{\partial u}{\partial y} \cdot dy \cdot dt}{dy}$$

Rate of rotation,

$$\frac{d\beta}{dt} = - \frac{\frac{\partial u}{\partial y} \cdot dy}{dy}$$

Angular velocity, $\omega_z = \frac{d\theta}{dt} = \frac{1}{2} \left[\frac{d\alpha}{dt} + \frac{d\beta}{dt} \right]$
(z-axis)



$$\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right]$$

$$\omega_x = \frac{1}{2} \left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right]$$

$$\omega_y = \frac{1}{2} \left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right]$$

Also, $\Omega = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$

$$\Omega = \frac{1}{2} \nabla \times \vec{v}$$

If angular velocity is zero $\rightarrow$ Flow is IRROTATIONAL.

Q) A flow field is given by

$$u = xy \hat{i} + 2yz \hat{j} - (yz + z^2) \hat{k}$$

- Determine whether this is possible steady incompressible fluid flow or not.
- If so, calculate the acc'n and angular velocity at point (1, 2, 3)

Sol: - (a) For steady incompressible fluid flow, continuity eqn must be satisfied.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Souvik 2021

$$u = xy, \quad \frac{\partial u}{\partial x} = y \quad v = 2yz \quad w = -(yz + z^2)$$
$$\frac{\partial v}{\partial y} = 2z \quad \frac{\partial w}{\partial z} = -(y + 2z)$$

$$y + 2z - y - 2z = 0.$$

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which is true. Hence it is possible.

(a) acceleration

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

VORTICITY (ξ).

- It is defined as twice of angular velocity.

$$\xi = 2\Omega$$

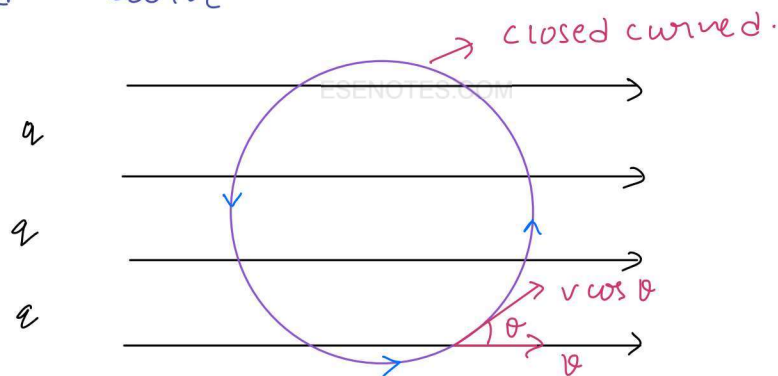
$$\xi = 2\Omega = 2 \times \frac{1}{2} \vec{\nabla} \times \vec{v}$$

$$\xi = \vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

- If vorticity of a point is zero, the fluid is irrotational.

CIRCULATION (τ) Souvik 2021

- It is defined as line integral of tangential component of velocity vectors along a closed curve.

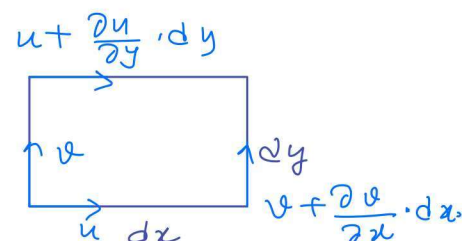


$$\tau = \oint \vec{v} \cdot d\vec{s}$$

$$\begin{aligned} \tau &= \oint (u\hat{i} + v\hat{j} + w\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k}) \\ &= \oint (u dx + v dy + w dz) \end{aligned}$$

Hence, if circulation at a point in fluid flow is zero, flow is termed as irrotational flow.

Let flow be two-dimensional.



$$\tau = u \cdot dx + \left(v + \frac{\partial v}{\partial x} \cdot dx \right) \cdot dy + \left\{ - \left(u + \frac{\partial u}{\partial y} \cdot dy \right) \right\} dx + (-v) \cdot dy$$

$$= \frac{\partial v}{\partial x} \cdot dx \cdot dy - \frac{\partial u}{\partial y} \cdot dy \cdot dx$$

$$\tau = \underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)}_{2\omega_z} \cdot dx \cdot dy$$

$$\tau = 2\omega_z \cdot A$$

$$\boxed{\frac{\tau}{A} = 2\omega_z} \quad \text{Also,} \quad \boxed{\frac{\tau}{A} = \xi}$$

circulation per unit area is termed as vorticity.

Souvik
2021

Aug 12 8) Find the circulation around a closed curve defined by $y=1$, $x=2$, $y=4$, $x=4$. when velocity field is given by.
 $u = 16y - 8x$, $v = 8y - 7x$.

Sol

$$\tau = \oint \vec{v} \cdot d\vec{s}$$

$$= \oint (u dx + v dy)$$

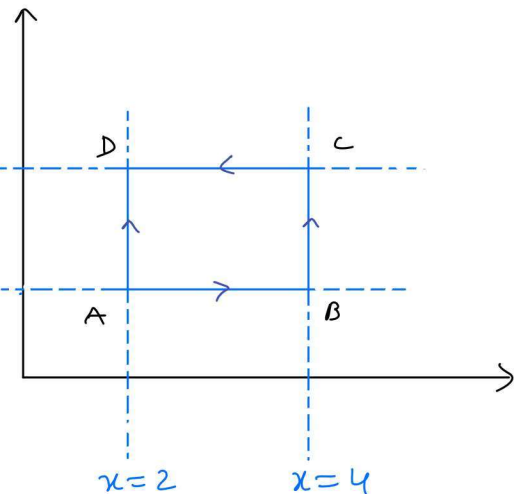
$$\tau = \int_2^4 \left(16y - 8x \right)_{y=1} dx + \int_1^4 \left(8y - 7x \right)_{x=4} dy + \int_4^2 \left(16y - 8x \right)_{y=4} dy + \int_4^2 \left(8y - 7x \right)_{x=2} dy$$

$$= 16 \times (2) - \frac{8}{2} (16 - 4) + \frac{8}{2} (15) - 28 \times 3 + 64 \times (2 - 4) - \frac{8}{2} (4 - 16) + \frac{8}{2} (1 - 16) - (4 \times (-3))$$

$$= 32 - 48 + 60 - 84 - 128 + 48 - 60 + 42$$

$$= 32 - 84 - 128 + 42 = -138$$

$$\begin{aligned} \kappa, \quad \tau &= \xi \cdot A = (\vec{\nabla} \times \vec{v}) \cdot \vec{A} = A \begin{vmatrix} \hat{i} & \hat{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 16y - 8x & 8y - 7x \end{vmatrix} \\ &= A (-7 - 16) = 2 \times 3 \times (-23) = -138 \end{aligned}$$



Q) If velocity is given by $v = \frac{c}{r}$ given the variation of tangential velocity with radius r , in a vortex formed in a wash basin. Determine the circulation.

(a) Around a closed curve formed by two stream line $r = R_1$ and $r = R_2$ and two radial lines with an angle of θ b/w them.

(b) Around a closed curve in form of concentric circle of radius R_1 .

(c) Solve the above problem in forced vortex flow. ($v = \omega r$).

Sol (a) $\Gamma = \oint \vec{v} \cdot d\vec{s}$

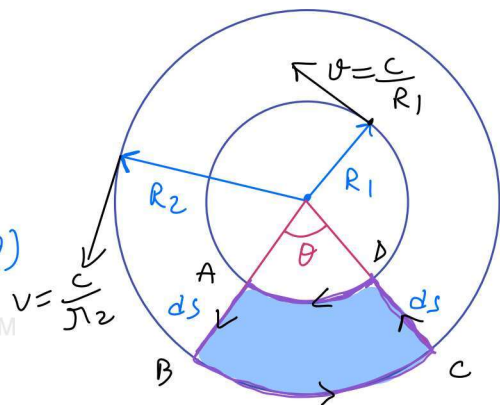
Souvik
2021

$$\Gamma = \int_{(AB)} 0 \cdot d\vec{s} + \frac{c}{R_2} (R_2 \cdot \theta)$$

$$+ \int_{(CD)} 0 \cdot d\vec{s} + \left(-\frac{c}{R_1}\right) (R_1 \cdot \theta)$$

$$\Gamma = c \cdot \theta - c \theta = 0.$$

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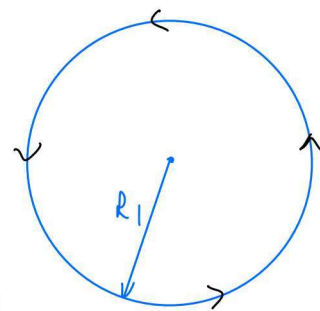
i.e flow is irrotational.

i.e Free vortex flow is irrotational.

(b) $\Gamma = \oint \vec{v} \cdot d\vec{s}$

$$= \frac{c}{R_1} \times 2\pi R_1 = 2\pi c \neq 0.$$

i.e Rotational.



NOTE: (**) If in case of concentric circle Γ is not zero, flow seems to be rotational. However even for smaller radius of the closed path including center, circulation is still $2\pi c$ i.e all Γ in free vortex condⁿ is confined near centre and flow away from centre is essentially irrotation.

(c) For forced vortex motion.

$$(i) \quad \Gamma = \oint \vec{v} \cdot d\vec{s}$$

$$= 0 \cdot ds + \omega R_2 \cdot R_2 \theta + 0 \cdot ds + (-\omega R_1) \cdot R_1 \theta$$

$$= \omega R_2^2 \theta - \omega R_1^2 \theta$$

$$= \omega \theta (R_2^2 - R_1^2) \neq 0$$

forced vortex flow is rotational.

$$(ii) \quad \Gamma = \oint \vec{v} \cdot d\vec{s} = \omega R_1 \cdot 2R_1 \pi = 2\omega R_1^2 \pi \quad \text{i.e. rotational}$$

VELOCITY POTENTIAL / POTENTIAL FUNCTION :- (ϕ)

- velocity potential is the scalar function of space and time such that its negative derivative wrt any direction ^{Souvik 2921} gives velocity of flow in that direction.

$$\phi = f(x, y, z, t)$$

$$-\frac{\partial \phi}{\partial x} = u \quad -\frac{\partial \phi}{\partial z} = w$$

$$-\frac{\partial \phi}{\partial y} = v$$

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Similarly in polar co-ordinate system.

$$-\frac{\partial \phi}{\partial r} = v_r \quad -\frac{\partial \phi}{\partial \theta} = r v_\theta$$

$$-\frac{\partial \phi}{\partial z} = v_z$$

- velocity potential exist only for ideal and irrotation flow.

$$\begin{aligned} \text{Angular velocity } \omega_z &= \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] \\ &= \frac{1}{2} \left[\frac{\partial}{\partial x} \left(-\frac{\partial \phi}{\partial y} \right) - \frac{\partial}{\partial y} \left(-\frac{\partial \phi}{\partial x} \right) \right] \\ &= \frac{1}{2} \left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x} \right] \end{aligned}$$

$$\omega_z = 0 \quad \text{i.e. flow is irrotation.}$$

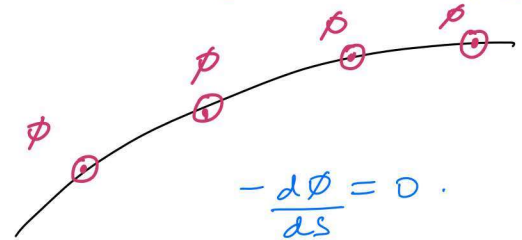
hence velocity potential exist for ideal, irrotation flow.

- If ϕ is constant in any direction, then velocity in that direction will be zero.

$$\phi = c$$

$$-\frac{\partial \phi}{\partial x} = u = 0.$$

- Line joining the points of equal vel. potential is termed as "equipotential line".
- Along equipotential line flow cannot occur as the change in potential on this line is zero, because potential is same in this line.



- For steady, incompressible flow.

Souvik 2021

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

$$-\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = 0.$$

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$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0.$$

$\nabla^2 \phi = 0.$

 $\rightarrow$ Also known as Laplace Eqn.

- Hence if any velocity potential function (ϕ) satisfies Laplace eqn, it will represent steady incompressible flow.

(***) velocity potential helps in integration of Euler's eqn to find out Bernoulli's eqn.

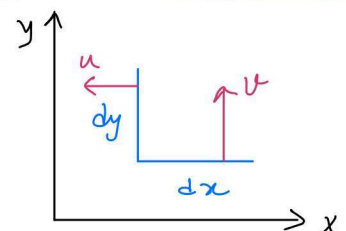
STREAM FUNCTION :- (ψ)

- It is a scalar function of space and time, such that its partial derivative w.r.t any direction gives the vel. comp. at right angle in anticlockwise direction.

$$\psi = f(x, y, t)$$

$$\frac{\partial \psi}{\partial x} = v_y = v.$$

$$\frac{\partial \psi}{\partial y} = -v_x = -u$$



- For irrotational flow, $\omega_z = 0$.

$$\omega_z = \frac{1}{2} \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] = 0.$$

$$\Rightarrow \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = - \frac{\partial^2 \psi}{\partial y^2}$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0.$$

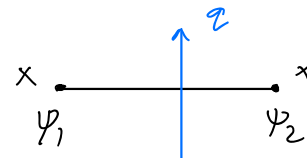
↳ Laplace eqn in 2D.

- Hence if stream fn (ψ) satisfies Laplace eqn, flow is irrotational, otherwise rotational.

Aug 14

- The difference of stream function b/w two points is equal to flow across the line joining these two points.

$\psi_1 - \psi_2 = \text{flow across line joining } x \text{ and } y.$



- Hence if two points are on a stream line, there cannot be any flow across the streamline, $\psi_1 - \psi_2 = 0$.
i.e. $\psi = \text{const.}$ i.e. stream function is constant for a stream line.

NOTE:- Potential function (ϕ) exists only for irrotational flow, whereas stream function (ψ) exists for both rotational & irrotational flow.

- For irrotational flow, both potential fn ψ satisfies Laplace eqn.

- For irrotational, incompressible flow

$$\left. \begin{aligned} u &= - \frac{\partial \phi}{\partial x} = - \frac{\partial \psi}{\partial y} \\ v &= - \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} \end{aligned} \right\} \text{Cauchy-Riemann Eqn}$$

- 8) stream function is given by $\psi = 2x^2y + (x+1)y^2$.
Find the flow across the line joining points A (0,2) and B (3,0).

Sol

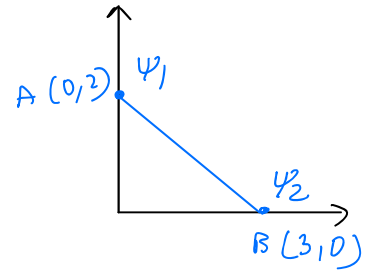
For flow across the line

$$\psi_1 - \psi_2 = \text{flow across AB.}$$

$$\psi_1 \Big|_{(0,2)} = 4$$

$$\psi_2 \Big|_{(3,0)} = 0$$

$$\text{flow} = 4 = \psi_{AB}$$



- 9) If $\phi = 3xy$, find x and y components of velocity at (1,3) and (3,3), also find the discharge b/w stream lines passing through this points.

Sol

$$x\text{-component, } u = -\frac{\partial \phi}{\partial x} = -3y$$

$$u \Big|_{(1,3)} = -9, \quad u \Big|_{(3,3)} = -9.$$

$$y\text{-component, } v = -\frac{\partial \phi}{\partial y} = -3x$$

$$v \Big|_{(1,3)} = -3, \quad v \Big|_{(3,3)} = -9.$$

$$v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} = -3x$$

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} = -3y$$

$$\partial \psi = -3x \cdot dx$$

$$\psi = -\frac{3x^2}{2} + f(y)$$

→ so that if we diff. ψ wrt x , it becomes 0.

$$\frac{\partial \psi}{\partial y} = f'(y).$$

$$3y = f'(y)$$

$$\therefore \int 3y dy = f(y)$$

$$f(y) = \frac{3y^2}{2} + C.$$

$$\psi = -\frac{3x^2}{2} + \frac{3y^2}{2} + C.$$

$$\psi_{AB} = \psi_A - \psi_B.$$

$$\psi_A(1,3) = -\frac{3}{2} + \frac{27}{2} + C$$

$$= 12 + C$$

$$\psi_B(3,3) = -\frac{27}{2} + \frac{27}{2} + C = 0$$

$$\psi_{AB} = 12$$

8) $\phi = x(2y-1)$, find the velocity at $(3,4)$.

Sol $u = -\frac{\partial \phi}{\partial x} = -(2y-1)$

$$u(3,4) = -7$$

$$v = -\frac{\partial \phi}{\partial y} = -2x$$

$$v(3,4) = -6$$

$$\therefore \text{velocity, } \vec{v} = -7\hat{i} - 6\hat{j}$$

$$|\vec{v}| = \sqrt{49+36}$$

$$= 9.21$$

9) $\phi = \frac{(y-x)xy}{3} + (y^2 - x^2)$, check whether flow is possible or not.

Sol If Laplace eqn is satisfied the flow is possible.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\phi = \frac{xy^2}{3} - \frac{x^2y}{3} + y^2 - x^2$$

$$\frac{\partial \phi}{\partial x} = \frac{y^2}{3} - \frac{2xy}{3} - 2x \quad \left| \quad \frac{\partial \phi}{\partial y} = \frac{2xy}{3} - \frac{x^2}{3} + 2y \quad \right| \quad \frac{\partial \phi}{\partial z} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{2y}{3} - 2 \quad \left| \quad \frac{\partial^2 \phi}{\partial y^2} = \frac{2x}{3} + 2 \quad \right| \quad \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \neq 0 \quad \text{Hence flow is not possible}$$

Q) If stream fn $\psi = x^2 - y^2$, what is the shape of the stream line.

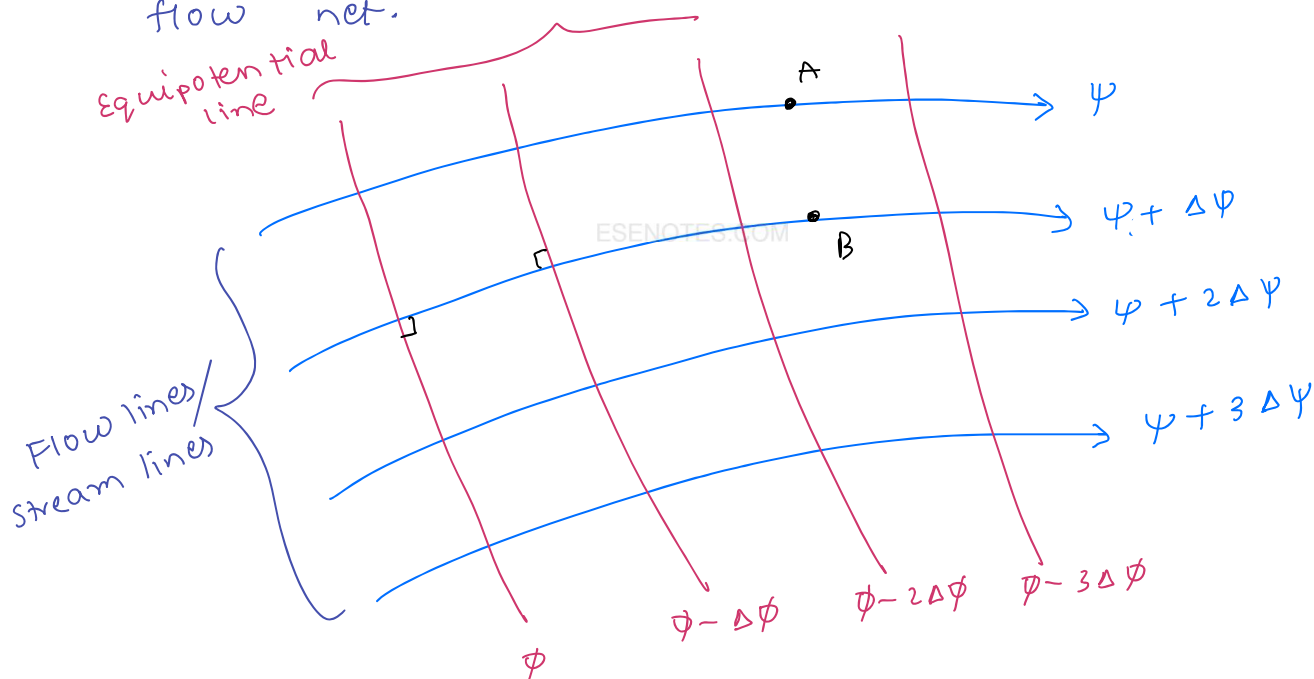
Sol/ For stream line, $\psi = \text{const.}$

$$x^2 - y^2 = c$$

i.e Rectangular hyperbola.

FLOW NET :-

- A line along which stream function is termed as stream line ($\psi = c$) and the line along which velocity fn is constant is called as equipotential line. ($\phi = c$).
- A grid obtained by drawing series of equipotential line and stream line is termed as flow net.



- Flow net is drawn in such a way that diff b/w stream fn of two successive streamline are same.
- Hence discharge b/w two successive stream line is constant in a flow net.
- Flow occurs in the direction of decreasing potential function
- Equipotential & stream line are orthogonal to each other.

let ψ = stream function
 ϕ = velocity function

$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy$$

Since $\psi = \text{const}$ for stream line.

$$0 = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\left. \frac{dy}{dx} \right|_{\psi} = - \left(\frac{\partial \psi}{\partial x} \right) \cdot \left(\frac{\partial y}{\partial \psi} \right)$$

$$= \frac{-v}{-u} = \frac{v}{u}$$

similarly.

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

since $\phi = \text{const}$.

$$\left. \frac{dy}{dx} \right|_{\phi} = \frac{-\partial \phi}{\partial x} \cdot \frac{1}{\left(\frac{\partial \phi}{\partial y} \right)} = \frac{u}{-v} = -\frac{u}{v}$$

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$$\left. \frac{dy}{dx} \right|_{\psi} \times \left. \frac{dy}{dx} \right|_{\phi} = \frac{v}{u} \cdot \frac{(-u)}{v} = -1$$

$$m_1 \cdot m_2 = -1$$

Hence, they are orthogonal.

FLUID DYNAMICS

- It is the study of flow of fluid along with the forces responsible for it.
- Hence Newton's second law ($F=ma$) of motion is used to analyze dynamic behaviour of fluid flow.
- In the fluid flow various forces acting on it are as follows:-
 - (i) Gravity force
 - (ii) Pressure force.
 - (iii) Viscous force.
 - (iv) Force due to turbulence.
 - (v) " " compressibility.
 - (vi) " " ^{Souvik 2021} Surface Tension. (Minor)

Aug 15

NOTE:- when all forces are taken into account, the equation of motion is termed as **Newton's eqn of motion**. ESENOTES.COM

- But when compressibility and surface tension is not considered then it is termed as **Reynold's eqn of motion**.
- when compressibility, surface tension and force due to turbulence is neglected (i.e. only gravity, pressure, viscous force is considered) it is called **NAVIER-STOKES EQUATION**.
- when only gravity and pressure force is considered, it is termed as **euler's eqn of motion**. (It will be used in dynamics for analysis of fluid flow).

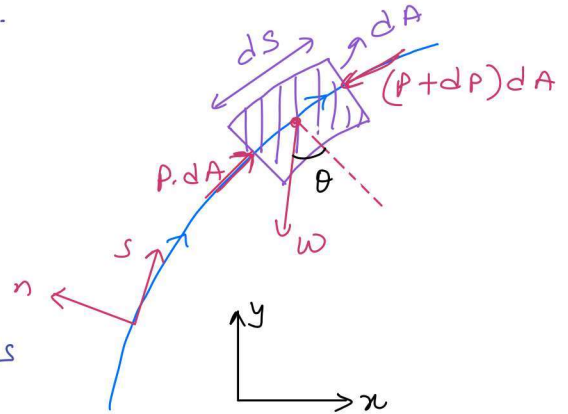
BERNOULLI'S EQUATION :-

- It is based upon conservation of Energy.
- It is obtained by integration of Euler's eqn of motion along a stream line under steady incompressible flow.

As per Newton's second law of motion.

$$\Sigma F_s = m \cdot a_s$$

(Euler's eqn)
 $\rightarrow P \cdot dA - (P + dP) dA - (w \sin \theta) = m a_s$



$$-dP \cdot dA - (\rho w \, dv) \sin \theta = (\rho w \, dv) \left(\frac{dv_s}{dt} \right)$$

$v_s \rightarrow$ tangential velocity
 $v \rightarrow$ volume

$$-dP \cdot dA - \rho w \cdot (dA \cdot ds) \sin \theta = \rho w \, ds \cdot dA \left[\frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t} \cdot \frac{dt}{dt} \right]$$

For steady flow, $\frac{\partial v}{\partial t} = 0$.

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$$-dP - \rho w \, ds \sin \theta = \rho w \, ds \left[\frac{\partial v}{\partial s} \cdot v_s + 0 \right]$$

$$-dP - \rho w \cdot dy = \rho w \, v_s \cdot (dv)$$

$$v_s \cdot dv + g \, dy + \frac{dP}{\rho w} = 0$$

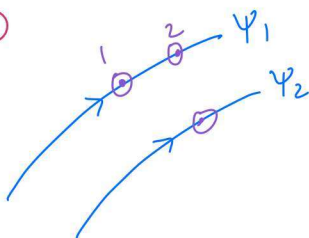
Integrating both side.

$$\boxed{\frac{v^2}{2} + g y + \int \frac{dP}{\rho w} = C} \rightarrow \text{general eqn}$$

For incompressible flow, $\rho = \text{const.}$

$$\boxed{\frac{v^2}{2} + g y + \frac{P}{\rho w} = \text{const.}} \rightarrow \text{specific eqn}$$

NOTE:- ①



The value of constant in above eqn is same along a stream line, However if flow is irrotational, the value of constant is same along all the streamlines.

If flow is rotational.

$$(const)_1 = (const)_2 \neq C_3$$

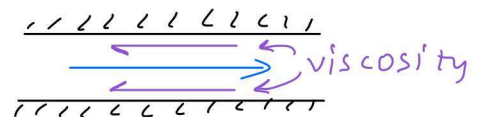
$$\frac{P_1}{\rho_w} + g y_1 + \frac{v_1^2}{2} \neq \frac{P_2}{\rho_w} + g y_2 + \frac{v_2^2}{2} \neq \frac{P_3}{\rho_w} + g y_3 + \frac{v_3^2}{2}$$

If flow is irrotational.

$$\frac{P_1}{\rho_w} + g y_1 + \frac{v_1^2}{2} = \frac{P_2}{\rho_w} + g y_2 + \frac{v_2^2}{2} = \frac{P_3}{\rho_w} + g y_3 + \frac{v_3^2}{2}$$

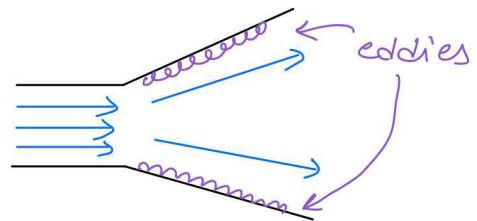
- ② while analysing Bernoulli's eqn, only gravity and pressure forces are considered, hence if any other force except these two are present in system, then this eqn will not be applicable.

Eg:- It is not applicable for long narrow flow passage because viscous force will come in action.



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- ③ It is not applicable beyond Mach No. > 3 , as compressibility would also come into the picture.
- ④ In diverging flow section, it is not applicable as flow separation and energy loss occurs.



- ⑤ It is not applicable in flow section that involves fan, turbines or any other mechanical unit as it involves energy loss or gain or work transfer.
- ⑥ It is not applicable in flow sections that involve temperature change as compressibility comes into the picture.

FORMS OF BERNOULLI'S EQU :-

(a) $\frac{p}{\rho} + gy + \frac{v^2}{2} = \text{const. (C}_1)$ [Here diff. forms of energy is reported per unit mass]

$\frac{p}{\rho}$ $\downarrow$ Pressure En. / mass
 gy $\downarrow$ Potential En. / mass
 $\frac{v^2}{2}$ $\downarrow$ Kinetic En. / mass

(b) $\frac{p}{\rho g} + y + \frac{v^2}{2g} = \text{const. (C}_2)$ [Reported per unit weight also termed as head.]

$\frac{p}{\rho g}$ $\downarrow$ Pr. energy / wt.
 y $\downarrow$ Potential En. / wt.
 $\frac{v^2}{2g}$ $\downarrow$ Kinetic En. / wt.

(c) $P + \gamma y + \frac{\rho v^2}{2} = \text{const. (C}_3)$ [Reported in terms of pressure]

P $\downarrow$ Static pres. [Actual pr. in fluid]
 γy $\downarrow$ Hydrostatic pres. [Due to weight]
 $\frac{\rho v^2}{2}$ $\downarrow$ Dynamic pressure. [pressure developed when a liq. in motion is brought to rest].

Kinetic energy of liq. = Pressure developed per unit wt

$$\frac{v^2}{2g} = \frac{p}{\gamma}$$

$$p = \frac{\rho v^2}{2} \rightarrow \text{dynamic pressure.}$$

NOTE:- Here sum of static pressure (P) and dynamic pressure ($\rho v^2/2$) is termed as **STAGNATION pressure**

$$P_{\text{stag}} = P + \frac{\rho v^2}{2}$$

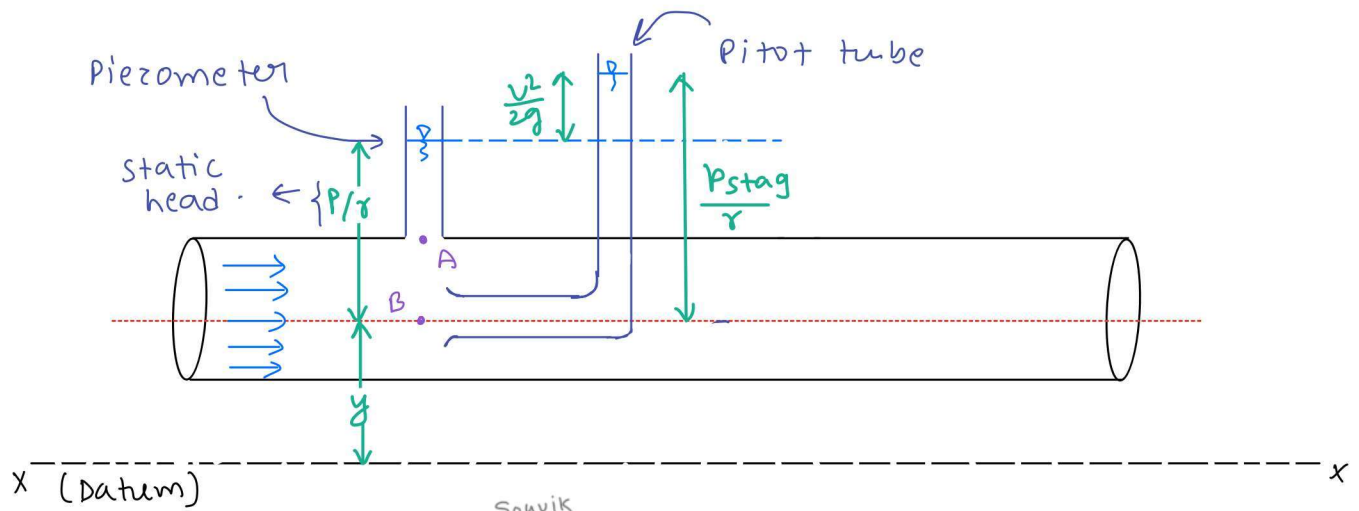
Measurement of stagnation pressure helps in finding the velocity in any system. and velocity can be found out as.

$$\frac{\rho}{2} (P_{\text{stag}} - P) = \frac{\rho v^2}{2}$$

$$v = \sqrt{\frac{2}{\rho} (P_{\text{stag}} - P)}$$

NOTE:- Here sum of static pressure and hydrostatic pressure is termed as **PIEZOMETRIC Press.**

$$p_{\text{piez}} = p + \gamma y$$



Datum head at B = y

Pressure " " B = p/γ

piezometric head at B = $y + p/\gamma$

Datum head @ B $\neq$ Datum head @ A

press. " @ B $\neq$ press. " @ A

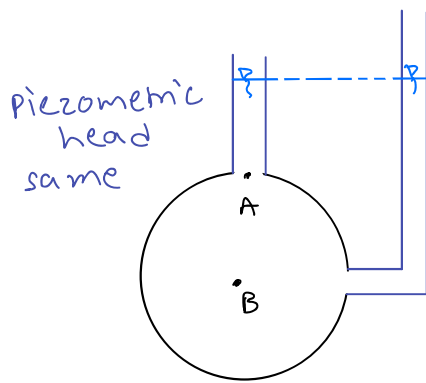
But, (piezometric head)_B = (piezometric head)_A

i.e. piezometric head is the head of water in piezometer wrt. **datum.**

But in general, the head in piezometer above concerned point will give pressure head, but the head in piezometer wrt datum will give piezometric head.

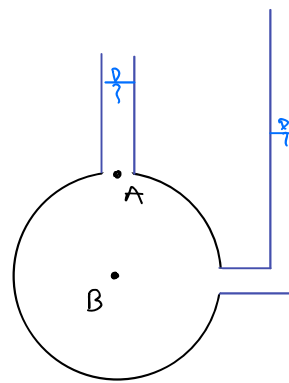
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- Piezometric head is const. at all the depths, at a particular section, if streamline are straight.
- But if streamline are curved, the piezometric head will vary across the depth at a particular section.



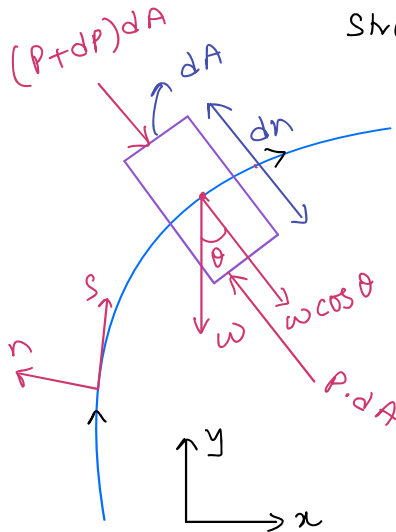
piezometric head same

straight pipe



Diff. piezometric head.

curved pipe.



normal direction :-

From newton's second law.

$$F = ma.$$

$$p \cdot dA - (p + dp) \cdot dA - w \cos \theta = m a_n.$$

$$-dp \cdot dA - r dA \cdot dn \cos \theta = \rho dA \cdot dn a_n$$

$$-dp - r \cdot dy = \rho dn \cdot a_n.$$

$$dp + r \cdot dy + \rho dn \cdot a_n = 0.$$

$$\frac{dp}{\rho} + g dy + dn \cdot a_n = 0.$$

$$\frac{dp}{dn} \cdot \frac{1}{\rho} + g \frac{dy}{dn} = -a_n$$

$$a_n = -\frac{d}{dn} \left[\frac{p}{\rho} + gy \right]$$

conclusion :-

Here, differential of piezometric head is not zero for curved streamline i.e. piezometric head changes from point to point.

(piezometric head) per unit mass.

But be it straight streamline, $a_n = 0$, i.e. differential of p-head is zero i.e. p-head is constant at all the point.

$$a_n = 0$$

$$\frac{p}{\rho} + gy = \text{const.}$$

NOTE:- In real situations where energy transfer (i.e. energy gain or loss) and head loss takes place, the bernoulli's eqn can be modified as follows.

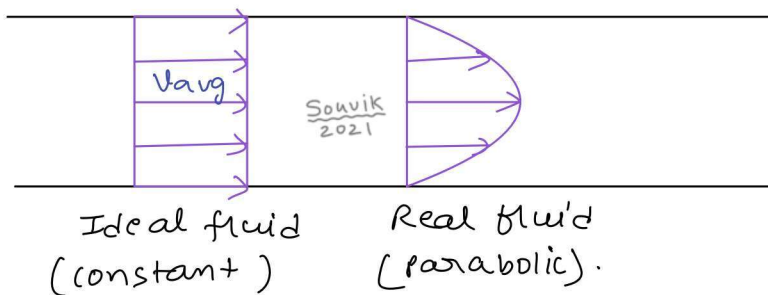
modified
form of
Bernoulli's
eqⁿ

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + y_1 + E_a = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + y_2 + E_L + h_L$$

E_a = gain of energy , h_f = head loss due to friction.
 E_L = loss of energy

CORRECTION FACTOR :-

- While analysing Bernoulli's eqⁿ fluid is considered to be ideal, hence vel. is taken to be const. across the section, but in real fluid vel. varies across the section.



- Hence, if we want to calculate the actual kinetic energy in terms of avg KE, a factor (α) termed as **Kinetic Energy correction factor** is to be used along it.

$$\alpha \cdot \left(\frac{1}{2} m v_{avg}^2 \right) = \frac{1}{2} m v_{actual}^2$$

$$\alpha = \frac{\text{Actual K.E}}{\text{Avg. K.E}}$$

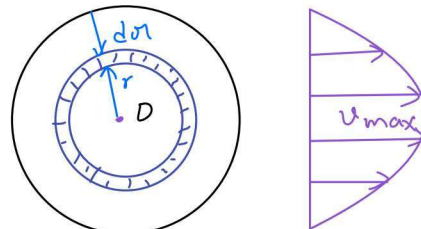
K.E of elemental area

$$d(K.E) = \frac{1}{2} \times dm \times v^2$$

$$= \frac{1}{2} \times \underbrace{(p \times dA \cdot v)}_{\text{Rate i.e per unit time}} \times v^2$$

$$KE = \int \frac{1}{2} p dA v^3$$

$$(KE)_{act} = \frac{p}{2} \int v^3 \cdot dA \quad - (i)$$



(KE) using avg. velocity.

$$\begin{aligned} &= \frac{1}{2} \times (\rho A v_{avg}) v_{avg}^2 \times \alpha \\ &= \frac{1}{2} \rho A v_{avg}^3 \times \alpha \quad \text{--- (ii)} \end{aligned}$$

From (i) & (ii)

$$\frac{1}{2} \rho \int v_{act}^2 \cdot dA = \alpha \frac{1}{2} \rho A v_{avg}^3$$

$$\alpha = \frac{\int v_{act}^2 \cdot dA}{A v_{avg}^3}$$

Here, to find v_{avg} use continuity eqn

$$Q = A v_{avg} = \int_A v \cdot dA$$

$$v_{avg} = \frac{\int_A v \cdot dA}{A}$$

Hence Bernoulli's eqn using KE correction factor is given by

$$\frac{p}{\rho} + \frac{\alpha v_{avg}^2}{2g} + y = \text{const}$$

- A similar "Momentum correction factor" (β) is defined in momentum eqn

$$\beta \left[\overset{\text{Rate of mass}}{m} \cdot v_{avg} \right] = \int_A dm \cdot v_{act} \quad Q = Av$$

$$\beta \left[\rho A \cdot v_{avg} \right] v_{avg} = \int \rho \cdot dA \cdot v_{act} \cdot v_{act}$$

$$\beta = \frac{\int v_{act}^2 \cdot dA}{A v_{avg}^2}$$

Here also,

$$v_{avg} = \frac{\int_A v \cdot dA}{A}$$

8) velocity profile in circular pipe is given by .

$$v = v_0 \left(1 - \frac{r^2}{R^2} \right).$$

flow is laminar. compute α & β .

Sol (a) KE correction factor (α) :-

$$\alpha = \frac{\int v_{act}^3 \cdot dA}{A \cdot v_{avg}^3}$$

$$v_{avg} = \frac{\int_A v \cdot dA}{A}$$

$$= \frac{1}{A} \int v_0 \left(1 - \frac{r^2}{R^2} \right) \cdot 2\pi r \cdot dr$$

$$= \frac{2\pi}{A} \int v_0 \left(1 - \frac{r^2}{R^2} \right) \cdot r \cdot dr$$

$$= \frac{2\pi}{A} \int_0^R v_0 \cdot r - v_0 \frac{r^3}{R^2} \cdot dr$$

$$= \frac{2\pi}{\pi R^2} v_0 \left(\frac{1}{2} R^2 - \frac{1}{4R^2} \cdot R^4 \right) = \frac{2v_0}{R^2} \left(\frac{R^2}{2} - \frac{R^2}{4} \right)$$

$$v_{avg} = \frac{v_0}{2}$$

$$\alpha = \frac{\int_0^R \left\{ v_0 \left(1 - \frac{r^2}{R^2} \right) \right\}^3 \cdot 2\pi r \cdot dr}{\pi R^2 \cdot (v_0/2)^3}$$

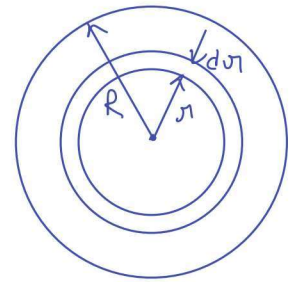
$$\alpha = \frac{16}{R^2} v_0^3 \cdot \int_0^R v_0^3 \left(1 - \frac{r^2}{R^2} \right)^3 \cdot r \cdot dr$$

$$\alpha = \frac{16}{R^2} \int_0^1 t^3 \cdot \frac{R^2}{2} dt$$

$$\alpha = 8 \cdot \frac{1}{4} (1) = 2$$

$$\boxed{\alpha = 2}$$

For laminar flow, in circular pipe.



$$\text{let, } 1 - \frac{r^2}{R^2} = t$$

$$- \frac{2r}{R^2} dr = dt$$

$$r \cdot dr = -\frac{R^2}{2} dt$$

r	0	R
t	1	0

NOTE:- • For laminar flow condition

(a) In circular pipe, $\alpha = 2$

(b) In parallel plate, $\alpha = 1.573$

• For turbulent flow condition

(a) In circular pipe, $\alpha = (1.03 - 1.06)$

• For ideal flow condition

For all pipes, $\alpha = 1$

(b) Momentum correction factor (β).

$$\beta = \frac{\int v_{act}^2 \cdot dA}{A v_{avg}^2}$$

$$\beta = \frac{\int v_0^2 \left(1 - \frac{r^2}{R^2}\right)^2 \cdot 2\pi r \cdot dr}{\pi R^2 \cdot (v_0/2)^2}$$

$$\beta = \frac{8}{R^2 \cdot v_0^2} \cdot v_0^2 \int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 \cdot r \cdot dr$$

$$\beta = \frac{8}{R^2} \int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 \cdot r \cdot dr$$

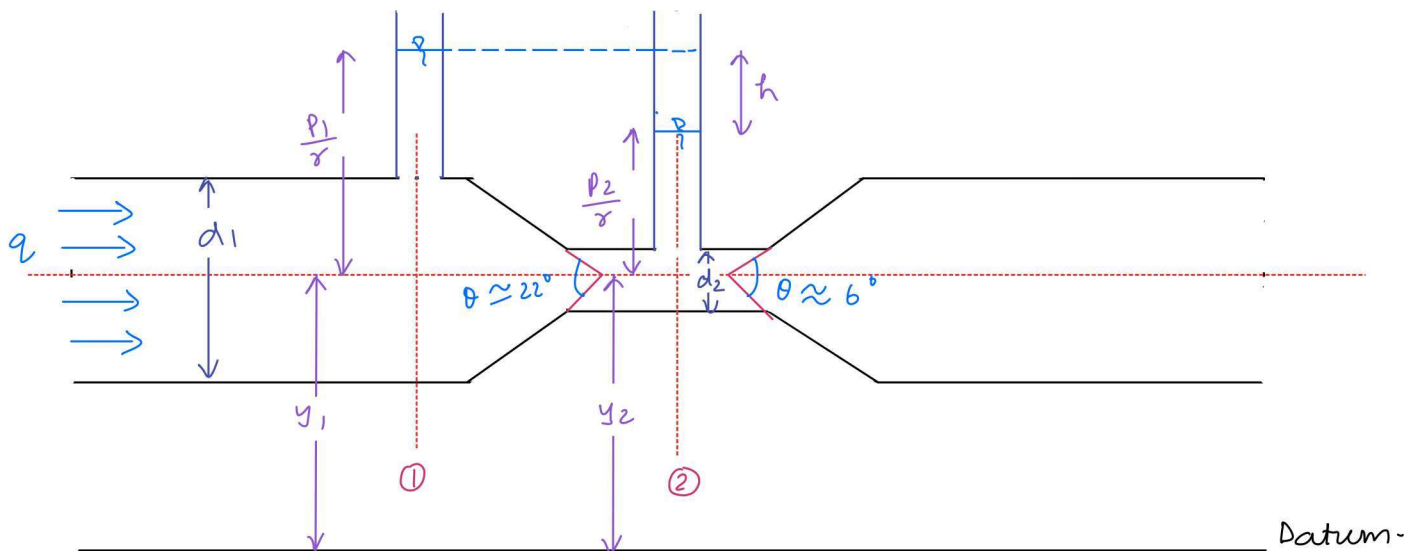
$$\beta = \frac{7}{3} (1) = 4/3$$

$$\boxed{\beta = 4/3}$$

Aug 18 # APPLICATION OF BERNOULLI'S EQN :-

(a) VENTURIMETER :-

- It is a device used to find out the discharge through a pipeline.
- working principle of venturimeter includes reduction in area at throat that results in increase in velocity in steady flow, which further causes decrease in pressure, that gives the discharge.



Applying Bernoulli's eqn at ① and ②.

$$\frac{P_1}{\rho} + y_1 + \frac{v_1^2}{2g} = \frac{P_2}{\rho} + y_2 + \frac{v_2^2}{2g}$$

$$\left(\frac{P_1}{\rho} + y_1 \right) - \left(\frac{P_2}{\rho} + y_2 \right) = \frac{v_2^2 - v_1^2}{2g} = h$$

h = piezometric head difference.

$$\frac{v_2^2 - v_1^2}{2g} = h$$

$$\left(\frac{Q}{A_2} \right)^2 - \left(\frac{Q}{A_1} \right)^2 = 2gh$$

$$Q^2 \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right) = 2gh$$

$$Q = \frac{2gh}{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right)} = \frac{2gh A_1^2 A_2^2}{A_1^2 - A_2^2}$$

$$Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

- In above discharge (theoretical) losses are not being considered, then a actual discharge is given by

$$Q_{act} = C_d Q_{the.}$$

C_d = coeff. of discharge which accounts for losses in pipe section.

$$Q_{act} = C_d A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}} \quad - (i)$$

here, $A_1 A_2 \sqrt{\frac{2g}{A_1^2 - A_2^2}} = K$, termed as venturi constant.

$$\therefore Q_{act} = C_d \cdot K \cdot \sqrt{h}$$

- To compute coeff. of discharge, losses can be considered in Energy eqn

$$\frac{p_1}{\gamma} + y_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + y_2 + \frac{v_2^2}{2g} + h_L$$

Souvik 2021

$$\left(\frac{p_1}{\gamma} + y_1\right) - \left(\frac{p_2}{\gamma} + y_2\right) = \frac{v_2^2 - v_1^2}{2g} + h_L$$

$$h = \frac{v_2^2 - v_1^2}{2g} + h_L$$

$$v_2^2 - v_1^2 = 2g(h - h_L)$$

$$\left(\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_1}\right)^2 = 2g(h - h_L)$$

$$Q^2 \left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2} \right) = 2g(h - h_L)$$

$$Q = 2g(h - h_L) \frac{A_1^2 A_2^2}{A_1^2 - A_2^2}$$

losses are considered $\leftarrow$

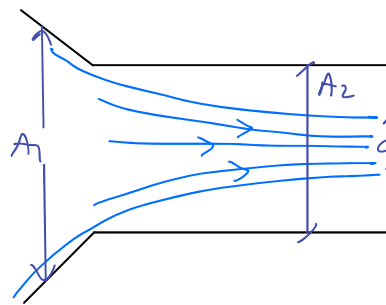
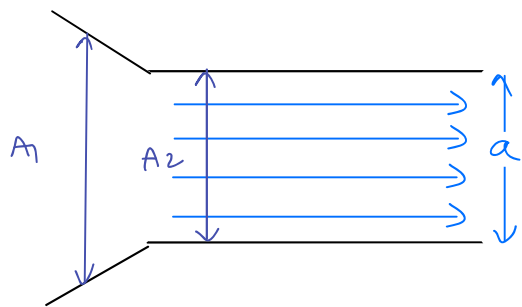
$$Q_{act} = A_1 A_2 \sqrt{\frac{2g(h - h_L)}{A_1^2 - A_2^2}} \quad - (ii)$$

From (i) + (ii)

$$\frac{C_d \cdot A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{A_1 A_2 \sqrt{2g(h - h_L)}}{\sqrt{A_1^2 - A_2^2}}$$

$$C_d = \sqrt{\frac{h - h_L}{h}}$$

- In venturimeter gradual contraction and expansion is ensured to avoid flow separation, hence
(a) a_2 can be taken as complete area of throat.



Area is varying due to flow separation

NOTE:- Here reduction in area of flow is expressed in terms of coeff of contraction.

In 1st case $\frac{Sovik}{2020} c_c = \frac{a}{a_1} = 1$

2nd case $c_c = \frac{a}{a_2} < 1$

(b) hence losses due to eddy formation does not take place (hence value of C_d is comparatively more ≈ 0.98). ESENOTES.COM

- Since tendency of flow separation is more during expansion than contraction, angle of divergence (6°) is kept less than angle of convergence (22°). and the section is not used for discharge measurement, rather contraction section is used.

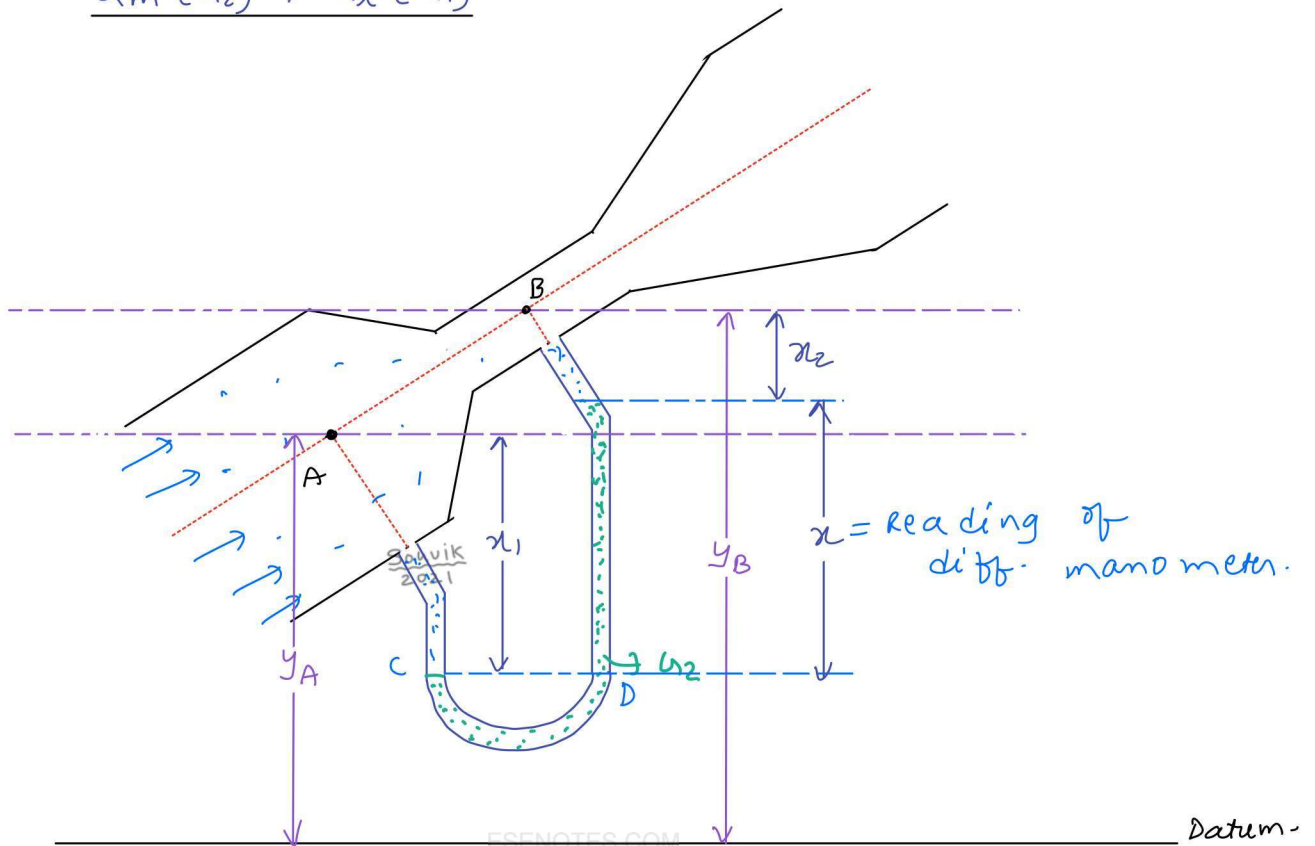
- Cross-sectional area of throat cannot be reduced beyond a certain limit as pressure may fall below vapour pressure leading to cavitation.

hence, $d_2 = \left(\frac{1}{3} \text{ to } \frac{3}{4}\right) d_1$.

d_2 = diam. of throat, d_1 = dia of pipe.

INCLINED VENTURI METER WITH DIFFERENTIAL MANOMETER

case (i) $\rho_m (\rho_2) > \rho_1 (\rho_1)$



Using pascal's law,

$$P_A + \rho_1 g x_1 = P_B + \rho_1 g x_2 + \rho_2 g x$$

$$\frac{P_A - P_B}{\rho_1 g} = (x_2 - x_1) + \frac{\rho_2}{\rho_1} x$$

Because " ρ_1 " sp-gravity liquid is flowing

$$\left(\frac{P_A}{\rho_1 g} + y_A \right) - \left(\frac{P_B}{\rho_1 g} + y_B \right) = (x_2 - x_1) + \frac{\rho_2}{\rho_1} x + y_A - y_B$$

Piezometric head diff

$$h = x_2 - x_1 + y_A - y_B + \frac{\rho_2}{\rho_1} x$$

$$h = -x + \frac{\rho_2}{\rho_1} x \quad \left[(y_B - x_2) - (y_A - x_1) = x \right]$$

$$h = \left(\frac{\rho_2}{\rho_1} - 1 \right) x$$

$$Q_{act} = C_d \times Q_{th} = \frac{C_d \cdot A_1 A_2 \sqrt{2g \left(\frac{h_2}{a_1} - 1 \right) x}}{\sqrt{A_1^2 - A_2^2}}$$

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case (ii) $\rho_m (h_2) < \rho_x (a_1)$:-

$$P_A - \rho_1 r_w x_1 = P_B - \rho_1 r_w x_2 - h_2 r_w x$$

$$P_A - P_B = \rho_1 r_w (x_1 - x_2) - h_2 r_w x$$

$$\frac{P_A - P_B}{\rho_1 r_w} = x_1 - x_2 - \frac{h_2}{\rho_1} x$$

$$\left(\frac{P_A}{\rho_1 r_w} + y_A \right) - \left(\frac{P_B}{\rho_1 r_w} + y_B \right) = \frac{\rho_1 x_1 + x_2}{\rho_1} - \frac{h_2}{\rho_1} x + y_A - y_B$$

$$h = x_1 - x_2 + y_A - y_B - \frac{h_2}{\rho_1} x$$

$$h = (x_1 + y_A) - (x_2 + y_B) - \frac{h_2}{\rho_1} x$$

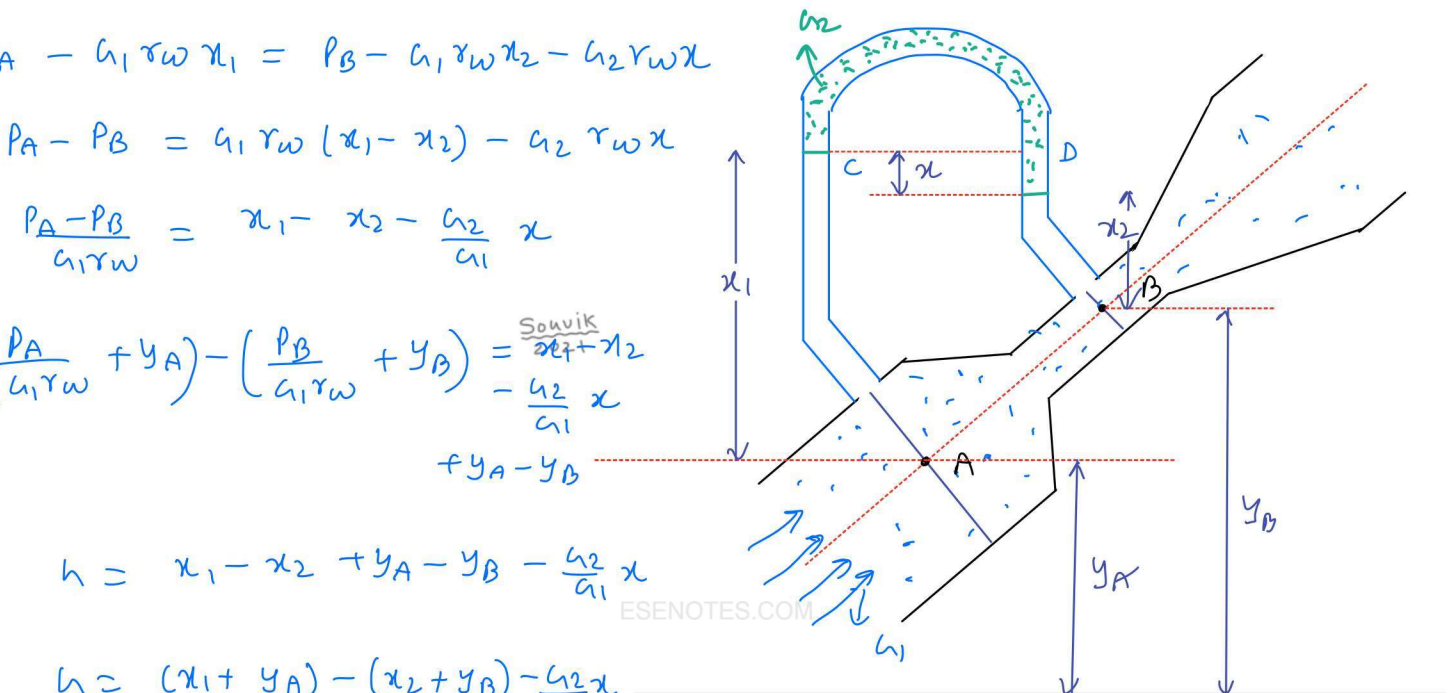
$$h = x - \frac{h_2}{\rho_1} x$$

$$h = x \left(1 - \frac{h_2}{\rho_1} \right)$$

x = Manometric Reading or difference in reading.

now,

$$Q_{act} = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g x \left(1 - \frac{h_2}{\rho_1} \right)}$$



- Q) A flow nozzle is used for measuring the discharge in laboratory. The approach flow pipe has 100 mm dia and throat dia of nozzle is 60 mm, $C_v = 0.9$. Diff. head manometer reading is 13.5 cm Hg. Find the discharge in pipe.

Sol :- piezometric head diff,

$$h = \left(\frac{h_2}{a_1} - 1 \right) x = \left(\frac{13.6}{1} - 1 \right) \times 13.5 \times 10^{-2}$$
$$h = 1.701 \text{ m.}$$

$$\frac{P_1}{\gamma} + y_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + y_2 + \frac{v_2^2}{2g}$$

$$\left(\frac{P_1}{\gamma} + y_1 \right) - \left(\frac{P_2}{\gamma} + y_2 \right) = \frac{v_2^2 - v_1^2}{2g}$$

$$h = \frac{v_2^2 - v_1^2}{2g}$$

$$v_2^2 = 2gh + v_1^2$$

$$\text{Sowvik 2021 } v_2 = \sqrt{2gh + v_1^2}$$

$$v_{2act} = C_v \cdot v_{2th}$$

$$\frac{Q}{A_2} = C_v \sqrt{2gh + \left(\frac{Q}{A_1} \right)^2}$$

$$\left(\frac{Q}{A_2} \right)^2 = C_v^2 \left(2gh + \left(\frac{Q}{A_1} \right)^2 \right)$$

$$Q^2 \left(\frac{1}{A_2^2} - \frac{C_v^2}{A_1^2} \right) = C_v^2 2gh$$

$$Q = 0.01575 \text{ m}^3/\text{s.}$$

- 8) A venturimeter is installed in a pipe line 500 mm dia, the throat pipe dia ratio is $\frac{1}{2}$. The pressure in the pipe line is 1.5 kg/cm^2 and at throat is 40 cm Hg (vacuum). If 4% of differential head is lost between the gauges. compute the flow in pipeline.

Sol

$$D_1 = 500 \text{ mm} \quad p_A = 1.5 \text{ kg/cm}^2$$
$$D_2 = 250 \text{ mm}$$

1st approach :- (Cd - approach)

$$\frac{P_1}{\gamma} + y_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + y_2 + \frac{v_2^2}{2g}$$

$$\left(\frac{P_1}{\gamma} + y_1 \right) - \left(\frac{P_2}{\gamma} + y_2 \right) = \frac{v_2^2 - v_1^2}{2g} = h$$

Assuming $y_1 = y_2$ i.e horizontal.

$$P_1 = 1.5 \text{ kg/cm}^2$$

$$P = \rho g h$$

$$\frac{P_1}{\rho_1} = \frac{1.5 \text{ (kg)} \frac{1}{(10^{-2})^2 \text{ m}^2} 1000 \text{ kg}}{1000 \text{ kg}} = 15 \text{ m}$$

$$\frac{P_2}{\rho_2} = \frac{-40 \times 10^{-2} \text{ m} \times 13.6 \times \rho_w}{\rho_w} = -5.44 \text{ m}$$

$$\text{Diff. head, } h = 15 - (-5.44) = 20.44 \text{ m}$$

$$\text{Now, } h_L = 4\% \text{ of } h$$

$$C_d = \sqrt{\frac{h - h_L}{h}} = \sqrt{\frac{1 - 0.04}{1}} = 0.98$$

$$Q = C_d \cdot \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$Q = 0.98 \times \frac{\pi}{4} (0.5)^2 + \frac{\pi}{4} (0.5)^2 \sqrt{\frac{2 \times 9.81 \times 20.44}{\frac{\pi}{4} (0.5^2 - \frac{0.5^2}{4})}}$$

$$Q = 2.22 \text{ m}^3/\text{s}$$

IInd approach - Energy ESENOTES.COM

$$\frac{P_1}{\rho_1} + y_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho_2} + y_2 + \frac{V_2^2}{2g} + h_L$$

$$\text{Assume } y_1 = y_2, \left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_1} \right) = h = \frac{V_2^2 - V_1^2}{2g} + h_L$$

$$h = 20.44 \text{ m}$$

$$h_L = 4\% \text{ of } h = 0.04 h = 0.817 \text{ m}$$

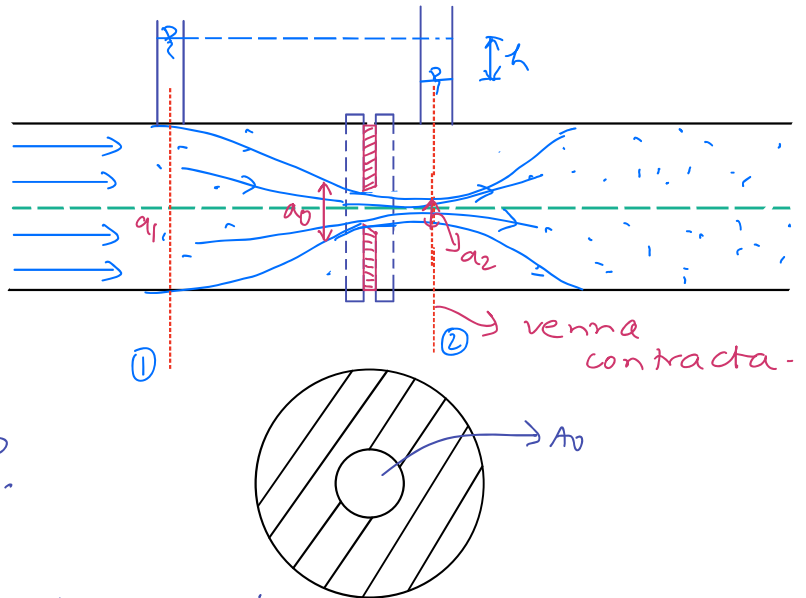
$$20.44 = \frac{Q^2}{2g} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right) + 0.817$$

$$385 = Q^2 \left\{ \frac{1}{\left(\frac{\pi}{4} \times 0.25^2 \right)^2} - \frac{1}{\left(\frac{\pi}{4} \times 0.5 \right)^2} \right\}$$

$$Q = 0.98 \text{ m}^3/\text{s}$$

(B) ORIFICE METER :-

- It is used to measure the discharge through pipe.
- In this case a circular plate with concentric sharp edged hole such that the plate is $\perp$ to the axis of pipe.



- It is comparatively cheaper than venturimeter as its size is small, hence it finds its application if there is space restriction. Souvik 2021
- As flow separation takes place in this case head loss is more, which also results in lower value of C_d (≈ 0.6).

NOTE :- 1) The region where flow area is minimum is termed as vena-contracta (vc).

2) As it is difficult to find area (a_2) at vena contracta, it is expressed in terms of parameter "coeff. of contraction."

$$C_c = \frac{a_2}{a_0}$$

Applying Bernoulli's eqn

$$\frac{P_1}{\gamma} + y_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + y_2 + \frac{v_2^2}{2g}$$

$$\left(\frac{P_1}{\gamma} + y_1 \right) - \left(\frac{P_2}{\gamma} + y_2 \right) = \frac{v_2^2 - v_1^2}{2g} = h.$$

$$\left(\frac{Q}{A_1} \right)^2 - \left(\frac{Q}{A_2} \right)^2 = 2gh.$$

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

But a_2 is unknown, hence, $a_2 = C_c \cdot a_0$

$$Q_{th} = \frac{a_1 c_c a_0 \sqrt{2gh}}{\sqrt{a_1^2 - (c_c a_0)^2}}$$

$$Q_{act} = \frac{c_d \cdot c_c a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - (c_c a_0)^2}}$$

Aug 20

$$V_{act} = C_v \cdot V_{theor.}$$

$$V_a = C_v \cdot V$$

$$\text{Also, } a_2 = c_c a_0$$

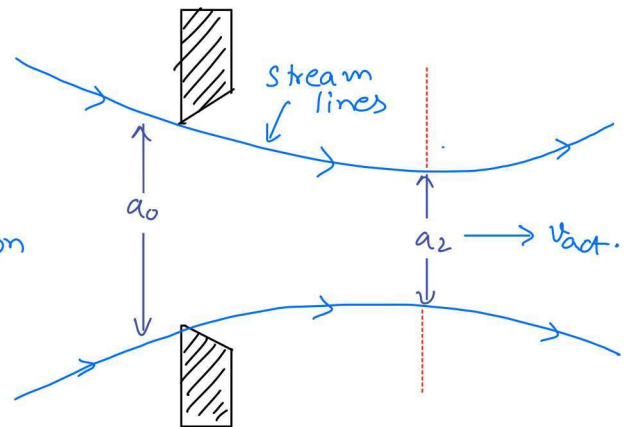
$$Q_{act} = V_a \times a_2$$

$$Q_a = c_c \cdot C_v \cdot a_0 \cdot V$$

$$Q_a = C_d Q_{th}$$

c_c = coeff of contraction

Souvik
2021



Here, C_d = coeff. of discharge at venna contracta

$$C_d = C_c \cdot C_v$$

coeff of discharge = coeff of contraction x coeff of velocity.

Q_{th} = theoretical discharge at orifice

$$Q_a = \frac{C_d \cdot a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

- If area of flow is not reduced after passing through orifice i.e. $a_0 = a_2$

$$C_c = 1$$

$$\therefore C_d = C_v$$

- At vena contracta, streamlines are straight lines hence piezometric head is constant throughout the section.

- Q3) An orificemeter is being used to measure flow rate of a liquid in a pipe. piezometer shows a pressure difference of "x" m of water column, when the flow rate is Q. If the flow rate is doubled the pressure diff in ht of water column (m) will be?

Sol

$$Q \propto \sqrt{h}$$

$$Q_1 = K \sqrt{h_1} \quad Q_2 = K \sqrt{h_2}$$

$$\frac{Q_2}{Q_1} = \frac{\sqrt{h_2}}{\sqrt{h_1}}$$

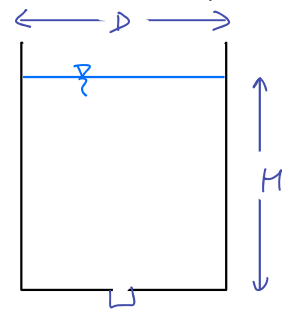
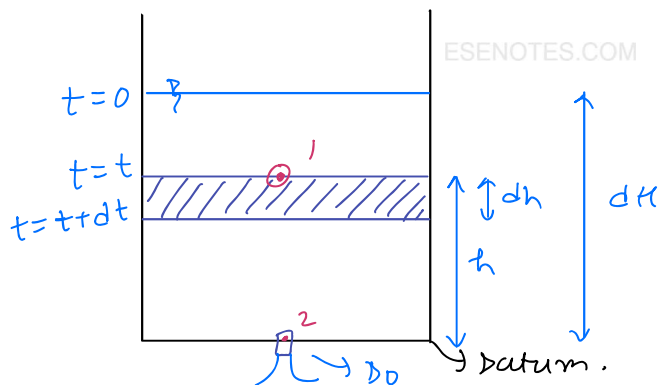
$$\text{Now, } Q_2 = 2Q_1 = 2Q$$

$$\frac{2Q}{Q} = \sqrt{\frac{h_2}{x}}$$

$$h_2 = 4x$$

- Q4) Find the time required to empty half cylindrical tank which is full initially to ht "H".

Sol



Applying Bernoulli's Eqn at ① & ②

Area is large

$$0 + \cancel{0} + h = 0 + \frac{v_2^2}{2g} + 0$$

point "2" at the interface.

$$P_2 = 0 \text{ (atmospheric)}$$

$$P_1 = 0 \text{ (atmospheric)}$$

$$v_2 = \sqrt{2gh}$$

vol. of water drained out from tank in time "dt"

$$dV = A \cdot dh = Q \cdot dt$$

$$-\frac{\pi D^2}{4} \cdot dh = \frac{\pi D_0^2}{4} \cdot \sqrt{2gh} \cdot dt$$

$$dt = \frac{D^2}{D_0^2} \frac{dh}{\sqrt{2gh}}$$

$$-\int_0^t dt = \left(\frac{D}{D_0}\right)^2 \frac{1}{\sqrt{2g}} \int_h^{H/2} \frac{dh}{\sqrt{h}}$$

$$-t = \left(\frac{D}{D_0}\right)^2 \frac{1}{\sqrt{2g}} \left[2\sqrt{h}\right]_H^{H/2}$$

$$t = -\left(\frac{D}{D_0}\right)^2 \frac{\sqrt{2}}{\sqrt{g}} \left(\sqrt{H/2} - \sqrt{H}\right)$$

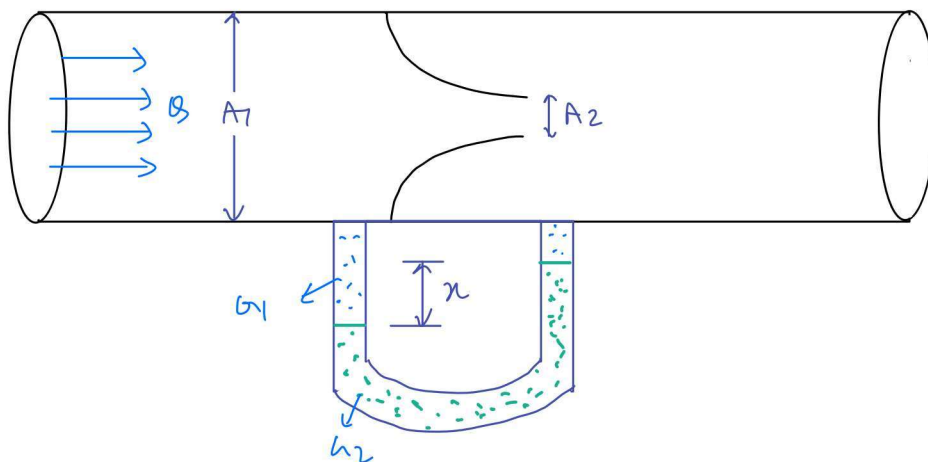
$$t = \left(\frac{D}{D_0}\right)^2 \frac{2}{\sqrt{2g}} \left(\sqrt{H} - \sqrt{H/2}\right)$$

$$t = \left(\frac{D}{D_0}\right)^2 \frac{1}{g} \left(\sqrt{2H} - \sqrt{H}\right)$$

NOTE, $t \propto D^2$ (Dia of tank)
 $t \propto \frac{1}{D_0^2}$ (Dia of orifice).
Souvik 2021

(c) NOZZLE METER :-

- The nozzle meter is truncated form of venturimeter without the diverging portion.
- It is simply a contraction with well rounded entrance placed in the pipe line.
- It is simpler than venturimeter and can be installed easily between flanges.
- Here $C_c = 1$. So, $C_d = C_v$.



$$Q_a = \frac{C_d \cdot a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$h = x \left(\frac{h_2}{a_1} - 1 \right)$$

C_d can be taken as C_v .

(D) ELBOW METER :-

- It is also used to measure discharge in pipe.
- It is based on principle when liquid moves along a pipe bend it follows "FREE VORTEX" condition.

i.e. $vR = \text{const}$, $v = \frac{c}{R}$

- Since $R_1 > R_2$
 $v_1 < v_2$

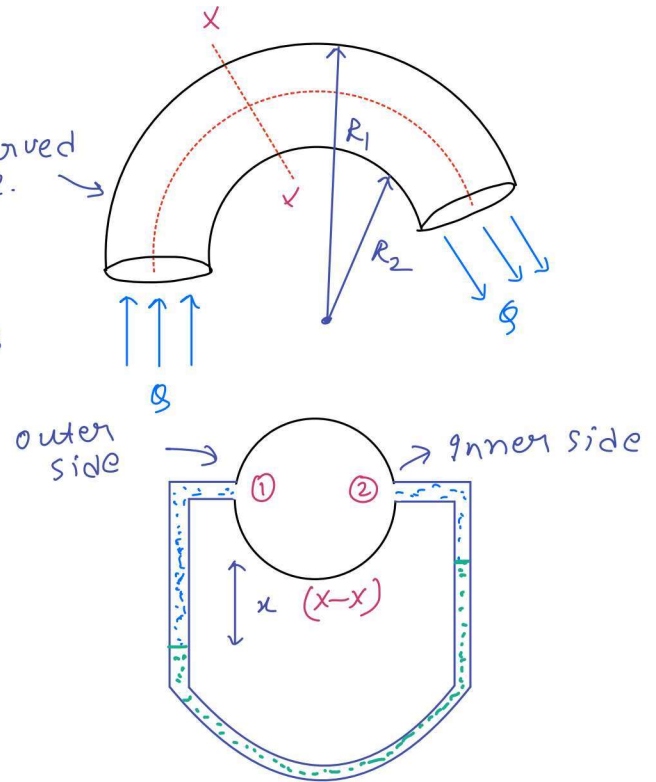
From energy conservation.

$$P_1 > P_2 \quad \text{Sowvik 2021}$$

$$Q_a = C_d A \sqrt{2gh}$$

$$h = x \left(\frac{v_2}{v_1} - 1 \right)$$

A = Area of pipe.



(E) PITOT TUBE :-

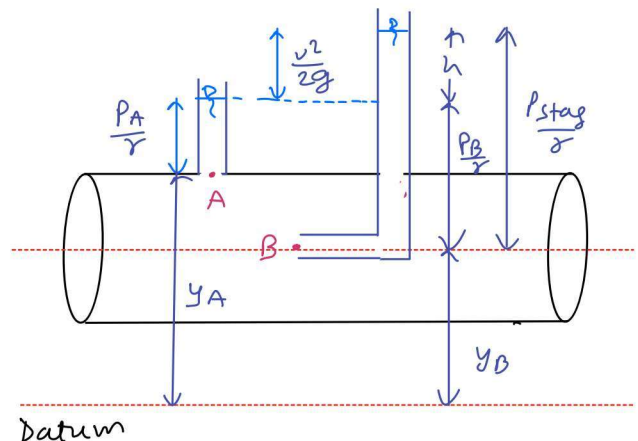
- It is used to measure velocity of fluid when section of the pipe is straight.
- Difference in reading of pitot tube and piezometer is noted which indicates the velocity head.

$$h = \frac{v_B^2}{2g} \quad v_B \rightarrow \text{velocity at B}$$

$$v_B = \sqrt{2gh}$$

$$v_{\text{act}} = C_v \sqrt{2gh}$$

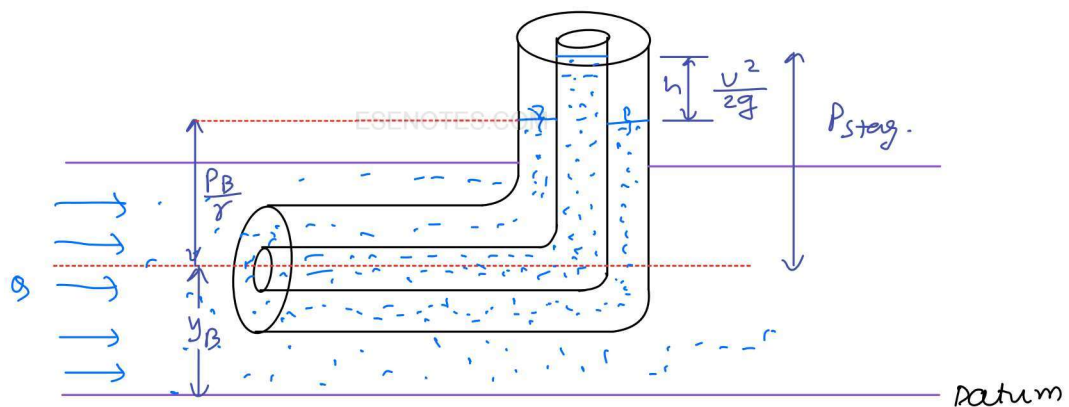
$$C_v \approx 0.98 \text{ for pitot tube}$$



NOTE:- **Anemometer** is used to measure air/gas velocity.

Aug 21 (F) PITOT STATIC TUBE / PRANDTL TUBE :-

- This is also used to measure the velocity at a point in fluid flow especially when streamlines are curved.
- As if streamlines are curved piezometric head is not constant throughout the section.
- Hence in such case, it is necessary to bound out piezometric head at same point at which velocity is to be measured.
- Hence pitot static tube / prandtl tube is used as it satisfies the above purpose.
- The front portion of the tube is rounded in order to ^{Souvik} avoid separation of flow and it shaft holes are provided at certain distance where stream line becomes parallel.



$$\frac{v^2}{2g} = h$$

$$v_{th} = \sqrt{2gh}$$

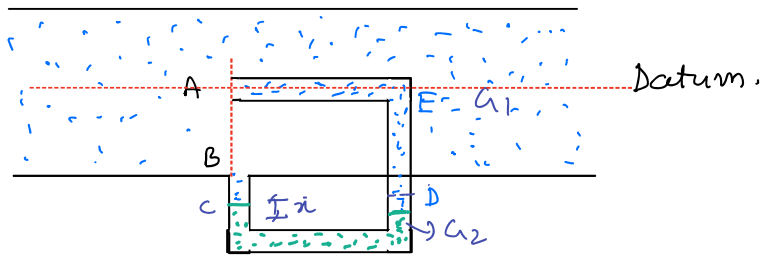
$$v_{act} = C_v \cdot v_{th}$$

$C_v \approx 0.99$ for prandtl tube.

8) For the given case compute velocity at point A.

$$\text{sol} = \frac{-V_A^2}{2g} = h = x \left(\frac{u_2}{u_1} - 1 \right)$$

$$V_A = \sqrt{2g x \left(1 - \frac{u_2}{u_1} \right)}$$



9) A pitot tube records reading of 7.85 kN/m^2 as the stagnation pressure when it is held at the centre of pipe of 250 mm conveying water. The static pressure in the pipe is 40 mm of mercury (gauge-vacuum). Compute the discharge through the pipe assuming that the mean velocity of flow is 0.8 times the max velocity ^{Souvik 2021}, $C_d = 0.98$.

sol =

$$P_{\text{stag}} = P_{\text{static}} + P_{\text{dynamic}}$$

$$P_{\text{static}} = \frac{P}{\rho_{\text{Hg}}} = -40 \text{ mm} ; \quad \frac{P}{\rho_w} = -40 \times 13.6 \times \frac{\rho_w}{\rho_w}$$

$$P_{\text{stag}} = \frac{P_s}{\rho_w} = \frac{7.85}{9.81} = 0.8 \text{ m} \quad \begin{aligned} &= -544 \text{ mm} \\ &= -0.544 \text{ m} \end{aligned}$$

$$P_{\text{dyn}} = P_{\text{stag}} - P_{\text{st.}}$$

$$\frac{V^2}{2g} = 0.8 - (-0.544)$$

$$V^2 = 26.36 \text{ m/s}$$

$$V_{\text{th}} = 5.1$$

$$\text{Now, } C_d = C_c C_v$$

$$\text{As } C_c = 1$$

$$\therefore C_d = C_v$$

$$V_{\text{act}} = C_v \cdot V_{\text{th}} = 0.98 \times 5.1$$

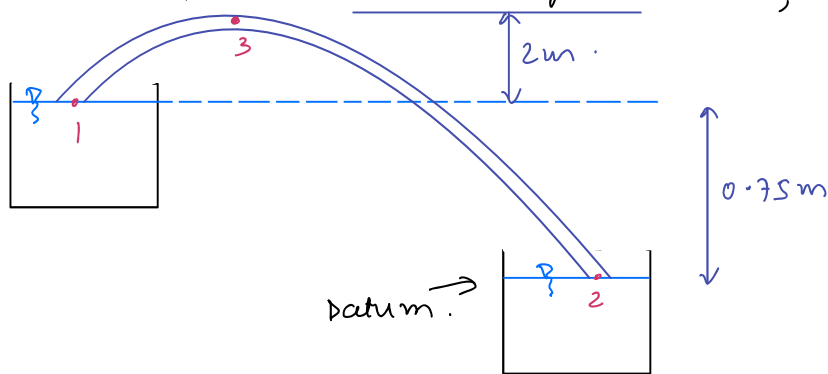
$$V_{\text{max}} = 5.03 \text{ m/s}$$

at centre.
and pitot
tube is installed
at centre

$$V_{\text{mean}} = 0.8 \times V_{\text{max}} = 4.02 \text{ m/s}$$

$$Q = A \cdot V_{\text{mean}} = \frac{\pi}{4} \times (250)^2 \times 4.02 = 0.197 \text{ m}^3/\text{s}$$

- 8) If siphon = 5mm, friction loss in siphon is neglected compute
 (i) determine the time of with draw 4 lit of petrol from ① to ②.
 (ii) Pressure at point ③, $\rho_{\text{petrol}} = 750 \text{ kg/m}^3$



Sol Applying energy eqn at ① and ②.

$$P_1 + z_1 + \frac{v_1^2}{2g} = P_2 + z_2 + \frac{v_2^2}{2g}$$

$$P_1 = 0 \text{ (atm)} \quad P_2 = 0$$

$$z_1 = 0.75 \text{ m} \quad z_2 = 0$$

$$v_1 = 0$$

$$0 + 0.75 + 0 = 0 + 0 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g \cdot 0.75} = 3.83 \text{ m/s.}$$

$$\text{Discharge, } Q = A \cdot v = \frac{\pi}{4} \times (0.005)^2 \times 3.83 = 7.52 \times 10^{-5} \frac{\text{m}^3}{\text{s}}$$

$$Q = 7.52 \times 10^{-5} \times 10^3 \text{ l/s.}$$

$$= 7.52 \times 10^{-2} \text{ l/s}$$

$$\text{For } V = 4 \text{ l.}$$

$$t = \frac{4}{7.52 \times 10^{-2}} \text{ s} = 53.2 \text{ sec.}$$

(ii) using energy eqn b/w pt ① and ③

$$P_1 + z_1 + \frac{v_1^2}{2g} = P_3 + z_3 + \frac{v_3^2}{2g}$$

$$0 + 0.75 + 0 = P_3 + 2.75 + (0.75)$$

$$P_3 = -2.75$$

$$\frac{\text{Pressure}}{\rho_{\text{petrol}}} = -2.75$$

$$(\text{Pressure})_3 = -2.75 \times 750 \frac{\text{kg}}{\text{m}^3} = -2062.5 \frac{\text{kg}}{\text{m}^2}$$

(since $A = \text{const}$).

$$\left. \begin{aligned} v_2 &= v_3 \\ \therefore \frac{v_2^2}{2g} &= \frac{v_3^2}{2g} = 0.75 \end{aligned} \right\}$$

Using Energy eqn b/w ② and ③.

$$P_2 + z_2 + \frac{v_2^2}{2g} = P_3 + z_3 + \frac{v_3^2}{2g}$$

$$0 + 0 + \frac{v_2^2}{2g} = P_3 + 2.75 + \frac{v_3^2}{2g}$$

since $A = \text{const}$, $v_2 = v_3$

$$\therefore P_3 = -2.75$$

$$(\text{Pressure})_3 = -2062.5 \text{ kg/m}^3$$

NOTE:- ① if $P_3 < \text{vapour pressure}$ boiling will start.

② if in que., it is asked to find that height where boiling will start.

put, $P_3 = \text{vapour pressure}$ and assume height "h".

Aug 23

Q) A siphon pumps water from a large reservoir to a lower tank that is initially empty. The tank also has a rounded orifice 6m below the reservoir surface where water leaves the tank. Both the siphon and orifice dia are 5 cm. Ignoring friction loss find the height up to which the water will rise in the tank at equilibrium.

Sol = Applying Bernoulli's eqn b/w ① and ③

$$\frac{P_1}{\rho_w} + z_1 + \frac{v_1^2}{2g} = \frac{P_3}{\rho_w} + z_3 + \frac{v_3^2}{2g}$$

$$P_1 = 0, \quad P_3 = \rho_w h$$

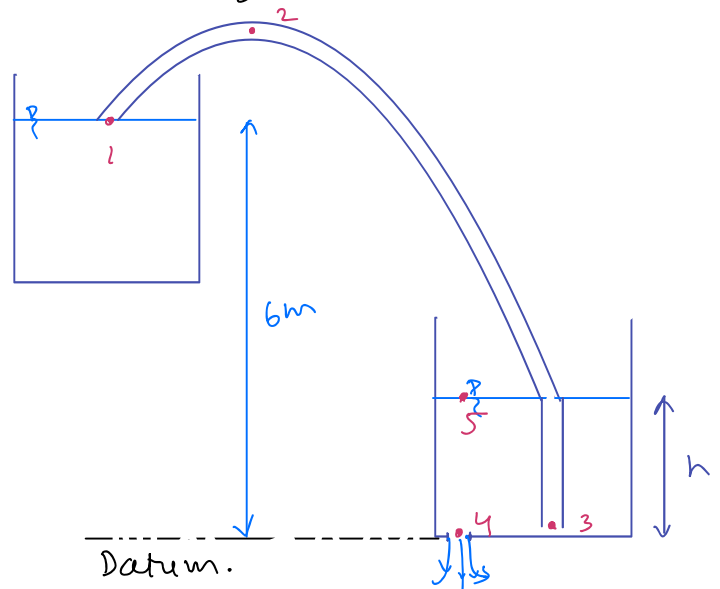
$$z_1 = 6, \quad v_1 = 0.$$

$$0 + 6 + 0 = \frac{\rho_w h}{\rho_w} + 0 + \frac{v_3^2}{2g}$$

$$v_3 = \sqrt{(6-h)2g}$$

Applying Bernoulli's eqn b/w 5 & 4

$$\frac{P_5}{\rho_w} + z_5 + \frac{v_5^2}{2g} = \frac{P_4}{\rho_w} + z_4 + \frac{v_4^2}{2g}$$



$$P_5 = 0, \quad z_5 = h, \quad v_5 = 0$$

$$P_4 = 0, \quad z_4 = 0,$$

$$0 + h + 0 = 0 + 0 + \frac{v_4^2}{2g}$$

$$v_4 = \sqrt{2gh}$$

Now, for the tank to be in equilibrium, discharge from orifice and syphon should be same. Since their A is constant

Hence, velocity must be same.

$$\text{i.e. } v_3 = v_4$$

$$\sqrt{2g(6-h)} = \sqrt{2gh}$$

Souvik
2021

$$6-h = h$$

$$h = 3\text{m}$$

- 8) A air velocity in the duct of a heating system is to be measured by pitot static tube being inserted in to the duct parallel to the flow, if the differential height b/w the water column connected to the two outlet of the probe is 2.5cm, compute.

(a) Flow velocity.

(b) pressure rise at the tip of the probe.

Sol :- $h = 2.5\text{cm}$

From pascals law.

$$P_D = P_E$$

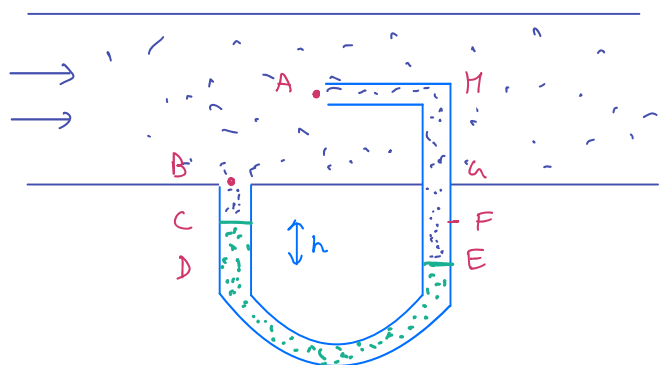
$$P_B + \rho_1 r_w (BC) + \rho_2 r_w (h) = P_A + \rho_1 r_w (ME)$$

$$\frac{P_B}{\rho_w} + \rho_1 (BC) + \rho_2 h = \frac{P_A}{\rho_w} + \rho_1 (ME)$$

$$ME = MA + MF + FE$$

$$\frac{P_B}{\rho_w} + \rho_1 (BC) + \rho_2 h = \frac{P_A}{\rho_w} + \rho_1 (MA + MF + FE)$$

$$\frac{P_B}{\rho_w} - \frac{P_A}{\rho_w} = \rho_1 (MA) + (\rho_1 - \rho_2) h \quad - (i)$$



Applying Bernoulli's eqn b/w A and B.

$$\frac{P_A}{\rho w} + Z_A = \frac{P_B}{\rho w} + Z_B + \frac{V_B^2}{2g}$$

$$Z_A = AB, \quad Z_B = 0, \quad V_B = 0.$$

$$\frac{P_A}{\rho w} + \frac{V_A^2}{2g} + AB = \frac{P_B}{\rho w}$$

$$-\left\{h_1 (1+h) + (h_1 - h_2)h\right\} = \frac{V_A^2}{2g} + AB$$

$$1+h = AB$$

$$\frac{V_A^2}{2g} + AB(1+h_1) + (h_1 - h_2)h = 0.$$

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MOMENTUM EQUATION & ITS APPLICATION :-

- Momentum is a vector quantity and its direction is given by the direction of velocity.
- To apply the momentum eqn for a flowing fluid, we have to consider the volume of fluid termed as control volume.
- control volume is selected in such a way that the force needed to be calculated becomes the external force to the control volume.
- Now according to momentum eqn,

$$\text{Net force on CV in given direction} = \text{change in momentum flux in same direction} \\ (\Sigma F) \quad \left(\Delta \frac{mv}{t} \right)$$

$$\Sigma F = \left(\text{Momentum flux going out of CV in given direction} \right) - \left(\text{Momentum flux coming into the CV in given direction} \right)$$

$$\Sigma F = \frac{m_2 v_2}{t} - \frac{m_1 v_1}{t}$$

$$\frac{m_1 v_1}{t} = \frac{\rho V v_1}{t} = \rho g v_1$$

$$\therefore \Sigma F = \rho_2 g v_2 - \rho_1 g v_1$$

- The above exp is valid for steady flow condition.

- If flow is unsteady, add one more term

$\frac{d(mv)}{dt}$ on right hand side.

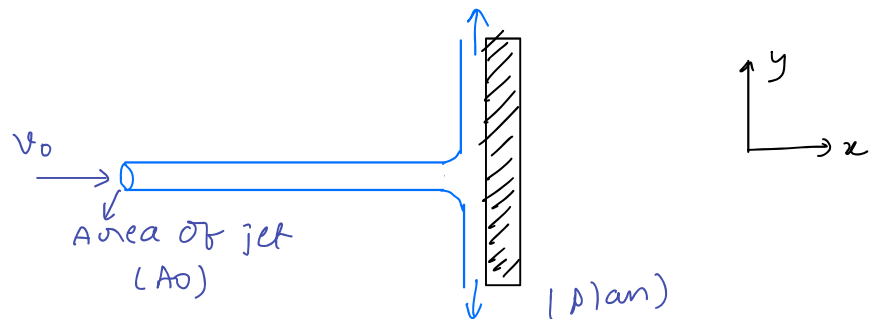
m = mass of fluid in the cv
 v = velocity of cv in given direction.
 mv = momentum of cv in given direction.

v_1 and v_2 are velocity component for outgoing and incoming fluid in given direction.

NOTE:- 1) v_2 and v_1 can be absolute velocity or relative velocity.

2) velocity w.r.t ground is called ABSOLUTE velocity and velocity w.r.t some other reference is termed as ^{Souvik 2021} RELATIVE velocity.

Q) Find the force exerted by the jet on the stationary plate assuming (a) plate is friction (b) impact loss is negligible (c) movement of jet is in horizontal direction (d) pressure throughout the jet is atmospheric.

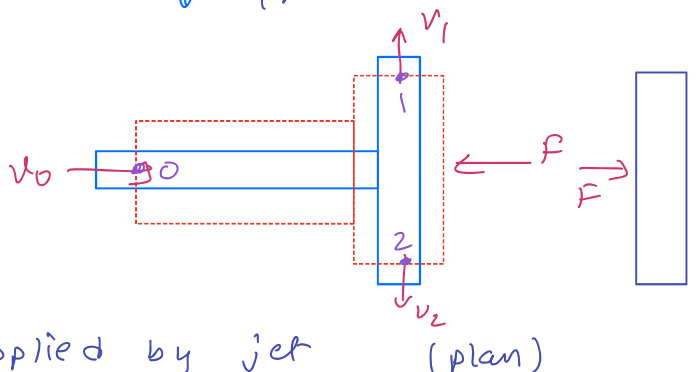


Sol Applying momentum eqn

$$\sum F_x = 0 - \rho_1 P Q_0 v_0$$

$$-F = -\rho Q_0 v_0$$

$$\boxed{F = \rho A v_0^2} \rightarrow \text{force applied by jet over the plate.}$$



using energy eqn b/w pt. 0 and 1.

$$0 + 0 + \frac{v_0^2}{2g} = 0 + 0 + \frac{v_1^2}{2g}$$

$$v_0 = v_2 \quad \text{Similarly, } v_0 = v_2, v_1 = v_2$$

mom. eqn in y-dirn.

$$\Sigma F_y = (p \theta_1 v_1 - p \theta_2 v_2) - 0.$$

↳ Since the plate is frictionless, $\therefore$ Force in y-dirn is zero.

$$0 = (p \theta_1 v_1 - p \theta_2 v_2)$$

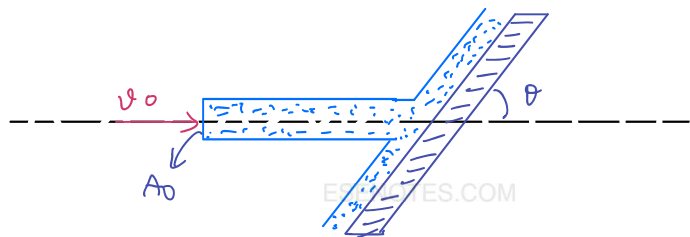
$$\theta_1 v_1 = \theta_2 v_2 \quad (v_1 = v_2)$$

$$\theta_1 = \theta_2 \quad - (i)$$

$$\text{At (v), } \theta = \theta_1 + \theta_2$$

$$\frac{\theta}{2} = \theta_1 = \theta_2$$

Aug 24 8) Find the force on the given force system.



sol For the control volume,

$$\Sigma F_x = 0 - p v_0 \theta_0 \sin \theta$$

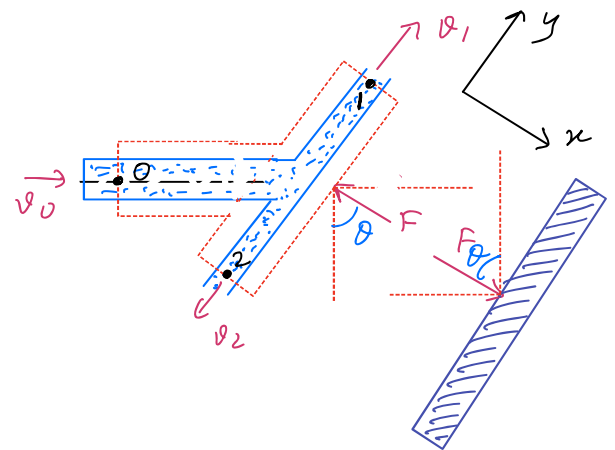
and from energy eqn

$$v_0 = v_1 = v_2$$

$$\Sigma F_x = -p v_0 \theta_0 \sin \theta$$

$$-F = -p v_0 A v_0 \sin \theta$$

$$F = p A v_0^2 \sin \theta$$



Since $\theta_1 < \theta_2$, $\therefore \theta_1 > \theta_2$

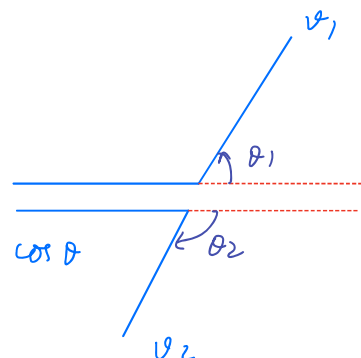
$$\Sigma F_y = (p \theta_1 v_1 - p \theta_2 v_2) - p v_0 \theta_0 \cos \theta$$

$$0 = p v_0 (\theta_1 - \theta_2 - \theta_0 \cos \theta)$$

$$\theta_1 - \theta_2 = \theta \cos \theta$$

$$\theta_1 + \theta_2 = \theta$$

$$2\theta_1 = \theta (1 + \cos \theta) \quad \theta_1 = \frac{\theta}{2} (1 + \cos \theta)$$



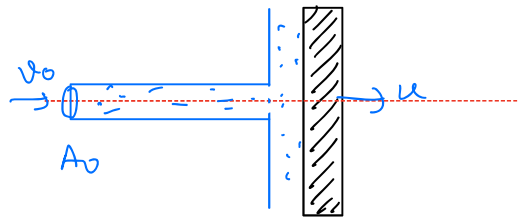
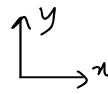
$$\theta_2 = \frac{\theta}{2} (1 - \cos \theta)$$

$$\therefore (\theta_1 > \theta_2) \checkmark$$

if $\theta = 90^\circ$, $\theta_1 = \theta_2$.

8) Relative velocity

Applying mom eqn along x-direction



$$\sum F_x = (\text{mom. flux})_0 - (\text{mom. flux})_i \quad A_0$$

$$-F = \rho \theta u - \rho \theta v_0$$

$$F = \rho \theta (v_0 - u) \quad (v_0 - u) \rightarrow \text{Relative velocity}$$

$$\theta = A (v_0 - u)$$

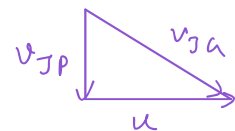
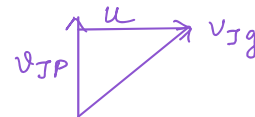
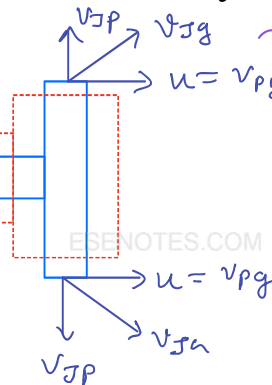
$$\therefore F = \rho A (v_0 - u)^2$$

IInd approach, Force ^{Solve it} in terms of absolute velocity.

v_{JG} = vel. of Jet wrt ground

$$v_0 = v_{JG}$$

$$v_{JG} = (v_0 - u) = v_{JP}$$



absolute vel. of jet. $\rightarrow \vec{v} = \vec{u} + \vec{v_J}$

$\vec{u}$ = vel. of plate wrt ground

$\vec{v_J}$ = vel. of jet wrt ground.

in terms of absolute velocity.

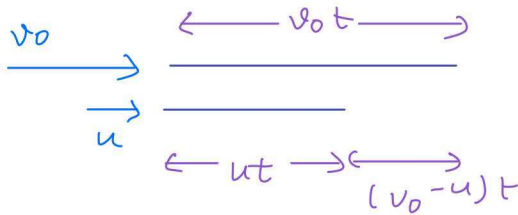
$$-F = (\rho \theta_1 u + \rho \theta_2 u) - \rho \theta v_0$$

$$F = \rho \theta v_0 - \rho u (\theta) \quad [\theta = \theta_1 + \theta_2]$$

$$F = \rho \theta (v_0 - u)$$

use in conventional.

Form & calculation



the length of the jet, momentum of which is changed by plate $(v_0 - u)t$

hence vol. of water whose momentum is changed.

$$V = A_0 (v_0 - u) t$$

hence discharge of water whose momentum is changed.

$$Q = \frac{V}{t} = A_0 (v_0 - u) \quad \checkmark \checkmark$$

hence, $F = \rho A (v_0 - u)^2$

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NOTE:- using energy eqn b/w 0 and 1 in terms of relative vel

$$0 + 0 + \frac{v_{j0}^2}{2g} = 0 + 0 + \frac{v_{j1}^2}{2g} + \cancel{K_L} + \text{work done by jet per unit weight (plate is stationary)}$$

$$v_{j0} = v_{j1} \quad \text{also,} \quad v_{j0} = v_{jp}$$

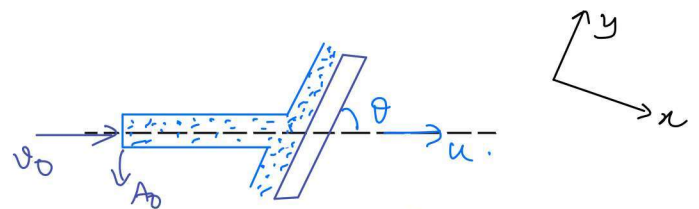
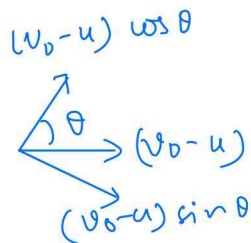
$$(v_0 - u) = v_{jp} \quad v_0 - u = v_{jp}$$

using energy eqn b/w 0 and 1 in terms of absolute.

$$0 + 0 + \frac{v_0^2}{2g} = 0 + 0 + \frac{v_{jp}^2}{2g} + \cancel{K_{L0-1}} + \text{work done by jet per unit weight}$$

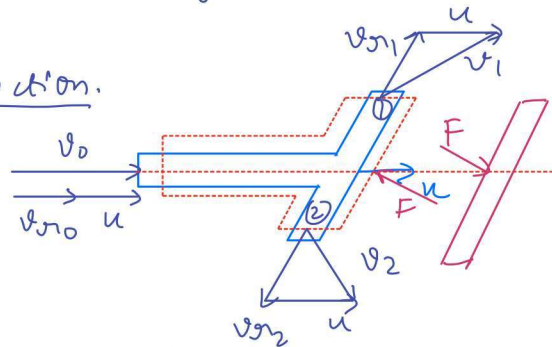
$$\frac{v_0^2}{2g} = \frac{v_{jp}^2}{2g} + \frac{F \cdot u}{\gamma Q}$$

8)



(i) Absolute velocity in x-direction.

$$-F = (\rho \theta_1 u \sin \theta + \rho \theta_2 u \sin \theta) - \rho \theta v_0 \sin \theta$$



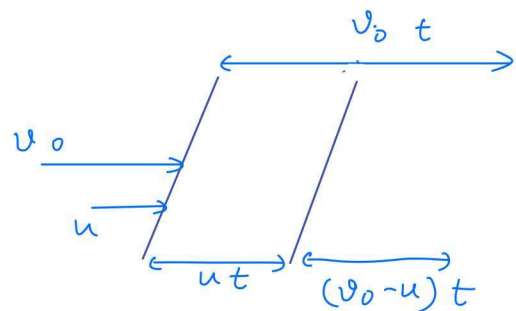
$$F = \rho \theta v_0 \sin \theta - (\rho u \sin \theta (\theta_1 + \theta_2))$$

$$= \rho \theta v_0 \sin \theta - \rho u \theta \sin \theta$$

$$F = \rho \theta \sin \theta (v_0 - u)$$

$$F = \rho \frac{(v_0 - u)t \cdot A_0}{t} (v_0 - u) \sin \theta$$

$$F = \rho A_0 (v_0 - u)^2 \sin \theta$$



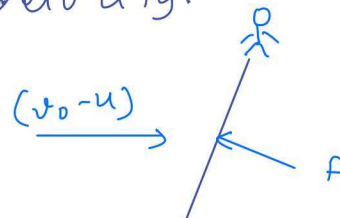
(ii) In terms of relative velocity.

$$-F = 0 - \rho \theta (v_0 - u) \sin \theta$$

$$F = \rho \theta (v_0 - u) \sin \theta$$

$$\theta = A \cdot (v_0 - u)$$

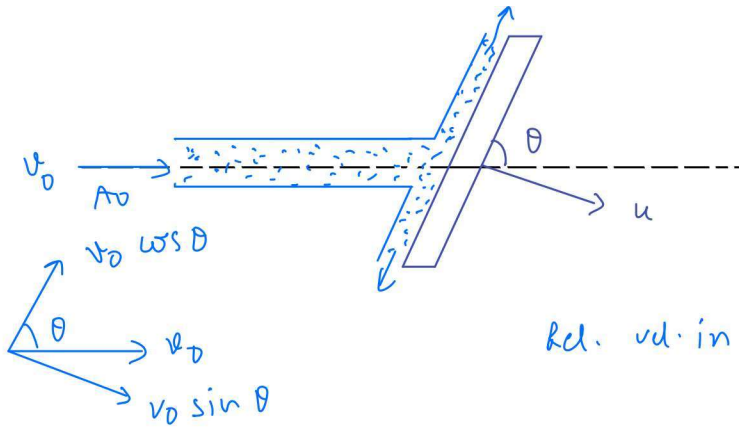
$$F = \rho (v_0 - u)^2 \sin \theta$$



Aug 28

Q)

sol



Rel. vel. in x-direction = $v_0 \sin \theta - u$

$$\Sigma F_x = (\text{Momentum})_o - (\text{Momentum})_i$$

$$-F = 0 - \rho \theta (v_0 \sin \theta - u)$$

$$F = \rho \theta (v_0 \sin \theta - u)$$

Absolute velocity

Souvik
2021

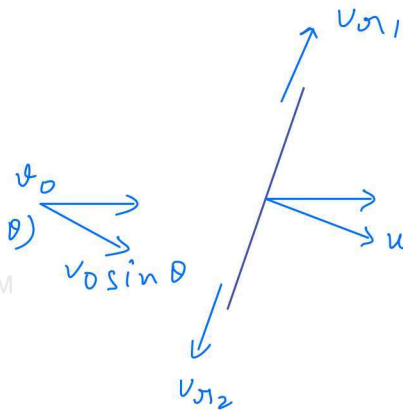
$$\Sigma F_x = (\text{mom flux})_o - (\text{mom flux})_i$$

$$-F = (\rho \theta_1 u + \rho \theta_2 u) - (\rho \theta v_0 \sin \theta)$$

$$-F = \rho u (\theta_1 + \theta_2) - \rho \theta v_0 \sin \theta$$

$$-F = \rho u \theta - \rho \theta v_0 \sin \theta$$

$$F = \rho \theta (v_0 \sin \theta - u)$$

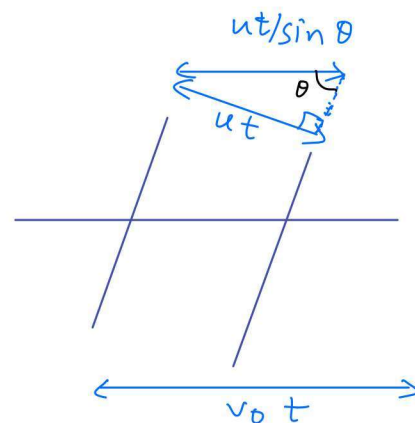


Discharge calculation

$$\text{length} = v_0 t - \frac{ut}{\sin \theta}$$

$$\text{vol} = A \left(v_0 t - \frac{ut}{\sin \theta} \right)$$

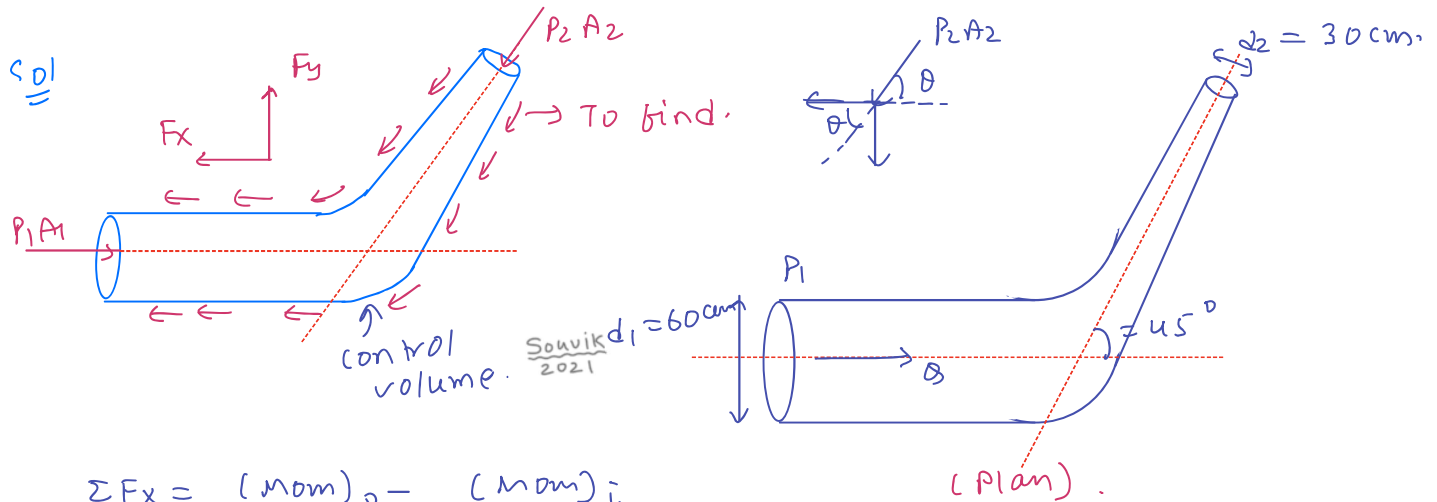
$$\theta = A \left(v_0 - \frac{u}{\sin \theta} \right)$$



$$\frac{ut}{\sin \theta} = \frac{ut}{h}$$

$$F = \rho A \left(v_0 - \frac{u}{\sin \theta} \right) (v_0 \sin \theta - u)$$

- Q) A 45° deflection angle defusing bend lies in a horizontal plane and tapers from 60 cm dia to 30 cm dia at the outlet. The pressure at the inlet is 15 kPa and discharge through the bend is $0.5 \text{ m}^3/\text{s}$. Assuming friction loss to be 20% of kinetic energy at inlet, compute the magnitude and direction of resultant force exerted by water on the bend.



$$\sum F_x = (\text{mom})_o - (\text{mom})_i$$

$$P_1 A_1 - P_2 A_2 \cos 45^\circ - F_x = \rho Q V_2 \cos 45^\circ - \rho Q V_1 - (i)$$

$$\sum F_y = (\text{mom})_o - (\text{mom})_i$$

$$F_y - P_2 A_2 \sin 45^\circ = \rho Q V_2 \sin 45^\circ - 0 - (ii)$$

$$Q = A_1 V_1 = A_2 V_2$$

$$0.5 = \frac{\pi}{4} \times (0.6)^2 V_1$$

$$V_1 = 1.77 \text{ m/s}$$

$$0.5 = \frac{\pi}{4} \times (0.3)^2 V_2$$

$$V_2 = 7 \text{ m/s}$$

Applying Bernoulli's eqn b/w two ends,

$$\frac{P_1}{\gamma} + y_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + y_2 + \frac{V_2^2}{2g} + h_L$$

$$\frac{15000}{9810} \text{ N/m}^2 + \frac{(1.77)^2}{2 \times 9.81} = \frac{P_2}{9810} + \frac{(7)^2}{2 \times 9.81} + 0.2 \times \frac{(1.77)^2}{2 \times 9.81}$$

$$P_2 = -8260 \text{ N/m}^2 = -8.26 \text{ kPa}$$

$$15 \times \frac{\pi}{4} \times (0.6)^2 - [-8.26] \pi \frac{(0.3)^2}{4} \cos 45^\circ - F_x = 10^3 \times 0.5 \times 7 \times \frac{1}{\sqrt{2}}$$

$$F_x = 3.05 \text{ (kN)}$$

Similarly.

$$F_y = (-8.26) \frac{\pi (0.3)^2}{4} \times \frac{1}{\sqrt{2}} = 1000 \times 0.5 \times 7 \times \frac{1}{\sqrt{2}}$$

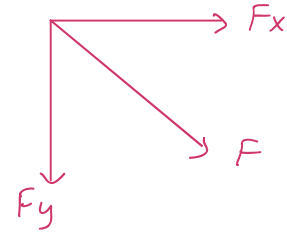
$$F_y = 2.47 \text{ (kN)}$$

NOTE :- Resultant force on bend

$$F = \sqrt{F_x^2 + F_y^2}$$

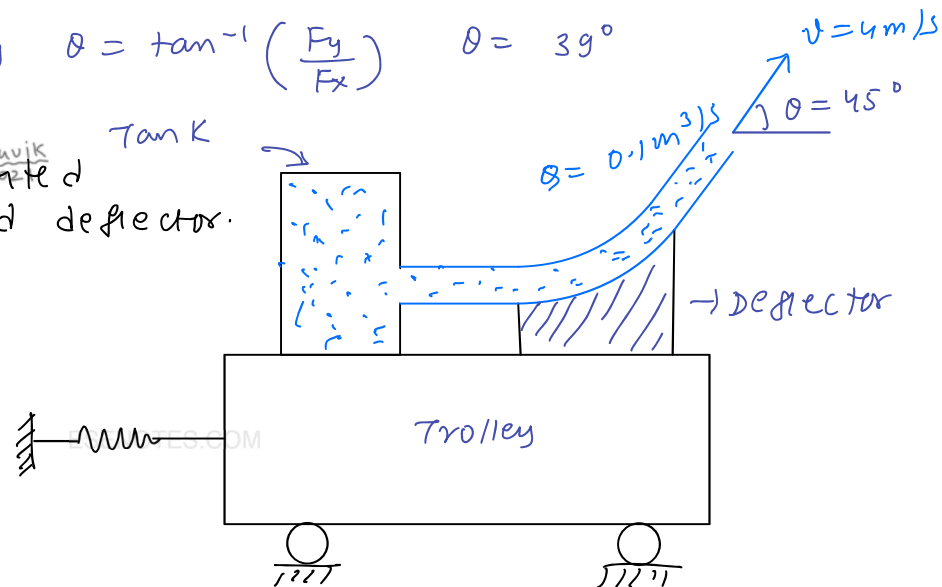
$$= \sqrt{3.05^2 + 2.47^2}$$

$$= 3.92 \text{ (kN)}$$



$$\tan \theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) \quad \theta = 39^\circ$$

8) Find the force exerted on the spring and deflector.



Sol $\Sigma F_x = (mom)_o - (mom)_i$

$$F = \rho Q v \cos 45^\circ - 0$$

$$= 1000 \frac{\text{kg}}{\text{m}^3} \times 0.1 \text{ m}^3/\text{s} \times 4 \frac{\text{m}}{\text{s}} \times \frac{1}{\sqrt{2}}$$

$$= 282.84 \text{ N}$$

Deflector

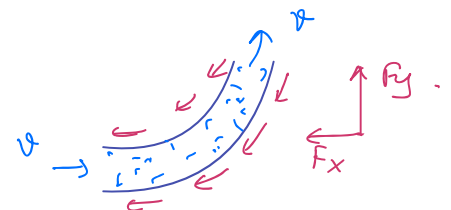
$$\Sigma F_y = (mom)_o - (mom)_i$$

$$F_y = \rho Q v \sin 45^\circ$$

$$F_y = 1000 \times 0.1 \times 4 \times \sin 45^\circ$$

$$F_y = 282.84$$

Resultant, $F = \sqrt{282.84^2 + 117.15^2}$
 $= 306.14 \text{ (N)}$



$$\Sigma F_x = (mom)_o - (mom)_i$$

$$-F_x = \rho Q v \cos 45^\circ - \rho Q v$$

$$= \rho Q v (1 - \frac{1}{\sqrt{2}})$$

$$= 1000 \times 0.1 \times 4 (1 - \frac{1}{\sqrt{2}})$$

$$F_x = 117.15 \text{ (N)}$$

Aug 30

- Q) A 30cm diameter 90° elbow has one limb vertical. The avg velocity of flow of water through the elbow is 5m/sec and the pressure intensity is 4 kPa. Find the vertical component of force required to keep the elbow in position.

Sol/ $p = 4 \text{ kPa}$ $v = 5 \text{ m/s}$

$$\Sigma F_y = (\text{Momentum})_o - (\text{mom})_i$$

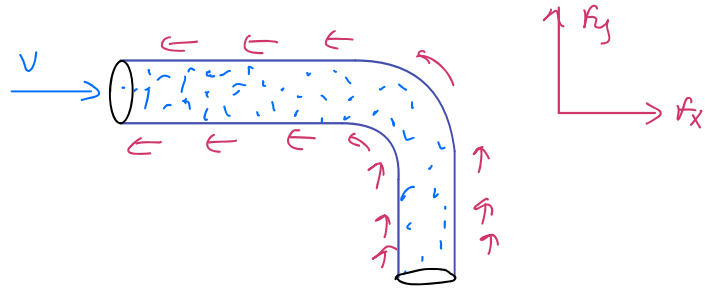
$$-PA + F_y = \rho Q v - 0$$

$$F_y = \rho A v^2 + PA$$

$$= 1000 \times \frac{\pi}{4} \times 0.3^2 \times 5^2 + 4000 \frac{\text{N}}{\text{m}^2} \times \frac{\pi}{4} \times (0.3)^2$$

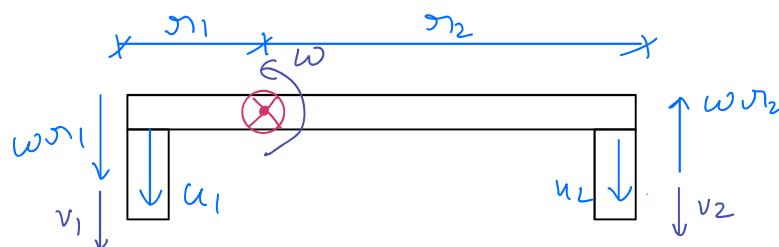
$$F_y = 2050 \text{ N} = 2.05 \text{ (kN)}$$

↳ force to be applied.



APPLICATION OF ANGULAR MOMENTUM :-

- According to this net torque on fluid in any direction is equal to change in angular momentum flux in same direction.
- It has its application in sprinkler problem.



u_1 & u_2 → relative vel. of jet at point ① and ②.
i.e. velo. of jet wrt sprinkler.

v_1 & v_2 → Absolute velocity of Jet at point ① and ②.

ωr_1 & ωr_2 → absolute velocity of sprinkler at point ① & ②.

Torque on jet in anticlock = change in angular mom flux in anticlockwise.

$$\Sigma T = [p \theta_1 v_1 r_1 - p \theta_2 v_2 r_2] - 0$$

$$T = p (A_1 u_1) v_1 r_1 - p (A_2 u_2) v_2 r_2$$

$$v_1 = \omega r_1 + u_1$$

$$v_2 = \omega r_1 - u_2$$

$$T = p (A_1 u_1) (\omega r_1 + u_1) r_1 - p (A_2 u_2) (\omega r_1 - u_2) r_2$$

also Torque applied by jet on sprinkler is same in magnitude but opposite in direction.

also, if ^{Souvik} sprinkler is frictionless then torque will be 0

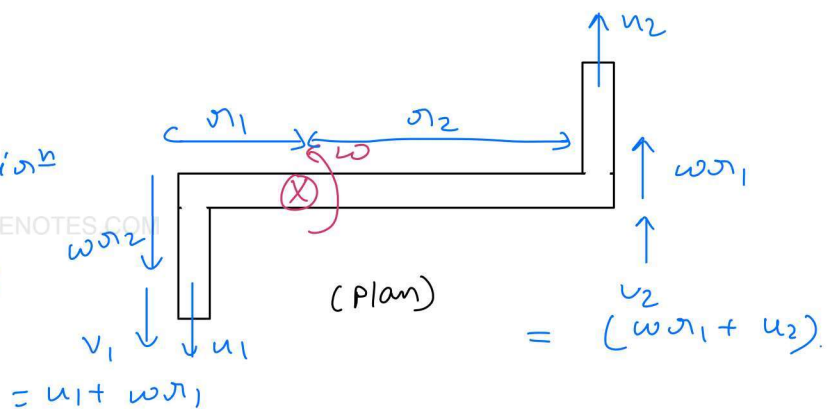
(b)

Torque on jet in ACW dirⁿ

$$= [p \theta_1 v_1 r_1 + p \theta_2 v_2 r_2]_0 - [0]_i$$

$$\theta_1 = A_1 u_1$$

$$\theta_2 = A_2 u_2$$



(c)

$$v_1 = u_1 \sin \theta_1 - \omega r_1$$

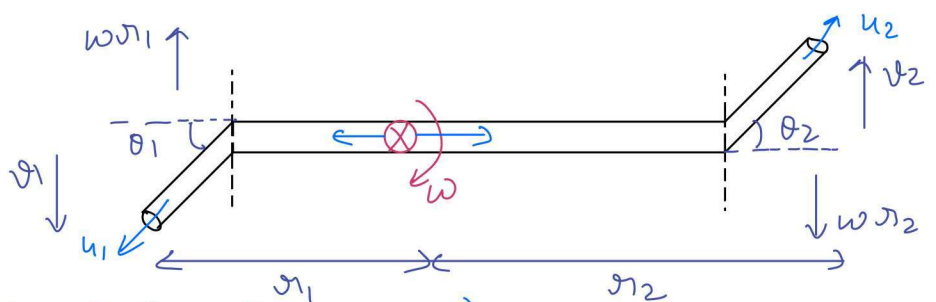
$$v_2 = u_2 \sin \theta_2 - \omega r_2$$

$$(\text{Torque})_{cw} = (\text{change in Ang. mom})_{cw}$$

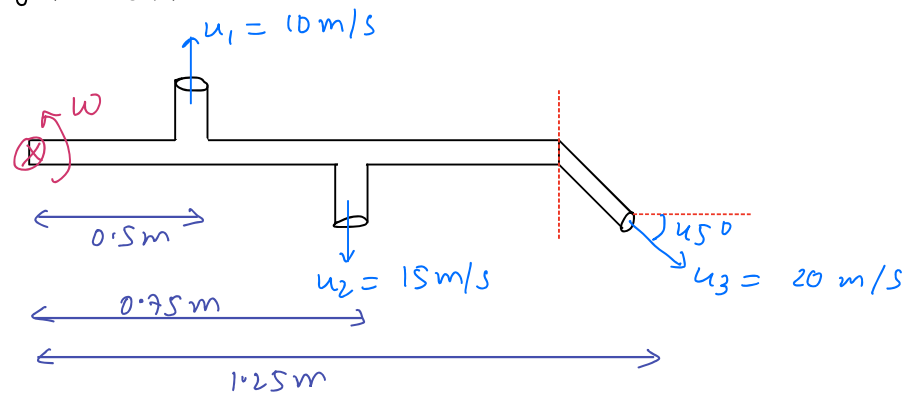
$$\Sigma T = - (p \theta_1 v_1 r_1 + p \theta_2 v_2 r_2) - (0)$$

$$\theta_1 = A_1 u_1$$

$$\theta_2 = A_2 u_2$$



- 8) Fluid passing through each nozzle is same for the relative velocity of flow through the nozzle as shown. Find the speed of rotation if system is frictionless.



Sol

$\Sigma \text{ Torque in ACW} =$
change in mom
flux in ACW

Souvik
2021

$$v_1 = \omega r_1 + u_1 = \omega 0.5 + 10$$

$$v_2 = \omega r_2 - u_2 = 0.75\omega - 15$$

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$$v_3 = \omega r_3 - u_3 \sin 45 = 1.25\omega - 10\sqrt{2}$$

$$\Sigma T = \rho g v_1 r_1 + \rho g v_2 r_2 + \rho g v_3 r_3$$

$$= \rho g r_1 (0.5\omega + 10) + \rho g r_2 (0.75\omega - 15) + \rho g r_3 (1.25\omega - 10\sqrt{2})$$

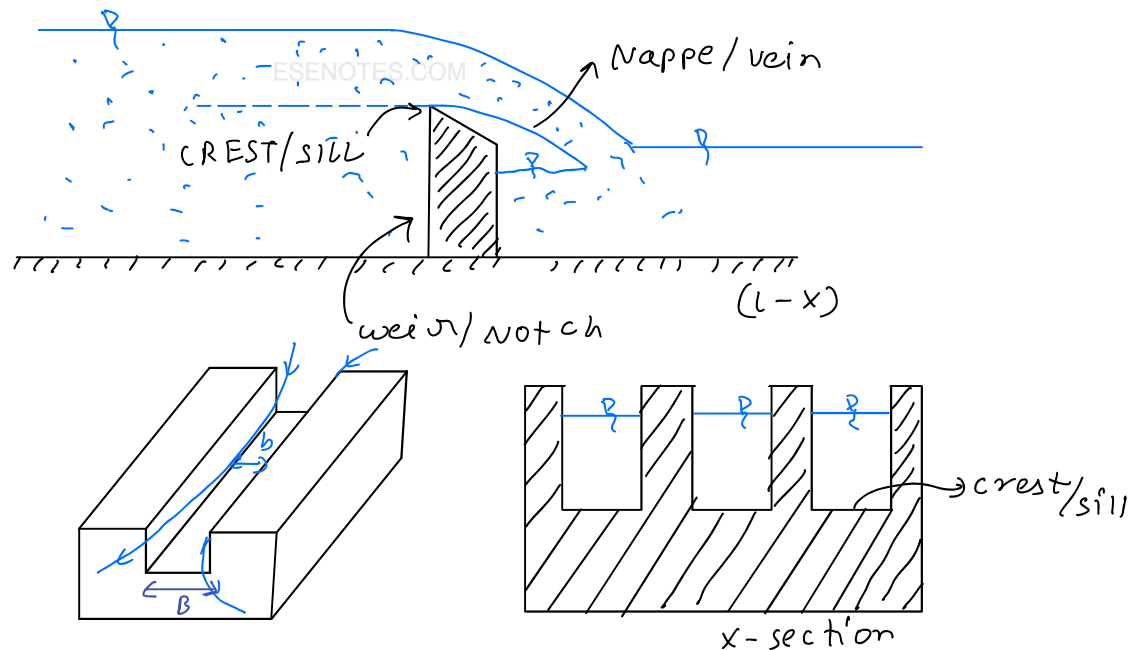
As the system is frictionless hence, $\Sigma T = 0$

$$0 = -23.92 + 2.375\omega$$

$$\omega = 10.07 \text{ rad/s.}$$

NOTCHES AND WEIRS

- A Notch is a device used for measuring the rate of flow through a small channel or a tank.
- It may be termed as an opening in side of channel or tank in such a way that the liquid surface in tank or channel is below the top edge of the opening.
- On the other hand a weir is a concrete or masonry structure, placed in a channel over which flow occurs.
- It is in form of vertical wall with sharp edge at the top.
- Notch is small in size and made up of metallic plate, whereas weir is of bigger size made of concrete or masonry.



NOTE:- Nappe/vein $\rightarrow$ It is the sheet of water flowing through the notch or over a weir.

CREST/SILL $\rightarrow$ The bottom edge of a notch or top of weir over which water flows is termed as crest

Aug 31 # TYPES OF WEIRS :-

- weirs can be classified as follows :-

(a) According to the shape of crest :-

- Sharp crested weir.
- Broad " "
- Narrow " "
- Ogee " "

(b) According to the shape of opening :-

- Rectangular weir.
- Trapezoidal weir / Cipolletti weir.

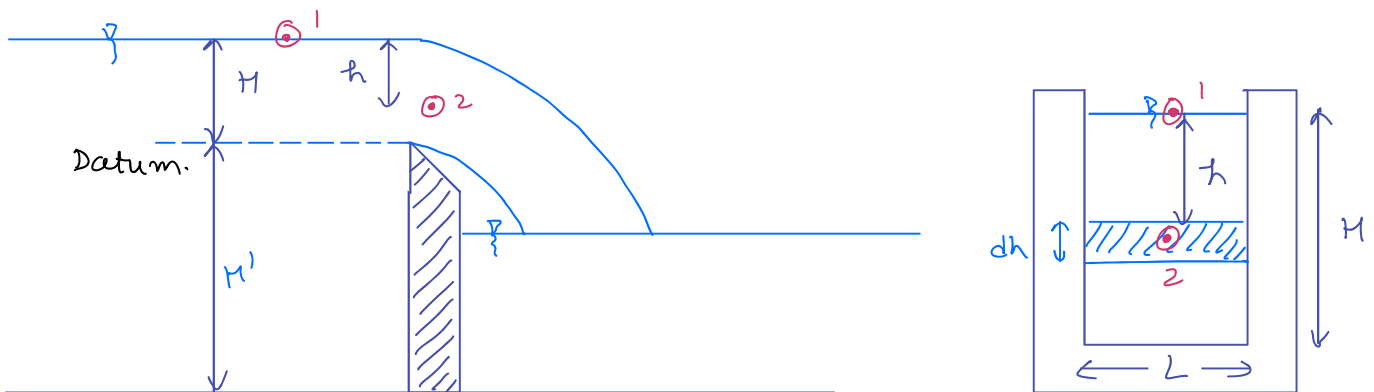
(c) According to the effect of sides on the emerging nappe

- weir with end contraction. $\rightarrow$ Suppressed
- weir without " "

(d) According to nature of discharge :-

- ordinary weir.
- submerged weir / drowned weir.

1) SHARP-CRESTED Rectangular Suppressed weir :-



H = head of water above the crest.

Using energy eqn b/w ① and ②.

$$0 + \frac{va^2}{2g} + H = 0 + \frac{v^2}{2g} + (H-h)$$

$$\frac{v^2}{2g} = \frac{va^2}{2g} + h$$

now, $v_a = \frac{Q}{(H+h)L}$ and $\frac{v_a^2}{2g} = h_a \rightarrow \text{Kinetic head}$

$$\therefore \frac{v^2}{2g} = h + h_a$$

$$v = \sqrt{2g(Lh + h_a)}$$

$$dQ = v \cdot dA = \sqrt{2g(Lh + h_a)} \cdot L \cdot dh$$

$$Q = \int dQ = \int_0^H \sqrt{2g(Lh + h_a)} L \cdot dh$$

$$= \sqrt{2g} L \int_0^H \sqrt{h + h_a} dh$$

$$Q = L \sqrt{2g} \left[(h + h_a)^{3/2} \cdot \frac{2}{3} \right]_0^H$$

$$Q = \frac{2}{3} \sqrt{2g} L \{ (H + h_a)^{3/2} - h_a^{3/2} \}$$

To account for losses due to contraction.

$$Q_{act} = \frac{2}{3} L \sqrt{2g} \{ (H + h_a)^{3/2} - h_a^{3/2} \} \cdot C_d \quad \text{--- (i)}$$

$$\{ Q \propto H^{3/2} \}$$

If area of approach is large i.e. y is large, then approach velocity v_a can be neglected.

then, $Q_a = \frac{2}{3} C_d \sqrt{2g} L H^{3/2} \quad \text{--- (ii)}$

$$C_d \approx 0.62$$

NOTE:- ① For calculation of discharge Q following steps are adopted.

(i) v_a is neglected or h_a is neglected and Q is computed from eqn (ii)

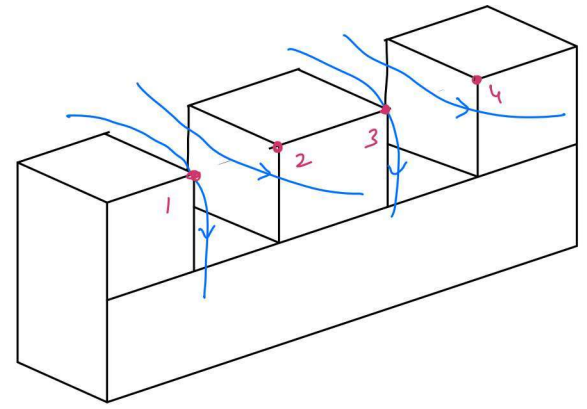
(ii) Now compute h_a and Q from eqn (i).

(iii) Repeat the above steps until Q is within 1% of preceding value.

② If there is end contraction, then for each end contraction " L " is reduced by **0.1 H**

$$L_{eff} = L - 4 \times (0.1H)$$

$$L_{eff} = L - n(0.1H)$$



4-end contraction.

③ when there is no ventilation of nappe, discharge increases as nappe is pulled down due to negative pressure created in zone below the nappe.

- This -ve pr. is developed as air in that portion is ^{Sucked} ~~drawn~~ by flowing water.

- Hence $Q' = (1.06 - 1.25)Q$.

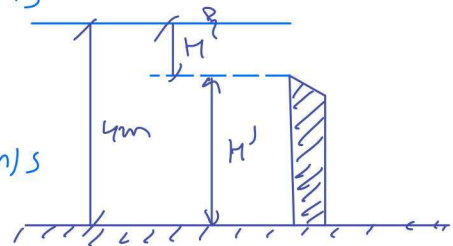
Q) A stream has a width of 30m, depth of 3m and a mean velocity of 1.25 m/s. Find the height of a weir to be built on this stream floor to raise the water level by 1m. Assume $C_d = 0.95$

sol_n $L = 30 \text{ m} \quad v = 1.25 \text{ m/s} \quad \{ Q = \text{const} \}$

$$Q = A \cdot v = (30 \times 3) \cdot 1.25 = 112.5 \text{ m}^3/\text{s}$$

Approach velocity,

$$v_a = \frac{Q}{A} = \frac{112.5}{(30 \times 4)} = 0.9375 \text{ m/s}$$



$$\text{Head, } h_a = \frac{v_a^2}{2g} = \frac{(0.9375)^2}{2 \times 9.81} = 0.044 \text{ m}$$

$$Q = \frac{2}{3} C_d \sqrt{2g} L \{ (H + h_a)^{3/2} - h_a^{3/2} \}$$

$$112.5 = \frac{2}{3} \times 0.95 \sqrt{19.62} \times 30 \{ (H + 0.04)^{3/2} - (0.04)^{3/2} \}$$

$$H = 1.175 \text{ m}$$

$$\text{Height above datum} = 4 - 1.175 = 2.83 \text{ m}$$

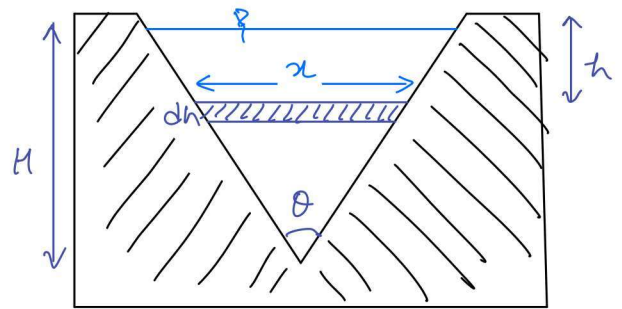
② Flow over Triangular weir or notch :-

$$dQ = v dA = v \cdot x \cdot dh$$

$$v = \sqrt{2g(h+h_a)}$$

$$\tan \frac{\theta}{2} = \frac{x/2}{H-h}$$

$$x = 2(H-h) \tan(\theta/2)$$



$$Q = \int dQ = \int \sqrt{2g(h+h_a)} \cdot 2(H-h) \tan \theta/2 dh$$

$$Q_a = C_d \cdot Q_{th}$$

$$Q_a = C_d 2 \tan \theta/2 \sqrt{2g} \int_0^H (h+h_a)^{1/2} (H-h) dh$$

$$Q = K \int_0^H [H(h+h_a)^{1/2} - h(h+h_a)^{1/2}] dh$$

$$Q_a = \frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g} [(H+h_a)^{3/2} - h_a^{3/2}]$$

$$\} Q \propto H^{3/2}$$

Here, $C_d = 0.52$

Since in this case end contraction is not taken into account, hence its effect is included in value of C_d . Thus lower value of C_d .

Advantage of Δ notch over Rect. weir :-

(i) It gives more accurate result for small discharge.

(ii) C_d is almost constant with depth, whereas it varies with head in Rect. weir.

(iii) In V-notch, head is large even for small discharge, hence flow is not affected much by viscosity and surface tension.

Sept 1

- Q) If head measured over a triangular notch as $H = 20 \text{ cm}$. Find the approach velocity which can be neglected, if discharge is to be measured within 3% accuracy.

Sol

$$H = 20 \text{ cm}$$

$$\% \text{ of accuracy} = \frac{Q_a - Q}{Q_a} \times 100 \leq 3\%$$

$$\text{we know, } Q \propto H^{5/2}$$

$$\frac{K[(h+ha)^{5/2} - ha^{5/2}] - K[h^{5/2}]}{K[(h+ha)^{5/2} - ha^{5/2}]} \leq 0.03$$

$$1 - \frac{h^{5/2}}{[(h+ha)^{5/2} - ha^{5/2}]} \leq 0.03$$

$$\frac{h^{5/2}}{[(h+ha)^{5/2} - ha^{5/2}]} \geq 0.97$$

$$\frac{0.2^{5/2}}{[(0.2 + ha)^{5/2} - ha^{5/2}]} \geq 0.97$$

$$ha = 2.458 \times 10^{-3} = 2.458 \text{ mm}$$

$$\frac{v_a^2}{2g} = ha, \quad v_a = 0.2196 \text{ m/s.}$$

Hence if $v_a \leq 0.2196 \text{ m/s}$, we can neglect it and the accuracy will be within 3%.

3) Trapezoidal weir/notch :-

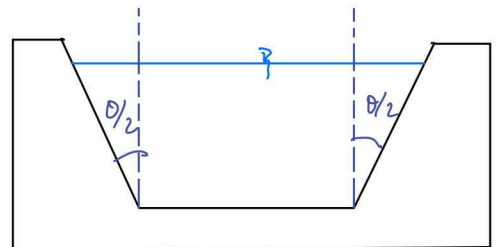
Triangular notch + Rectang. notch = Trap. notch

$$Q_T = Q_R + Q_T$$

$$Q_T = \frac{2}{3} C_{d1} \sqrt{2g} L H^{3/2}$$

$$+ \frac{8}{15} C_{d2} \sqrt{2g} \tan \frac{\theta}{2} H^{5/2} \quad [\text{neglecting } ha]$$

$$\text{But } C_{d1} = C_{d2}$$



$$Q = \frac{2}{3} C_d \sqrt{2g} H^{3/2} \left[L + \frac{4}{5} \tan \frac{\theta}{2} H \right]$$

NOTE :-

Special case :-

(i) CIPOLLETTI- WEIR

- It is a trapezoidal weir sides slopes provided are 1H : 4V

$$\tan \theta/2 = 1/4$$

$$\theta = 28^\circ$$

then, decrease in discharge over a rectangular weir due to end contraction is equal to increase in discharge through two triangular portion.

- Hence discharge of Cipolletti weir is that of rectangular weir with no end contraction

$$Q = Q_R + Q_T$$

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$$= \frac{2}{3} C_d \sqrt{2g} H^{3/2} \left[L_{eff} + \frac{4}{5} \tan(\theta/2) H \right]$$

$$\text{Here, } L_{eff} = L - 0.1H \times 2 = L - 0.2H$$

$$\tan \theta/2 = 1/4$$

$$Q = \frac{2}{3} C_d \sqrt{2g} H^{3/2} \left[L - 0.2H + \frac{4}{5} \times \frac{1}{4} H \right]$$

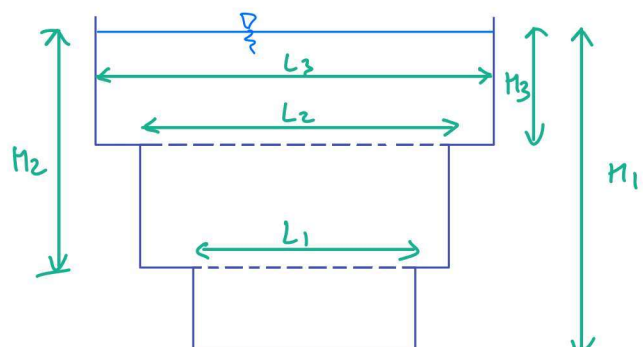
$$Q = \frac{2}{3} C_d \sqrt{2g} H^{3/2} L$$

→ Equal to Rectangular weir with no end contraction.

(ii) STEPPED NOTCH :-

- It is a combination of different rectangular notches.

$$Q = \sum Q_{Ri}$$



$$Q = \frac{2}{3} C_d \sqrt{2g} \sum_{i=1}^n L_i H_i^{3/2}$$

(4) BROAD-CRESTED WEIR :-

- Broad crested weir supports a nappe so that pressure variation is hydrostatic.

Using energy eqn at ① and ②.

$$0 + H + \frac{v_1^2}{2g} = (h - z) + z + \frac{v_2^2}{2g}$$

$$\frac{v_2^2}{2g} = H - h + h_a$$

$$v_2 = \sqrt{2g(H - h + h_a)} \rightarrow \text{constant}$$

Hence, $Q = A \cdot v_2$

$$= L \cdot h \sqrt{2g(H - h + h_a)}$$

$$Q_{act} = C_d L \cdot h \sqrt{2g(H - h + h_a)}$$

- In broad crested weir, flow adjust itself to have max discharge for the available head (H)

For Q_{max} , $\frac{dQ}{dh} = 0$.

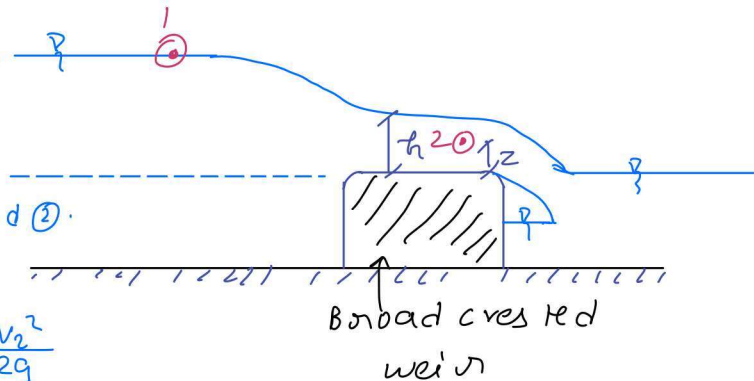
$$= C_d L \sqrt{2g} \left[\sqrt{2g(H - h + h_a)} + h \frac{1(-1)}{2\sqrt{2g(H - h + h_a)}} \right]$$

$$2(H - h + h_a) = h$$

$$h = \frac{2}{3} (H + h_a)$$

$$Q_{max} = C_d \cdot L \cdot \frac{2}{3} \cdot (H + h_a) \sqrt{2g(H - \frac{2}{3}(H + h_a) + h_a)}$$

$$\begin{aligned} Q_{max} &= C_d \cdot L \cdot \frac{2}{3} (H + h_a) \sqrt{\frac{2g}{3} (H + h_a)} \\ &= C_d \cdot L \cdot \frac{2}{3} \sqrt{\frac{2g}{3}} (H + h_a)^{3/2} \end{aligned}$$



$$Q_{\max} = 1.7 C_d \cdot L (H + h_a)^{3/2}$$

$$C_d = 0.85 - 1.$$

NOTE:- (a) Effect on discharge due to error in head measurement.

$$Q \propto H^n$$

$$Q = K H^n$$

$$dQ = K n H^{n-1} \cdot dH$$

$$\% \text{ change in } Q = \frac{dQ}{Q} \times 100 = \frac{K n H^{n-1} \cdot dH}{K H^n} = n \frac{dH}{H} \times 100$$

$$\frac{dQ}{Q} \times 100 = n \frac{dH}{H} \times 100$$

$\swarrow$ $\searrow$
 % error in discharge % error in measurement of head

$$\% \text{ error in discharge} = n \times \% \text{ error in measurement of head}$$

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→ For small error (< 5%) in head measurement, the above expression can be used to find error in discharge measurement.

→ But when error > 5%, then error in discharge measurement is given as,

$$\frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{K H_2^n - K H_1^n}{K H_1^n} \quad \%$$

(b) If there is error in the measurement of angle in V-notch, then corresponding error in discharge is given by -

$$Q \propto \tan(\theta/2)$$

$$dQ = K \sec^2(\theta/2) d\theta$$

$$\frac{dQ}{Q} \times 100 = \frac{\frac{K \sec^2(\theta/2)}{2 K \tan(\theta/2)} d\theta}{1} \times 100$$

$$\frac{dQ}{Q} \times 100 = \frac{1 \cos \theta/2}{2 \cos^2 \theta/2 \sin \theta/2} d\theta$$

$$\frac{dQ}{Q} \times 100 = \frac{d\theta}{\sin \theta} \times 100$$

$$\frac{dQ}{Q} \times 100 = \left(\frac{\theta}{\sin \theta} \right) \frac{d\theta}{\theta} \times 100 \quad \theta = \text{radian}$$

% error in discharge

% error in angle measurement.

Sept 2 (5) PROPORTIONAL WEIR :-

As we know, $Q \propto H^n$

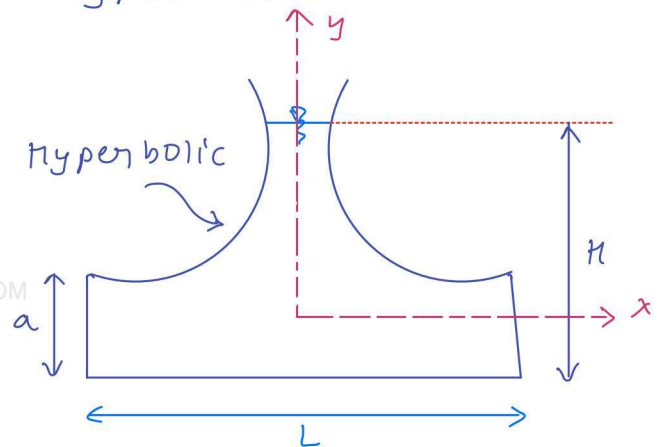
if $n=1$, $\rightarrow$ the weir is termed as proportional weir.

Surface of which is hyperbolic, given as.

$$\frac{2x}{L} = 1 - \frac{2}{\pi} \tan^{-1} \left(\sqrt{\frac{y}{a}} \right)$$

$$Q = K \left(H - \frac{a}{3} \right)$$

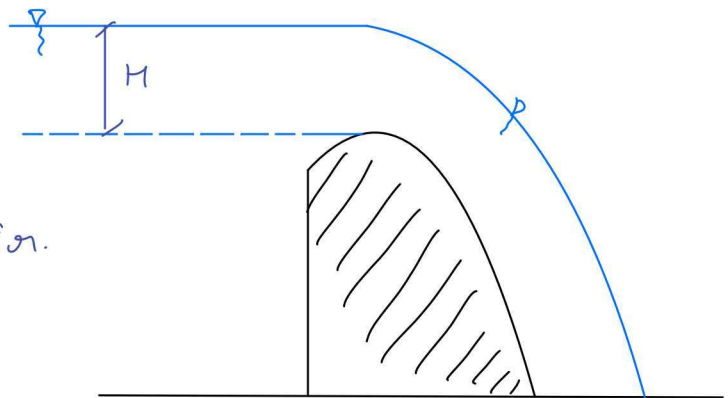
$$K = C_d \cdot L \sqrt{2ga}, \quad C_d = 0.6 - 0.65$$



(6) Ogee Spillway :-

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$\hookrightarrow$ Same as rectangular weir.



FLOW THROUGH ORIFICE & MOUTHPIECE :-

- Orifice is an opening in the tank where as mouthpiece is short length pipe/tube with length $\approx 2-3$ dia.

(a) orifice :-

Using energy eqn b/w ① and ②.

$$0 + h + 0 = 0 + 0 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2gh}$$

$$v_{2,act} = c_v \cdot \sqrt{2gh}$$

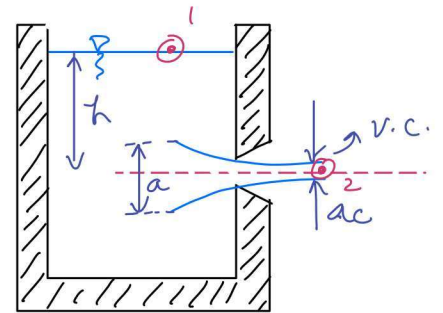
$$Q_{ac} = c_v \sqrt{2gh} \cdot a_c$$

$$\text{Now, } c_c = \frac{a_c}{a}, \quad a_c = c_c \cdot a$$

$$Q_{ac} = c_v \cdot c_c \cdot a \sqrt{2gh}$$

$$Q_{ac} = c_d \cdot a \sqrt{2gh}$$

$$c_d = c_v \cdot c_c$$



NOTE:- Losses through orifice is accounted in terms of coeff. of resistance.

$$C_{or} = \frac{\text{Loss of KE through orifice}}{\text{Actual KE.}}$$

$$C_{or} = \frac{\frac{v^2}{2g} - \frac{(c_v \cdot v)^2}{2g}}{\frac{(c_v \cdot v)^2}{2g}} = \frac{1 - c_v^2}{c_v^2}$$

$$C_{or} = \frac{1 - c_v^2}{c_v^2}$$

- Here C_{or} can be found by measuring c_v as follows.

$$x\text{-axis, } S = ut + \frac{1}{2}at^2$$

$$x = vt \quad \text{--- (i) } (a = 0 \text{ as } v = \text{const})$$

$$y\text{-axis, } S = ut + \frac{1}{2}at^2$$

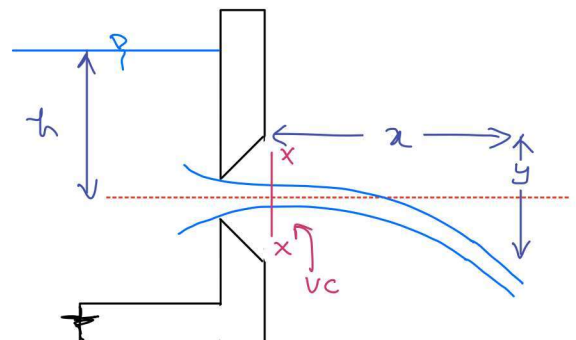
$$y = \frac{1}{2}gt^2 \quad \text{--- (ii)}$$

$$t = \frac{1}{g} \sqrt{2y}$$

$$x^2 = \frac{2v^2 y}{g}$$

$$v = \sqrt{\frac{gx^2}{2y}}$$

(Actual velocity).
Test is performed



Also, $v_{act} = C_v \cdot v_{th}$
 $v_{ac} = C_v \cdot \sqrt{2gh}$

$$\sqrt{\frac{gx^2}{2y}} = C_v \sqrt{2gh}$$

$$C_v = \sqrt{\frac{x^2}{4yh}}$$

now, C_c can be calculated.

- 8) In an experiment on determination of hydraulic coeff. of sharp edged orifice 2.5 cm of diameter. It is found that the jet issuing horizontally under a head of 1m travelled a horizontal distance of 1.6m from vena-contracta in a course of vertical drop of 0.7m from same point. Further if a flat plate were held normal to the jet at vena-con. the force of 5.6N would be exerted on the plate. Find C_c , C_v , C_d .

Sol

$$a = \frac{\pi}{4} \times (0.025)^2 = 4.9 \times 10^{-4} \text{ m}^2$$

$$h = 1\text{m}, x = 1.6\text{m}, y = 0.7\text{m}$$

$$C_v = \sqrt{\frac{x^2}{4yh}} = \sqrt{\frac{1.6^2}{4 \times 0.7 \times 1}} = 0.95$$

$$C_c = \frac{a_c}{a}$$

Applying mom. eqn

$$-F = 0 - \rho B V_c$$

$$v_c = \text{vel. at } v_c$$

$$F = \rho a_c \cdot v_c^2$$

$$5.6 \frac{\text{kg-m}}{\text{s}^2} = 1000 a_c \cdot (4.20)^2 \frac{\text{m}^2}{\text{s}^2} \cdot \frac{\text{kg}}{\text{m}^3} \quad \text{also, } v_c = \sqrt{2gh} C_v$$

$$a_c = 3.174 \times 10^{-4} \text{ m}^2 \quad v_c = 0.95 \sqrt{2 \times 9.81 \times 1} = 4.20 \text{ (m/s)}$$

$$C_c = \frac{a_c}{a} = \frac{3.174 \times 10^{-4}}{4.9 \times 10^{-4}} = 0.647$$

$$C_d = C_c \cdot C_v = 0.95 \times 0.647 = 0.614$$

- 8) Two orifices are placed in a vertical wall in such a way that lower one is 30cm above the ground, while upper one is 6m above the first one. Horizontal distance travelled by the jet from top orifice is three times than the one travelled by lower jet. Both distances are measured at ground level, $C_v = 0.98$ for both orifice. Find the head of water behind the wall.

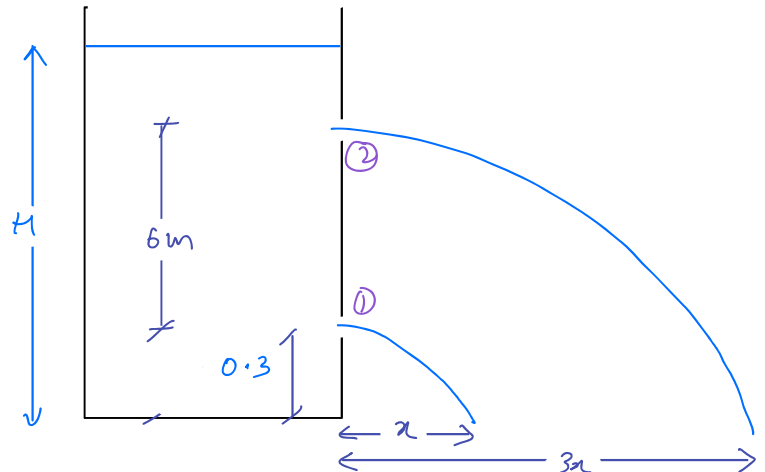
Sol

For ① orifice.

$$V_{ac} = C_v \cdot \sqrt{2gh}$$

$$= 0.98 \sqrt{2 \times (H - 0.3)}$$

Also, $C_v = \frac{x^2}{\sqrt{4yh}}$



Here, $y = 0.3m$, $h = H - 0.3$

$$0.98 = \sqrt{\frac{x^2}{4 \times 0.3 \times (H - 0.3)}} \quad \text{--- (i)}$$

For ② orifice

$y = 6.3$, $h = H - 6.3$

$$0.98 = \sqrt{\frac{9x^2}{4 \times 6.3 \times (H - 6.3)}}$$

(i) $\div$ (ii)

$$1 = \frac{x^2}{1.2 (H - 0.3)} \times \frac{25.2 (H - 6.3)}{9x^2}$$

$$10.8 H - 3.24 = 25.2 H - 158.76$$

$$14.4 H = 155.52$$

$$H = 10.8m$$

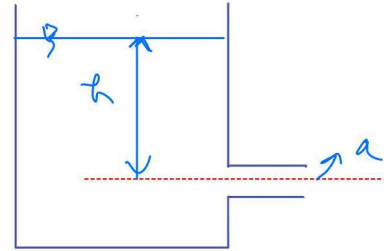
Head of water behind wall = 10.8m.

(b) MOUTHPIECE :-

$$Q = C_d \cdot a \sqrt{2gh}$$

$$C_d = 0.82$$

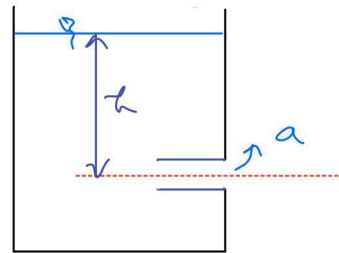
$$N.O., \quad V.C., \quad C_d = C_v.$$



(c) BORDA'S MOUTHPIECE

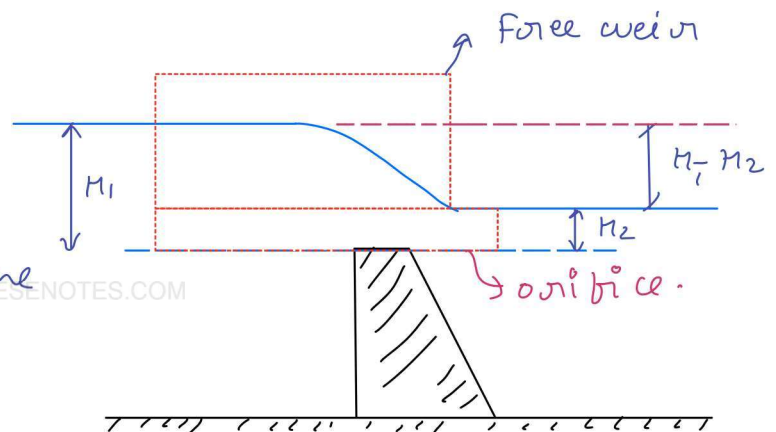
$$Q = C_d \cdot a \sqrt{2gh}$$

$C_d = 0.707$
head loss is more



(7) SUBMERGED WEIR :-

- when the water level on the downstream of the weir is above the crest of the weir, then the weir is said to be a submerged weir.



- During flood, weir constructed across river becomes submerged.
- Its discharge is more than free weir.
- For analysis point of view, it may be considered to be of two parts-

(a) The portion b/w the ups and d/s water surface may be treated as free weir (Q_1).

(b) The portion b/w d/s water surface and crest of weir may be treated as drowned orifice. (Q_2)

$$\text{Hence, } Q = Q_1 + Q_2.$$

$$Q_1 = \frac{2}{3} C_d \sqrt{2g} L \left[\left\{ (H_1 - H_2) + \frac{v_a^2}{2g} \right\}^{3/2} - \left(\frac{v_a^2}{2g} \right)^{3/2} \right]$$

$$Q_2 = C_d (L H_2) \sqrt{2g \left\{ (H_1 - H_2) + \frac{v_a^2}{2g} \right\}}$$

Sept 3 # DISCHARGE THROUGH LARGE RECTANGULAR ORIFICE

Applying energy eqn b/w ① and ②.

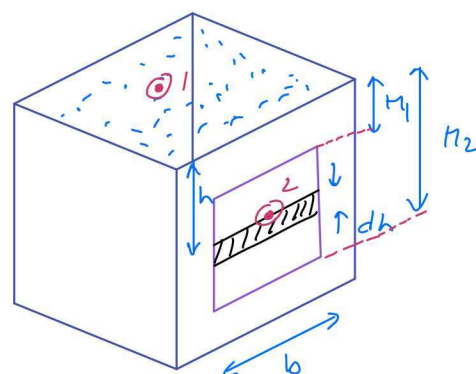
$$v_2 = \sqrt{2gh}$$

$$dQ = v_2 \cdot dA$$

$$= \sqrt{2gh} \cdot (b \, dh)$$

$$Q = \int dQ = \int_{H_1}^{H_2} \sqrt{2gh} \cdot b \, dh$$

$$Q_{act} = \frac{2}{3} C_d \sqrt{2g} b \{ H_2^{3/2} - H_1^{3/2} \}$$

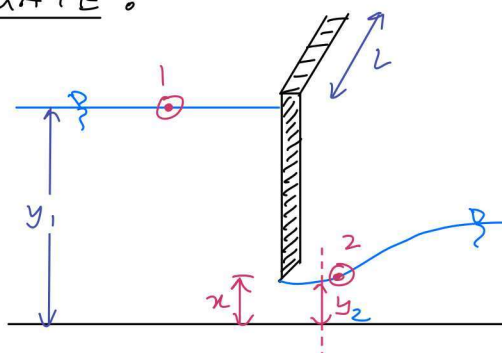


DISCHARGE THROUGH ^{Souvik} SLUICE GATE :-

using energy eqn (i) + (ii)

$$0 + y_1 + \frac{v_1^2}{2g} = 0 + y_2 + \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{2g(y_1 - y_2 + h_a)} \quad (h_a = \frac{v_1^2}{2g})$$



$$v_{2rac} = C_v \sqrt{2g(y_1 - y_2 + h_a)}$$

$$Q_{act} = A_2 \cdot v_{2rac} \rightarrow (C_v)$$

$$Q_a = (y_2 \cdot L) C_v \sqrt{2g(y_1 - y_2 + h_a)}$$

$$Q_a = C_c (xL) \cdot C_v \sqrt{2g(y_1 - y_2 + h_a)}$$

$$Q_a = C_d \cdot A \sqrt{2g(y_1 - y_2 + h_a)}$$

$A = (x \cdot L)$
Area of opening.

Since $y_1 \gg y_2 \Rightarrow y_1 - y_2 \approx y_1$ and $h_a \approx 0$.

$$Q_a = C_d A \sqrt{2g \cdot y_L}$$

NOTE:- coefficient of discharge (C_d) for rectangular weir can also be computed using Rehbock eqn.

$$C_d = 0.602 + 0.083 H/H_1$$

H = head over crest.

H_1 = height of crest/weir.

- 8) A large rectangular orifice 2m wide and 1m deep has water surface on either side 2m and 0.5m respectively above the lower edge of the opening. Estimate the discharge if $C_d = 0.6$ in both cases.

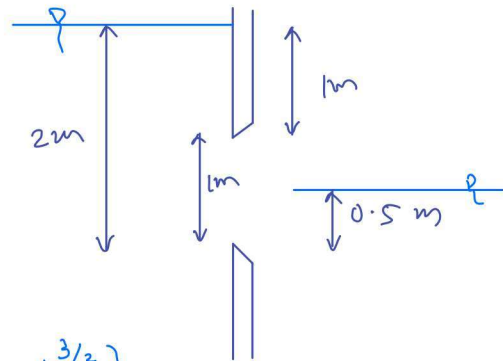
Sol large Rectangular orifice

$$Q_1 = \frac{2}{3} C_d \sqrt{2g} b [H_2^{3/2} - H_1^{3/2}]$$

$$b = 2m.$$

$$H_2 = 2 - 0.5 = 1.5m$$

$$H_1 = 1m.$$



$$Q_1 = \frac{2}{3} \times 0.6 \times \sqrt{2 \times 9.81} \times 2 [1.5^{3/2} - 1^{3/2}]$$

$$Q_1 = 2.96 \text{ m}^3/\text{s}.$$

→ Discharge due to orifice.

But another component will be discharge due to drowned orifice.

$$h = 2 - 1.5 = 0.5m$$

$$Q_2 = 0.6 \times 0.5 \times 2 \sqrt{2g \cdot 1.5} = 3.25 \text{ m}^3/\text{s}$$

$$Q = 2.96 + 3.25 = 6.22 \text{ m}^3/\text{s}.$$

- 9) A rectangular channel 3m wide carries a discharge of $1.5 \text{ m}^3/\text{s}$. Calculate the height of the rectangular suppressed weir which can pass this discharge, while maintaining the u/s depth of 1.5m.

$$Q = C_d \cdot \frac{2}{3} \sqrt{2g} L \{ (H + h_a)^{3/2} - h_a^{3/2} \} \quad | \quad H + H_1 = 1.5$$

From Rehbock eqn, $C_d = 0.602 + 0.083 \frac{H}{H_1}$

Assuming $C_d = 0.62$

$$1.5 = 0.62 \times \frac{2}{3} \sqrt{2 \times 9.81} \times 3 (H)^{3/2}$$

$$H = 0.42, \therefore H_1 = 1.08$$

$$\text{now, } C_d = 0.602 + 0.083 \frac{H}{H_1} = 0.602 + 0.083 \times \frac{0.42}{1.08}$$

$$C_d = 0.634$$

$$\text{now, } 1.5 = \frac{2}{3} \times 0.634 \times \sqrt{2 \times 9.81} \times 3 \times H^{3/2}$$

$$H = 0.414 \text{ m}$$

$$H_1 = 1.085 \text{ m}$$

$$C_d = 0.634$$

$$\text{Hence, } H_1 = 1.0853 \text{ m.}$$

Souvik
2021

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LAMINAR FLOW

- laminar flow is a flow in which liquid moves in layers, where one layer slides over another.
- In this case no mixing of different layers take place, hence momentum transfer is not observed. Hence shear force is due to **viscosity** only.
- In turbulent flow, due to continuous mixing b/w different layers, momentum transfer occurs which gives rise to additional shear.
- In laminar flow, viscous force predominant inertial force.
- For different mediums, nature of flow depends on value of ^{Souvik 2021} Reynolds number as follows:-

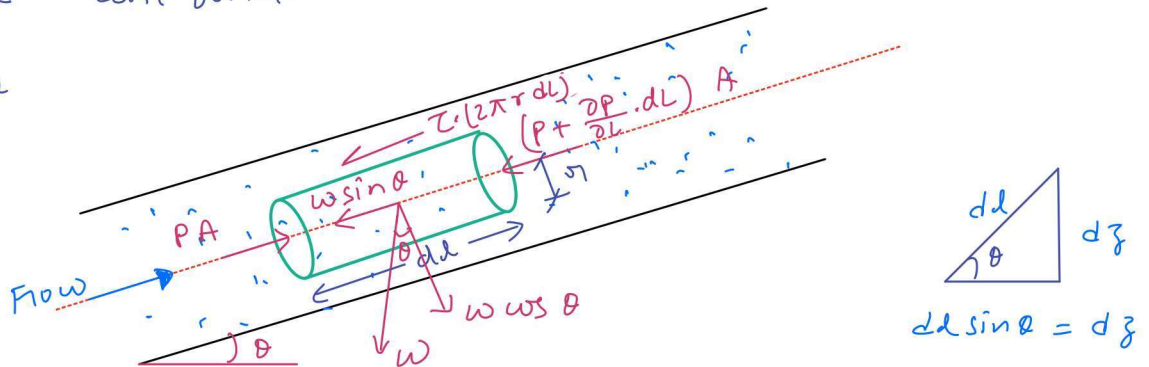
Medium	Laminar	nature of flow Transition	Turbulent.
1) PIPE	< 2000	$2000 - 4000$	> 4000
2) Parallel plate	< 1000	$1000 - 2000$	> 2000
3) Open channel	< 500	$500 - 2000$	> 2000
4) Soil	< 1	$1 - 2$	> 2

NOTE:- The values of Reynold's no. at which nature of flow changes are not well defined.

FLOW THROUGH CIRCULAR PIPES :-

- In this case flow is always assumed to be steady and uniform.

$$A = \pi r^2$$



$$P(\pi r^2) - (P + \frac{\partial P}{\partial L} \cdot dl) \cdot \pi r^2 - \gamma \pi r^2 \cdot dl \sin \theta - \tau (2\pi r \cdot dl) = 0.$$

$$-\frac{\partial P}{\partial L} \cdot dl \cdot \pi - \gamma \pi (dz) = \tau \cdot 2 \cdot dl$$

$$\tau = \frac{\eta}{2} \left[-\frac{\partial}{\partial r} (P + \gamma z) \right] \rightarrow \text{linear.}$$

- If pipe would have been horizontal, $\gamma = 0$.

$$\tau = \frac{\eta}{2} \left[-\frac{\partial P}{\partial r} \right]$$

Step 6

→ For steady and uniform flow.

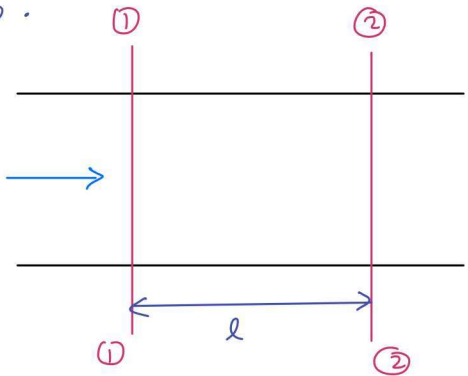
Applying Energy eqn ①-① and ②-②

$$\frac{P_1}{\gamma} + 0 + \cancel{\frac{V_1^2}{2g}} = \frac{P_2}{\gamma} + 0 + \cancel{\frac{V_2^2}{2g}} + h_L \rightarrow$$

$$\frac{P_1 - P_2}{\gamma} = h_L \quad \text{Sowvik 2021}$$

$$P_1 - P_2 = \gamma h_L$$

$$-\frac{dP}{dL} = \frac{P_1 - P_2}{L} = \frac{\gamma h_L}{L} = \text{const.}$$

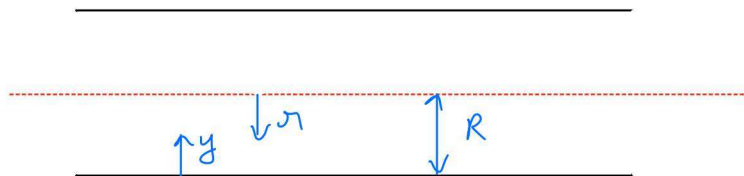


Hence, for steady and uniform flow, $-\frac{dP}{dL}$ is constant.

VELOCITY DISTRIBUTION IN PIPE FLOW (LAMINAR) :-

$$\tau = \frac{\eta}{2} \left[-\frac{dP}{dL} \right] \quad \text{--- (i) } \rightarrow \text{For all type of flow.}$$

$$\text{Also, } \tau = -\mu \frac{du}{dy} \quad \text{--- (ii) } \rightarrow \text{only for laminar flow}$$



$$y + r = R$$

$$dy + dr = 0$$

$$dy = -dr$$

From (i) & (ii)

$$\frac{\eta}{2} \left[-\frac{dP}{dL} \right] = -\mu \frac{du}{dy}$$

$$\frac{\eta}{2} \left[-\frac{dP}{dL} \right] = -\mu \frac{du}{dr}$$

$$du = -\frac{1}{2\mu} \left(-\frac{dP}{dx} \right) r \cdot dr.$$

$$u = \int du = -\frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \int r \cdot dr$$

$$u = -\frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \left(\frac{r^2}{2} \right) + c.$$

@ $r=R$, $u=0$.

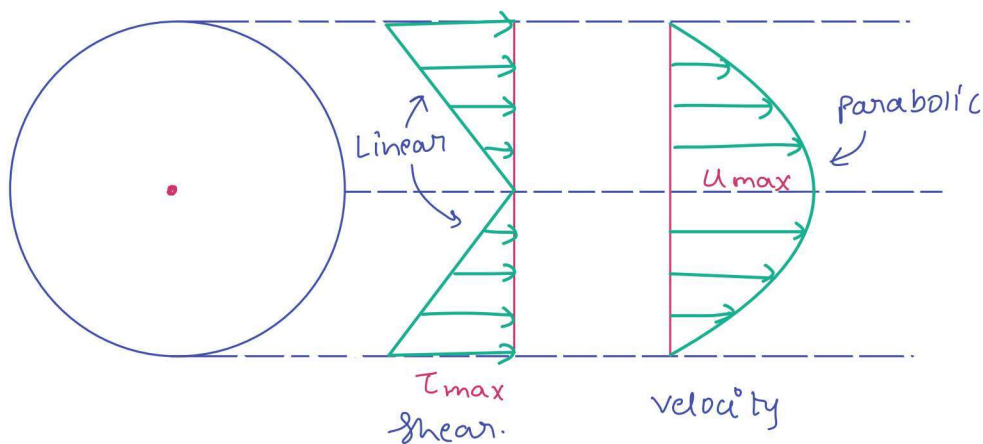
$$0 = -\frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \left(\frac{R^2}{2} \right) + c.$$

$$c = \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \left(\frac{R^2}{2} \right)$$

$$u = -\frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \frac{r^2}{2} + \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \left(\frac{R^2}{2} \right)$$

$$u = \frac{1}{4\mu} \left(-\frac{dP}{dx} \right) [R^2 - r^2] \rightarrow \text{Parabolic}.$$

→ But, remember when pipe will be moving, then
@ $r=R$, $u = \text{vel. of pipe}$.



For max. velocity, $\frac{du}{dr} = 0$.

$$\frac{du}{dr} = \frac{1}{4\mu} \left(-\frac{dP}{dx} \right) [0 - 2r] = 0.$$

$$r = 0$$

$$u_{max} = \frac{1}{4\mu} \left(-\frac{dP}{dx} \right) R^2$$

and,

$$T_{max} = \left(-\frac{dP}{dx} \right) \frac{R}{2}$$

Also, $u = \frac{1}{4\mu} \left(-\frac{dP}{dl} \right) R^2 \left(1 - \frac{r^2}{R^2} \right)$

$$u = u_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

Discharge through circular pipe for LAMINAR Flow :-

Discharge for element dr ,

$$dQ = dA \cdot du$$

$$dQ = (2\pi r \cdot dr) u_{\max} \left(1 - \frac{r^2}{R^2} \right)$$

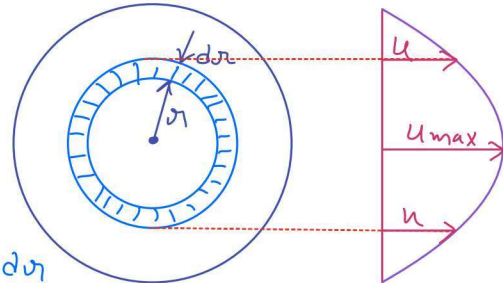
$$Q = \int dQ = 2\pi u_{\max} \int_0^R r \left(1 - \frac{r^2}{R^2} \right) dr$$

$$Q = 2\pi u_{\max} \int_0^R r - \frac{r^3}{R^2} dr$$

$$Q = 2\pi u_{\max} \left[\frac{r^2}{2} - \frac{1}{R^2} \frac{r^4}{4} \right]$$

$$Q = 2\pi u_{\max} \left[\frac{R^2}{4} \right]$$

$$Q = \pi u_{\max} \frac{R^2}{2}$$



Also, $u_{\max} = \frac{1}{4\mu} \left(-\frac{dP}{dl} \right) R^2$

$$\therefore Q = \frac{\pi R^2}{2} \frac{1}{4\mu} \left(-\frac{dP}{dl} \right) R^2$$

$$Q = \frac{\pi R^4}{8\mu} \left(-\frac{dP}{dl} \right)$$

All Equation of discharge is known as.

HAGEN POISEULLI EQN

Also,

Remember, $\frac{dP}{dl} = \frac{8\eta L}{L}$

$$Q = \frac{\pi D^4}{128\mu} \left(-\frac{dP}{dl} \right)$$

Now, $Q = A \cdot v_{\text{avg}}$

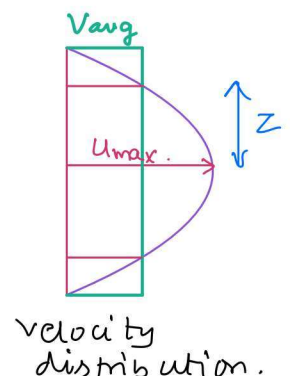
Also, $Q = \frac{A}{2} \cdot u_{\max}$

$$\therefore v_{\text{avg}} = \frac{u_{\max}}{2} \Rightarrow \text{For pipe flow}$$

avg velocity = $\frac{\text{max. velocity}}{2}$

$$v_{\text{avg}} = \frac{1}{8\mu} \left(-\frac{dP}{dl} \right) R^2$$

NOTE:- In all the above cases for inclined pipe replace $\frac{dP}{dl} = \frac{d}{dl} (P + \gamma z)$.



velocity distribution.

Distance at which where avg velocity = Actual velocity.

we know,

$$u = u_{max} \left(1 - \frac{r^2}{R^2}\right)$$

$$\text{For } u = u_{avg} = \frac{u_{max}}{2}$$

$$\frac{u_{max}}{2} = u_{max} \left(1 - \frac{r^2}{R^2}\right)$$

$$\frac{1}{2} = 1 - \frac{r^2}{R^2}$$

$$\frac{r^2}{R^2} = \frac{1}{2}$$

$$r = \frac{R}{\sqrt{2}} = 0.707 R$$

- LOSSES in Laminar Flow :-

$$-\frac{dP}{dL} = \frac{dF}{dL} \cdot \frac{1}{\text{Area}} = \text{Force per unit volume.}$$

Force required to maintain the flow through pipe.

$$= -\frac{dP}{dL} \times \text{volume} = -\frac{dP}{dL} \times \text{Area} \times \text{length.}$$

Power required, $P = \text{Force} \times V_{avg}$.

$$P = -\frac{dP}{dL} \cdot (AL) \cdot V_{avg}$$

$$P = \frac{P_1 - P_2}{L} \cdot L \cdot Q$$

Power required $\Rightarrow$ $P = (P_1 - P_2) \cdot Q$

$$\text{Also, } P_1 = P_2 + h_L$$

$$\therefore \text{Power} = h_L \cdot Q$$

(This entire power is required to overcome the frictional resistance).

NOTE:- Instead of pipe being horizontal if it is inclined.

$$\text{Power req, } P = \{(P_1 + \gamma z_1) - (P_2 + \gamma z_2)\} Q$$

$$P = (P_1 - P_2) Q + \gamma (z_1 - z_2) Q$$

Additional power to overcome weight.

case(i) Flow is taking place in upward dirⁿ

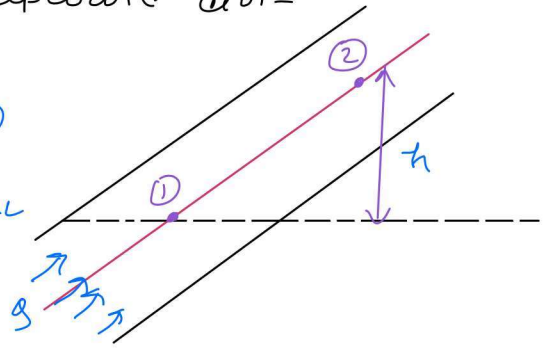
Applying Bernoulli's eqⁿ b/w ① & ②

$$\frac{P_1}{\gamma} + 0 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + h + \frac{V_2^2}{2g} + h_L$$

$$P_1 - P_2 = \gamma (h_L + h)$$

$$\therefore \text{Power} = (P_1 - P_2) \cdot Q$$

$$\boxed{\text{Power} = \gamma (h_L + h) Q} \quad \text{--- (i)}$$

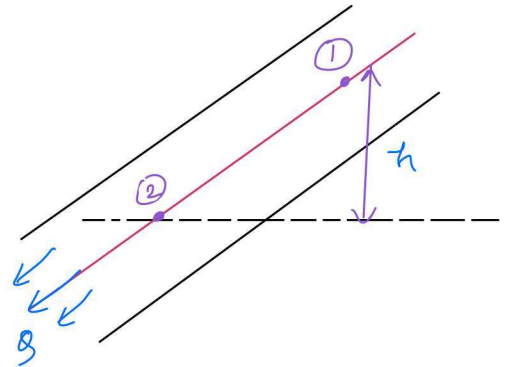


case(ii) when flow is occurring downward

$$\frac{P_1}{\gamma} + h + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$(P_1 - P_2) = \gamma (h_L - h)$$

$$\boxed{\text{Power required,} = \gamma Q (h_L - h)} \quad \text{--- (ii)}$$



i.e. (Power)_{eqⁿ (i)} > (Power)_{eqⁿ (ii)}

calculation of h_L for laminar flow :-

Head loss/pressure drop, h_L

$$u_{avg} = \frac{u_{max}}{2} = \frac{1}{32\mu} \left(-\frac{dP}{dz} \right) D^2$$

$$-\frac{dP}{dz} = \frac{32\mu u_{avg}}{D^2}$$

$$P_1 - P_2 = \frac{32\mu L u_{avg}}{D^2}$$

$$P_1 - P_2 = \frac{32\mu L u_{avg}}{D^2} \quad \text{--- (i)}$$

For inclined pipe.

$$(P_1 + \gamma z_1) - (P_2 + \gamma z_2) = \frac{32\mu L u_{avg}}{D^2}$$

$$\text{Also, } P_1 - P_2 = \gamma h_L \quad \text{--- (ii)}$$

From (i) & (ii)

$$h_L = \frac{32 \mu L (\dot{Q}/A)}{r D^2} \quad \text{--- } u_{avg.}$$

$$h_L = \frac{128 \mu \dot{Q} L}{\pi r D^4} \quad \left(h_L \propto \frac{1}{D^4} \right)$$

According to darcy-weisbach eqn

$$h_L = \frac{f L v^2}{2gD}$$

$$\therefore \frac{f L v_{avg}^2}{2gD} = \frac{32 \mu L v_{avg}}{r D^2}$$

friction factor
↓
depends on pipe surface.

$$f = \frac{64 \mu}{\rho D v}$$

$$f = \frac{64 \nu}{D v} = 64 \left(\frac{\nu}{D v} \right)$$

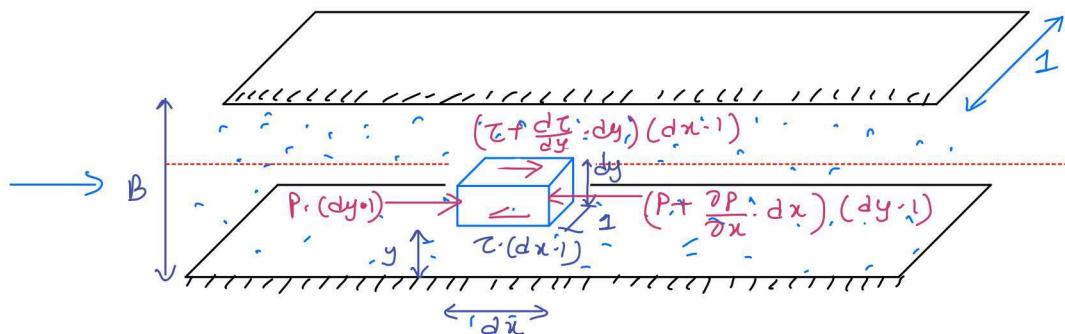
Here, $Re = \frac{v_{avg} D}{\nu}$ for pipe flow.

$$f = \frac{64}{Re}$$

f = friction factor.
 f' = coeff of friction

$$f = 4 f'$$

Sept-7 # LAMINAR FLOW BETWEEN PARALLEL PLATES :-

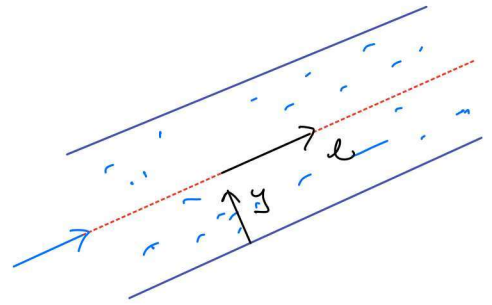


From static equilibrium eqn

$$\begin{aligned} \Sigma F_x = 0 &:- P(dy \cdot 1) - \left(P + \frac{\partial P}{\partial x} \cdot dx \right) (dy \cdot 1) + \left(\tau + \frac{\partial \tau}{\partial y} \cdot dy \right) (dx \cdot 1) \\ &\quad - \tau(dx \cdot 1) = 0 \\ &:- \frac{\partial P}{\partial x} \cdot dx \cdot dy + \frac{\partial \tau}{\partial y} \cdot dy \cdot dx = 0 \end{aligned}$$

$$\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

For inclined plate :- $\frac{\partial \tau}{\partial y} = \frac{\partial (p + \rho z)}{\partial x}$



Also, $\tau = \mu \frac{du}{dy}$

$$\frac{d\tau}{dy} = \mu \frac{d^2 u}{dy^2} = \frac{dp}{dx}$$

$$\int \mu \frac{d^2 u}{dy^2} = \int \frac{dp}{dx}$$

$$\frac{du}{dy} = \frac{1}{\mu} \left(\frac{dp}{dx} \right) y + c_1$$

$$u = \frac{1}{2\mu} \left(\frac{dp}{dx} \right) \frac{y^2}{2} + c_1 y + c_2$$

At $y=0$:- $u=0$.

$$0 = c_2$$

@ $y=B$, $u=0$.

$$0 = \frac{1}{2\mu} \left(\frac{dp}{dx} \right) \frac{B^2}{2} + c_1 B$$

$$c_1 = -\frac{1}{\mu} \left(\frac{dp}{dx} \right) \frac{B}{2}$$

$$u = -\frac{1}{2\mu} \left(\frac{dp}{dx} \right) [By - y^2] \Rightarrow \text{Parabolic.}$$

For $u = u_{max}$, $\frac{du}{dy} = 0$.

$$\frac{1}{2\mu} \left(-\frac{dp}{dx} \right) [B - 2y] = 0$$

$$y = B/2$$

[i.e. velocity is max at centre of plate]

$$u_{max} = \frac{1}{2\mu} \left(-\frac{dp}{dx} \right) \left[\frac{B^2}{2} - \frac{B^2}{4} \right]$$

$$u_{max} = \frac{1}{8\mu} \left(-\frac{dp}{dx} \right) B^2$$

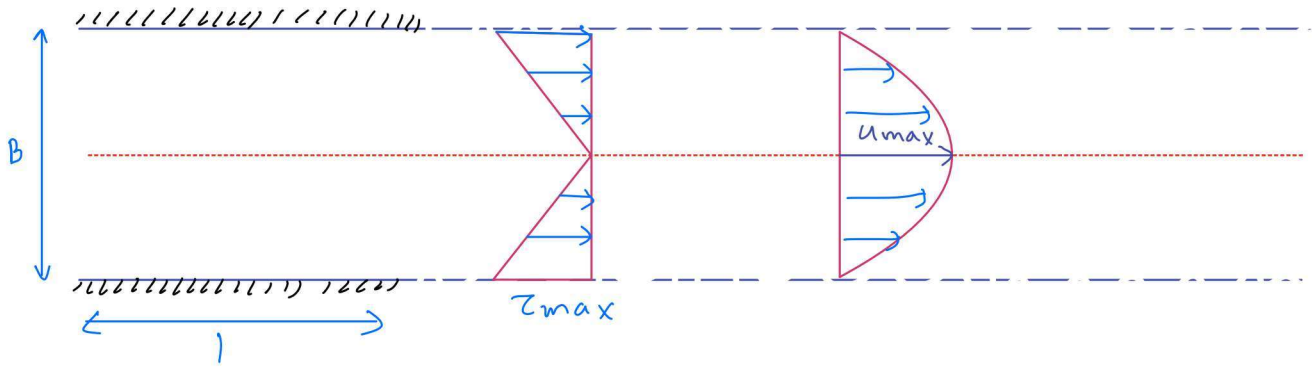
Also,
 $u = \frac{1}{8\mu} \left(-\frac{dp}{dx} \right) B^2 \left(\frac{y}{B} - \frac{y^2}{B^2} \right)$

$$u = 4u_{max} \left(\frac{y}{B} - \frac{y^2}{B^2} \right)$$

Shear, $\tau = \mu \frac{du}{dy} = \mu \cdot \frac{1}{2\mu} \left(-\frac{dp}{dx} \right) [B - 2y]$

$$\tau = \frac{1}{2} \left(-\frac{dp}{dx} \right) (B - 2y) \Rightarrow \text{Linear}$$

τ_{max} @ $y=0$, $\tau=0$ @ $y=B/2$.



Discharge through parallel plates:-

Discharge through this element = dQ .

$$dQ = dA \cdot u.$$

$$= (dy \cdot 1) \cdot \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) (By - y^2)$$

$$dQ = \frac{dQ}{\text{width}} = \frac{dQ}{1} = \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) (By - y^2) dy.$$

$$Q = \int dQ = \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \int_0^B By - y^2 dy$$

$$= \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \left\{ B \frac{B^2}{2} - \frac{B^3}{3} \right\}$$

$$Q = \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) \frac{B^3}{6}$$

Discharge per unit width.
(Here width=1).

$$\text{Now, } u = \frac{1}{8\mu} \left(-\frac{dP}{dx} \right) B^2$$

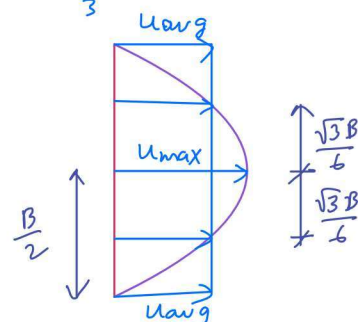
$$Q = \frac{1}{8\mu} \left(-\frac{dP}{dx} \right) B^2 \cdot \frac{8}{12} B.$$

$$Q = u_{\max} \frac{2B}{3}$$

$$Q = A \cdot v_{\text{avg}} = (B \cdot 1) \cdot v_{\text{avg}} = u_{\max} \frac{2B}{3}$$

$$\Rightarrow v_{\text{avg}} = \frac{2u_{\max}}{3}$$

For location where, $u_{\text{avg}} = u$.



$$\frac{2u_{max}}{3} = 4u_{max} \left(\frac{y}{B} - \frac{y^2}{B^2} \right)$$

$$\frac{1}{6} = \frac{y}{B} - \frac{y^2}{B^2}$$

$$6y^2 - 6yB + B^2 = 0.$$

$$y = \frac{6B \pm \sqrt{36B^2 - 24B^2}}{12}$$

$$y = \frac{6B \pm \sqrt{12}B}{12} = \frac{B}{2} \pm \frac{\sqrt{3}B}{6}$$

$$y = \frac{B}{2} + \frac{\sqrt{3}B}{6}$$

NOTE:- LAMINAR FLOW B/W PARALLEL PLATES with one plate stationary and other one moving (COHETTE FLOW)
Solvik 2021

$$\frac{dT}{dy} = \frac{\partial P}{\partial x} \quad - (i)$$

$$\tau = \mu \frac{du}{dy} \quad - (ii)$$

From (i) & (ii)

$$u = \frac{1}{2\mu} \left(\frac{dP}{dx} \right) y^2 + c_1 y + c_2$$

$$@ y=0, u=0 \therefore c_2 = 0.$$

$$@ y=B, u=v_0 \therefore v_0 = \frac{1}{2\mu} \left(\frac{dP}{dx} \right) B^2 + c_1 B$$

$$c_1 = \frac{v_0}{B} - \frac{1}{2\mu} \left(\frac{dP}{dx} \right) B$$

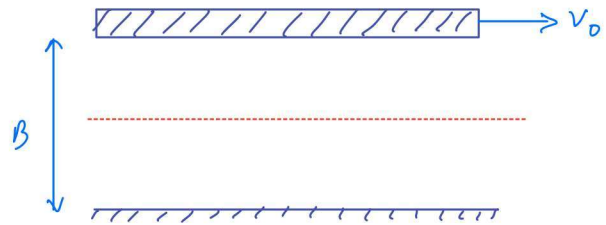
$$u = \frac{1}{2\mu} \left(\frac{dP}{dx} \right) y^2 + \left[\frac{v_0}{B} - \frac{1}{2\mu} \left(\frac{dP}{dx} \right) B \right] y + 0.$$

$$u = \frac{v_0 y}{B} + \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) [By - y^2]$$

$$\text{For max. } u, \frac{du}{dy} = 0.$$

$$\frac{du}{dy} = \frac{v_0}{B} + \frac{1}{2\mu} \left(-\frac{dP}{dx} \right) [B - 2y] = 0.$$

$$\frac{v_0}{B} = \frac{1}{2\mu} \left(\frac{dP}{dx} \right) B - \frac{1}{2\mu} \left(\frac{dP}{dx} \right) 2y$$



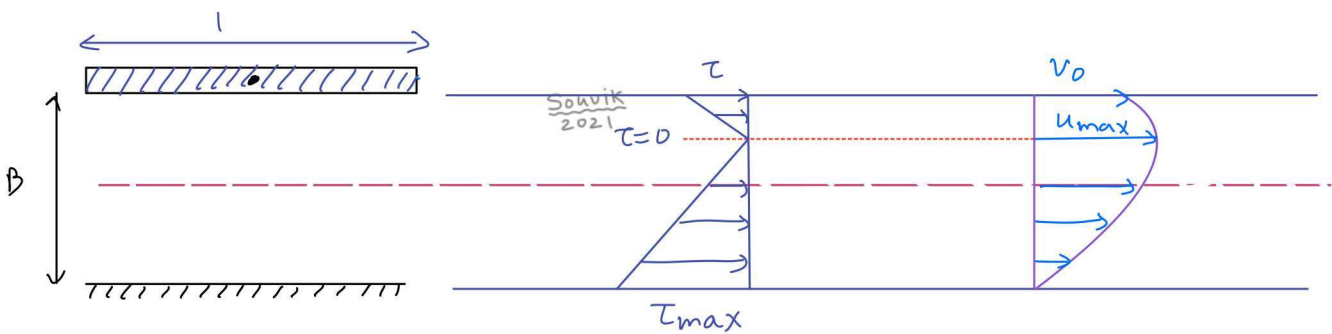
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$$y = \frac{-v_0}{\frac{B}{2\mu} \left(\frac{dp}{dx}\right)^2} + \frac{B}{2}$$

$$y = \frac{B + \frac{v_0}{\frac{B}{2\mu} \left(-\frac{dp}{dx}\right)}}{2}$$

Now, $\tau = \mu \frac{du}{dy} = \mu \left[\frac{v_0}{B} + \frac{1}{2\mu} \left(-\frac{dp}{dx}\right) (B-2y) \right]$

$$\tau = \frac{v_0 \mu}{B} + \frac{1}{2} \left(-\frac{dp}{dx}\right) (B-2y) \Rightarrow \text{linear variation.}$$



Sept 8 # LOSSES B/W PARALLEL PLATE :-

$$u_{\text{avg}} = \frac{2}{3} u_{\max} = \frac{1}{12\mu} \left(-\frac{dp}{dx}\right) B^2$$

$$v = \frac{1}{12\mu} \frac{(P_1 - P_2)}{L} B^2 = \frac{1}{12\mu} \frac{\gamma h_L}{L} B^2$$

head loss,

$$h_L = \frac{12\mu v_{\text{avg}} L}{\gamma B^2}$$

Also,

$$Q = A v_{\text{avg}} = (B \times 1) v_{\text{avg}}$$

$$v_{\text{avg}} = \frac{Q}{B}$$

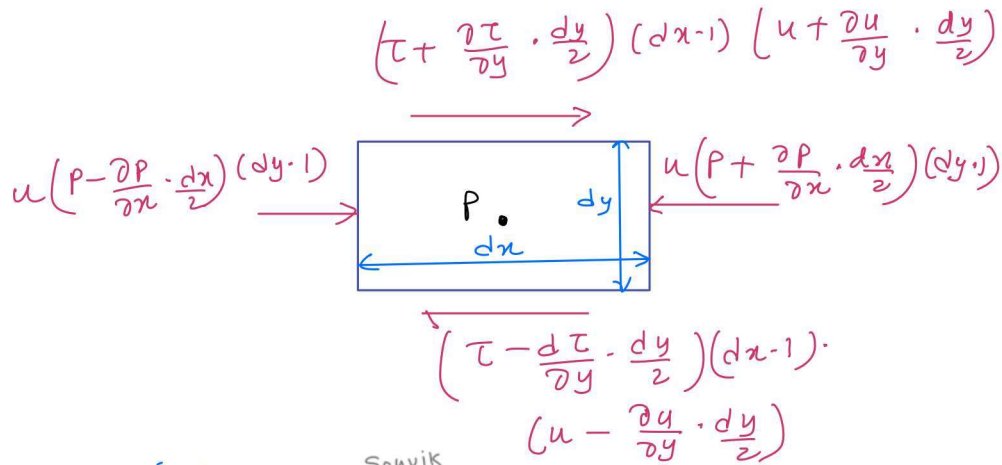
$$h_L = \frac{12\mu Q L}{\gamma B^3}$$

$$[h_L \propto \frac{1}{B^3}]$$

NET POWER INPUT PER UNIT VOLUME :-

For laminar flow condition, net power input per unit volume is given by $\tau \frac{du}{dy} = \mu \left(\frac{du}{dy} \right)^2 = \frac{\tau^2}{\mu}$

Power = Force $\times$ u



Net power input = $F \cdot u$

$$u \left(P - \frac{\partial P}{\partial x} \cdot \frac{dx}{2} \right) \cdot (dy) - u \left(P + \frac{\partial P}{\partial x} \cdot \frac{dx}{2} \right) (dy)$$

$$+ \left(\tau + \frac{\partial \tau}{\partial y} \cdot \frac{dy}{2} \right) (dx) \left(u + \frac{\partial u}{\partial y} \cdot \frac{dy}{2} \right) - \left(\tau - \frac{\partial \tau}{\partial y} \cdot \frac{dy}{2} \right) (dx) \left(u - \frac{\partial u}{\partial y} \cdot \frac{dy}{2} \right)$$

$$u \frac{\partial P}{\partial x} dx + \tau \frac{\partial u}{\partial y} \cdot dy \cdot dx + u \frac{\partial \tau}{\partial y} \cdot dy \cdot dx = F \cdot u$$

$$\Rightarrow \left[u \left(-\frac{\partial P}{\partial x} \right) + \tau \frac{\partial u}{\partial y} + u \frac{\partial \tau}{\partial y} \right] dx \cdot dy \cdot 1 = F \cdot u$$

$$\Rightarrow F \cdot u = \left[\tau \frac{\partial u}{\partial y} + u \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x} \right) \right] u$$

$$\Rightarrow \frac{\text{Net power input}}{\text{volume}} = \tau \frac{\partial u}{\partial y} \quad \left[\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x} \right]$$

For circular pipe, $\frac{\text{Net power input}}{\text{volume}} = \mu \left(\frac{\partial u}{\partial y} \right)^2 = \mu \left(\frac{\partial u}{\partial r} \right)^2$

$$u = u_{\max} \left(1 - \frac{r^2}{R^2} \right) \quad [\text{Here } dy = -dr]$$

$$\frac{\partial u}{\partial r} = u_{\max} \left(-\frac{2r}{R^2} \right) = -u_{\max} \frac{2r}{R^2}$$

$$\frac{N \cdot P \cdot I}{\text{vol}} = \mu u_{\max}^2 \frac{4r^2}{R^2}$$

$$\frac{NP I}{\text{vol}} = 4 \mu u_{\max}^2 \frac{r^2}{R^4}$$

- 8) Air flows through a circular pipe of 10mm dia, at an average velocity of 2m/s. Find the pressure drop in mm of water in a length of 5m. Assume density of air = 1.1774 kg/m^3 and $\mu_{\text{air}} = 1.982 \times 10^{-5} \text{ kg/m-s}$.

sd
=

$$u_{\text{avg}} = 2 \text{ m/s}$$

$$\text{nature of flow, } Re = \frac{\rho u_{\text{avg}} D}{\mu} = \frac{1.174 \times 10 \times 10^{-3} \times 2}{1.982 \times 10^{-5}} = 1188.1 < 2000.$$

Hence flow is laminar.

$$\text{Pressure drop, } P_1 - P_2 = \tau_a h_L$$

$$\text{pressure drop in terms of mm of water} = \frac{\tau_a h_L}{\rho_w}$$

$$\text{Now, } h_L = \frac{f L V^2}{2gD} \quad f = \frac{64}{Re} = \frac{64}{1188.1} = 0.053.$$

$$\begin{aligned} & \frac{32 \mu V L}{D^2} \\ &= \frac{32 \times 1.982 \times 10^{-5} \times 2 \times 5}{(0.01)^4} \\ &= \frac{63.424 \text{ N/m}^2}{10} = 6.34 \text{ m} \end{aligned}$$

$$h_L = \frac{0.053 \times 5 \times 2^2}{2 \times 9.81 \times 0.01}$$

$$h_L = 5.40 \text{ m}$$

$$\begin{aligned} \text{Pressure drop in terms of mm of water} &= \frac{5.40 \times 1.1774}{1} \\ &= 6.36 \text{ mm of water.} \end{aligned}$$

- 9) For a fully developed flow of glycerine through a 70m long, 4cm dia horizontal circular pipe, if flow velocity at the centre is measured as 0.6m/sec. Determine. $\mu = 0.3 \frac{\text{Ns}}{\text{m}^2}$, $k = 1.2$

(a) velocity profile.

(b) pressure diff across 70m length.

(c) pumping power required to maintain the flow.

(d) For the same pumping power input determine the % change of flow rate if

(i) pipe is inclined 15° downward

(ii) pipe is inclined 15° upward.

Sol

$$L = 70 \text{ m}$$

$$D = 4 \text{ cm}$$

$$u_{\max} = 6 \text{ m/s}$$

$$\mu = 0.3 \frac{\text{Ns}}{\text{m}^2} = 0.3 \frac{\text{kg m} \cdot \text{s}}{\text{s}^2 \text{ m}^2} = 0.3 \text{ kg/m} \cdot \text{s}$$

$$(a) \quad u = u_{\max} \left(1 - \frac{r^2}{R^2}\right)$$

$$u = 6 \left(1 - \frac{r^2}{\left(\frac{0.04}{2}\right)^2}\right) = 6 (1 - 2500 r^2)$$

Hence profile is parabolic.

$$(b) \quad \text{Type of flow, } Re = \frac{\rho u_{\text{avg}} D}{\mu} \quad u_{\text{avg}} = \frac{u_{\max}}{2} = 3 \text{ m/s}$$

$$Re = \frac{1.2 \times 1000 \times 3 \times 4 \times 10^{-2}}{0.3} = 480 (< 1000) \rightarrow \text{laminar flow}$$

$$P_1 - P_2 = \frac{32 \mu u_{\text{avg}} L}{D^3} \quad [\text{Always use this eqn in laminar flow}]$$

$$= \frac{32 \times 0.3 \times 3 \times 70}{(4 \times 10^{-2})^3} = 1260 \text{ kN/m}^2$$

$$(c) \quad \text{Pumping power required} = (P_1 - P_2) Q$$

$$= (P_1 - P_2) A u_{\text{avg}}$$

$$= 1260 \times \frac{\pi}{4} \times (0.04)^2 \times 3$$

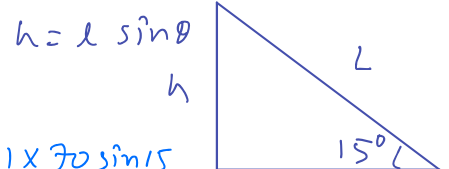
$$= 4.75 \text{ kW}$$

(d) when pipe is inclined downward.

$$P_1 - P_2 = \rho (h_1 - h_2)$$

$$\text{Power req} = \rho (h_1 - h_2) Q = \rho (h_1 - h_2) A u_{\text{avg}}$$

$$4.75 = \rho \left(\frac{128 \mu L Q}{\pi D^4} - L \sin 15^\circ \right) Q$$



$$4.75 = \frac{128 \times 0.3 \times 70 \times Q^2}{\pi \times (0.04)^4} - 1.2 \times 9.81 \times 70 \sin 15^\circ Q$$

$$4.75 = 3.34 \times 10^8 Q^2 - 2132.77 Q$$

$$Q =$$

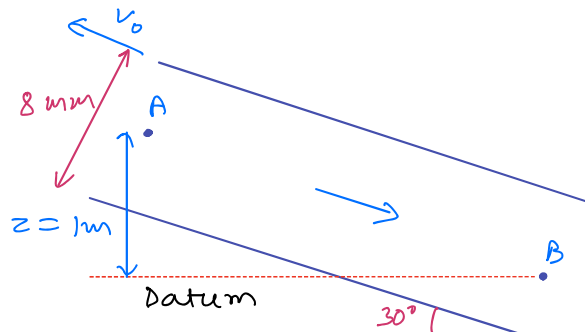
Sept 9

Q) Laminar flow of oil takes place down the slope b/w the parallel plates 8mm apart held inclined at 30° to the horizontal. The oil is of viscosity 0.92 kg/m-s and of mass density 1320 kg/m^3 . The pressure of two points, 1m vertically apart are 40 kN/m^2 and 170 kN/m^2 , when the upper plate moves at 0.3 m/s velocity relative to the lower plate in the direction opposite to the oil flow, find

- max. velocity of flow
- Shear stress at top plate.
- Rate of oil flow per meter width.

Sol. $\mu = 0.92 \text{ kg/m-s}$
 $\rho = 1320 \text{ kg/m}^3$ Souvik 2021

$P = 170 \text{ kN/m}^2$
 $P = 40 \text{ kN/m}^2$
 $V_0 = 0.3 \text{ m/s}$



vel. distribution for couple flow.

$$u = \frac{V_0 y}{B} + \frac{1}{2\mu} \left(-\frac{\partial P}{\partial x} \right) (By - y^2)$$

Now, flow is inclined and plate is moving in opposite direction.

Hence, $-\frac{\partial P}{\partial x} = -\frac{\partial(P + \rho z)}{\partial x}$

$$-\frac{\partial(P + \rho z)}{\partial x} = \frac{(P_1 + \rho z_1)_A - (P_2 + \rho z_2)_B}{z/\sin \theta}$$

$$= \frac{(P_1 - P_2) + \rho(z_1 - z_2)}{z/\sin \theta} \quad (z_2 = 0)$$

$$= \frac{(170 - 40) + 1320 \times 9.81 \times 10^3 \times 1}{1/\sin 30^\circ} = 71.5 \text{ kN/m}^2/\text{m}$$

$$u = -\frac{V_0}{B} + \frac{1}{2\mu} \left(-\frac{\partial P}{\partial x} \right) (By - y^2)$$

$$u = \frac{-0.3 y}{8 \times 10^{-3}} + \frac{1 \times 71.5 \times 10^3}{2 \times 0.92} \times (8 \times 10^{-3} y - y^2)$$

$$u = -37.5 y + 38858.7(8 \times 10^{-3} y - y^2)$$

$$u = 273.36 y - 38858.7 y^2$$

(a) For max velocity.

$$\frac{du}{dy} = 0 = 273.36 - 77717.4 y = 0$$

$$y = 3.517 \text{ mm}$$

$$u_{\max} = 273.36 \times 3.517 \times 10^{-3} - 38858.7 \times (3.517 \times 10^{-3})^2$$

$$= 0.48 \text{ m/s.}$$

(b) $\tau = \mu \frac{du}{dy}$

$$\tau = 0.92 \times 10^{-1} [273.36 - 77717.4 \times 8 \times 10^{-3}]$$

$$\tau = -320.50 \text{ N/m}^2$$

(c) $Q = \int dQ = \int u \cdot dA = \int_0^B u \cdot (dy \cdot 1)$

$$= \int_0^B (273.36 y - 38858.7 y^2) dy$$

$$= 273.36 \times \frac{B^2}{2} - 38858.7 \frac{B^3}{3}$$

$$= 2.112 \times 10^{-3} \text{ m}^3/\text{s.}$$

NOTE:-

(a) KINETIC ENERGY CORRECTION FACTOR :- (α)

$$\alpha = \frac{\int u^3 \cdot dA}{u_{\text{avg}}^3 \cdot A}$$

$\alpha = 2$, for laminar flow in circular pipe

$\alpha = 1.543$, laminar flow in parallel plates

$\alpha = 4/3$, for turbulent's flow (power law)

$\alpha = 1.03 - 1.06$ for turbulent's flow (log law).

(b) MOMENTUM CORRECTION FACTOR :-

$$\beta = \frac{\int u^2 dA}{u_{avg}^2 A}$$

$\beta = \frac{4}{3}$ for laminar flow in circular pipe.

$\beta = 1.2$ for " " in parallel plate.

$\beta = 1.015$ for turbulent flow for pipe with log. variation.

MEASUREMENT OF VISCOSITY :-

- Measurement of viscosity can be done by any of following methods.

(a) ROTATING CYLINDER :-

We measure the torque.

$$T = T_1 + T_2$$

$$T_1 = \frac{2\pi \omega \mu R_1^2 R_2 L}{(R_2 - R_1)}$$

$$T_2 = \frac{\pi \mu \omega R_1^4}{2b}$$

$$T = \pi \omega \mu R_1^2 \left[\frac{2R_2 L}{(R_2 - R_1)} + \frac{R_1^2}{2b} \right]$$

Now, Power, $P = T\omega$, Find " μ "

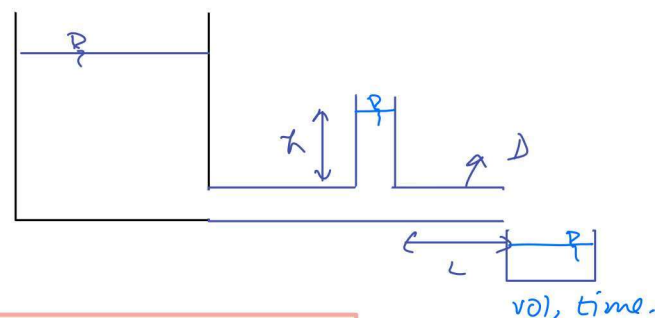
(b) Capillary tube viscometer :-

vol and time is noted.

$$h_L = \frac{128 \mu Q L}{\pi D^4} \quad \text{--- (i)}$$

$$\text{and } Q = \frac{Vol}{t} \quad \text{--- (ii)}$$

$$h_L = \frac{128 \mu \left(\frac{V}{t}\right) L}{\pi D^4}$$



$$\mu = \frac{h_L \pi D^4 t}{128 \mu V L}$$

NOTE:- For steady, uniform, laminar flow in circular pipe.

$$\tau = \frac{r}{2} \left(-\frac{dP}{dr} \right)$$

For boundary shear stress, τ_0 at $r=R$

$$\tau_0 = \frac{R}{2} \left(\frac{\delta h_L}{L} \right)$$

$$\tau_0 = \frac{R}{2} \left(\frac{\gamma}{L} \cdot \frac{f L V^2}{2gD} \right)$$

$$\tau_0 = \frac{f \rho V^2 R}{8}$$

$$\frac{\text{Solve it 2021 } \tau_0}{\rho} = \frac{f V^2}{8}$$

$$\sqrt{\frac{\tau_0}{\rho}} = v \sqrt{\frac{f}{8}}$$

shear velocity (u_*)

$$u_* = v \sqrt{\frac{f}{8}}$$

valid for both laminar and turbulent flow.

- 8) When water flows in a 2cm dia pipe. Compute the largest discharge up to which flow will be definitely laminar. $\rho_w = 1000 \text{ kg/m}^3$, $\nu = 0.0098 \text{ stokes}$. Also compute the boundary shear stress at this discharge.

Sol For flow to be laminar, $Re \leq 2000$.

$$\frac{VD}{\nu} \leq 2000, \quad V \leq \frac{2000\nu}{D} = \frac{2000 \times 0.0098 \times 10^{-4}}{2 \times 10^{-2}}$$

$$V \leq 0.098 \text{ m/s}$$

$$Q \leq V \cdot A \leq 0.098 \times \frac{\pi}{4} \times (2 \times 10^{-2})^2$$

$$Q \leq 3.07 \times 10^{-5} \text{ m}^3/\text{s}.$$

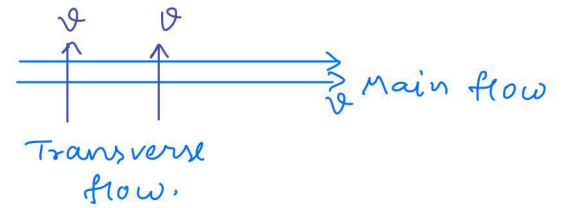
$$\text{shear velocity, } u_* = v \sqrt{\frac{f}{8}} \quad f = \frac{64}{Re} = \frac{64}{2100}$$

$$u_* = 0.098 \sqrt{\frac{64}{2000 \times 8}} = 6.198 \times 10^{-3} \text{ m/s},$$

$$\tau = u_*^2 \times \rho = (6.198 \times 10^{-3})^2 \times 10^3 = 0.0384 \text{ N/m}^2.$$

INSTABILITY OF LAMINAR FLOW :-

- In any fluid, disturbances of various magnitude are caused due to inlet, outlet of pipe, pump vibrations etc.



- These disturbances are in the form of transverse velocity component, superimposed on main flow velocity.
- Depending upon flow and fluid properties like mass, density, viscosity, velocity gradient these disturbances are either damped or amplified causing it laminar and Turbulent respectively.
- Instability of ^{Souvik}laminar flow is represented by a dimensionless number. χ (chi)

$$\chi = \frac{y^2 \rho}{\mu} \left(\frac{du}{dy} \right)$$

@ $y=0$ and at centre where $\frac{du}{dy} = 0$.

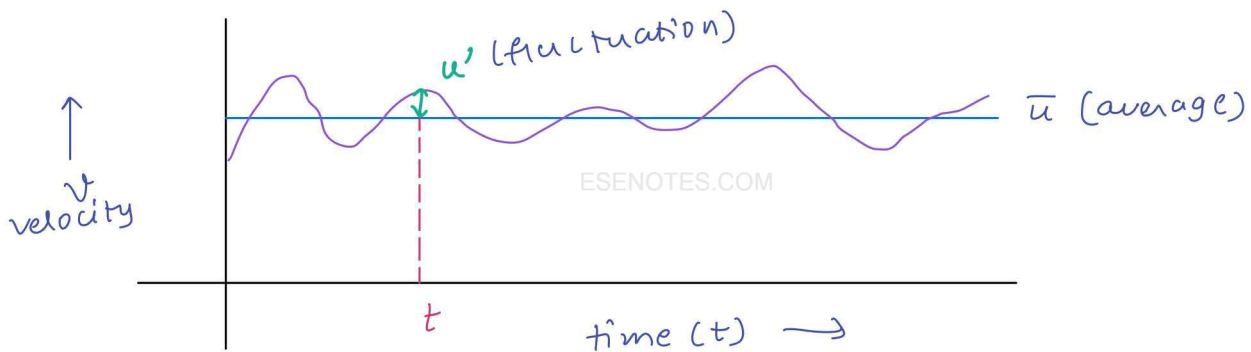
$$\chi = 0.$$

Hence, laminar flow becomes unstable b/w centre and boundary

Laminar flow becomes unstable when $\chi > 500$ at any point.

TURBULENT FLOW

- Turbulent flow develops from instability of laminar flow.
- The velocity in transverse direction of main flow (eddies) spread over the entire cross-section due to which large scale momentum and energy transfer takes place b/w the different layers of fluid.
- In this flow fluid parameters starts varying wrt time and space i.e. **unsteady and non-uniform**.
- But for analysis we assume that this parameters ^{Solve} are considered as combination of average and fluctuating component.



$$\text{velocity} = \bar{u} + u'$$

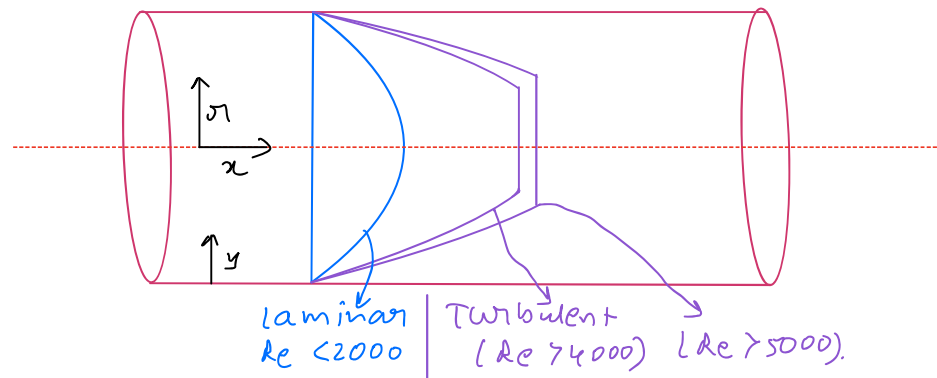
$\bar{u}$ = Temporal mean/avg velocity

u' = Fluctuating comp. of viscosity.

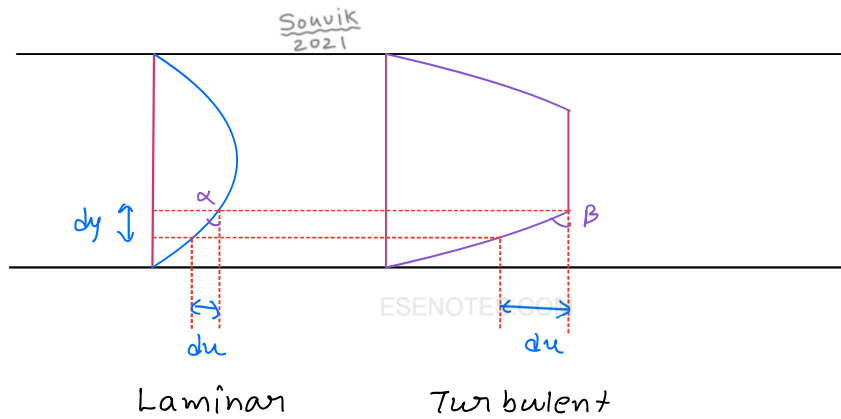
- Flow in pipes (circular) is considered to be turbulent if $Re > 4000$

$$Re = \frac{\rho V D}{\mu} > 4000 \quad (\text{not a fixed range}).$$

- velocity profile in case of turbulent is much flatter than in laminar flow and this flatness increases with increase in Re .
- This flatness of velocity profile is due to intermixing of liquid b/w different layers



- Shear stress at boundaries in case of turbulent flow is much more than that in laminar flow as velocity gradient (du/dy) near boundaries is large in turbulent flow.



$$\tan \alpha = \frac{du}{dy}, \quad \tan \beta = \frac{du}{dy}$$

$$\tan \beta > \tan \alpha.$$

Total shear stress in turbulent flow may be considered to be of two different components

- VISCOUS SHEAR.
- TURBULENT SHEAR.

$$\tau_{\text{total}} = \tau_{\text{laminar}} + \tau_{\text{turbulent}}.$$

(viscous shear stress) (Turbulent shear stress)

- Here laminar shear stress is due to viscosity and turbulent shear stress is due to momentum transfer b/w diff. layers.

TURBULENT SHEAR STRESS :-

- This being in different ways as follows :-

(a) BOUSSINESQ TH. :-

$$\tau_{\text{turbul}} = \eta \frac{d\bar{u}}{dy}$$

Also, $\frac{\eta}{\rho} = \text{edd kinematic viscosity.}$

$\bar{u}$ = avg. velocity
 η = eddy viscosity /
Turbulent mixing
coeff
(It decreases towards
wall. & becomes
zero at wall).

- Since " η " is flow property and
not fluid property, this theory is
not of much use.

(b) REYNOLD'S TH. :- Souvik 2021

- As per this th,

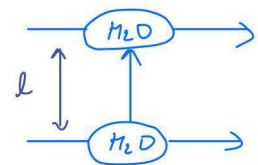
$$\tau_{\text{turbulent}} = \rho u'v'$$

u' = fluctuating component of velocity in
x-direction.

v' = fluctuating component of vel. in y-direct.

(c) PRANDTL'S TH :-

- This theory is based on concept of
mixing length. i.e the distance
travelled by fluid molecule
before striking another molecule.



$$u' = l \frac{du}{dy} \quad v' = l \frac{dv}{dy}$$

$$\tau_{\text{turb}} = \rho l \frac{du}{dy} \cdot l \frac{dv}{dy}$$

$$\tau_{\text{turbul}} = \rho l^2 \left(\frac{du}{dy} \right)^2$$

Also, mixing length, $l = ky$

$y \rightarrow$ measured from
wall of pipe.

here $k = 0.4$ (Karman const).

Hence, total stress,

$$\tau_{total} = \mu \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy} \right)^2$$

Since the effect of laminar/viscous shear stress is pre-dominant only near boundaries.

Hence

$$\tau_{total} = \tau_{turbulent} = \rho l^2 \left(\frac{du}{dy} \right)^2$$

VELOCITY DISTRIBUTION :-

$$\tau_{total} = \tau_{turb} = \rho l^2 \left(\frac{du}{dy} \right)^2$$

Also, $\frac{S_{uvik}}{224} \kappa y = 0.4 y$.

$$\tau = \rho (0.4 y)^2 \left(\frac{du}{dy} \right)^2$$

$$\left(\frac{du}{dy} \right)^2 = \frac{\tau}{\rho \kappa^2 y^2}$$

$$\frac{du}{dy} = \sqrt{\frac{\tau}{\rho}} \cdot \frac{1}{\kappa y}$$

For small values of "y" $\tau = \tau_0$ (boundary shear stress).

$$\frac{du}{dy} = \sqrt{\frac{\tau_0}{\rho}} \cdot \frac{1}{\kappa y}$$

$$\frac{du}{dy} = \frac{u^*}{\kappa y}$$

[u^* = shear velocity]

$$du = \frac{u^*}{\kappa y} dy$$

$$u = \frac{u^*}{\kappa} \ln(y) + C$$

$$@ y = R, u = u_{max}$$

$$u_{max} = \frac{u^*}{\kappa} \ln(R) + C$$

$$C = u_{max} - \frac{u^*}{\kappa} \ln(R)$$

$$u = \frac{u^*}{\kappa} \ln(y) + u_{max} - \frac{u^*}{\kappa} \ln(R)$$

$$u = \frac{u^*}{\kappa} \ln\left(\frac{y}{R}\right) + u_{max}$$

⇒ velocity profile in turbulent flow.

For $K=0.4$,

$$u = 2.5 u^* \ln \left(\frac{y}{R} \right) + u_{\max}$$

$$\text{or, } u = 5.75 u^* \log \left(\frac{y}{R} \right) + u_{\max}$$

NOTE :- The above eqn can also be written in non-dim. form.

{ Remember all the forms, as the cases diff. forms }

$$\frac{u_{\max} - u}{u^*} = -5.75 \log \left(\frac{y}{R} \right)$$

$$\frac{u_{\max} - u}{u^*} = 5.75 \log \left(\frac{R}{y} \right)$$

Here, $u_{\max} - u$ is termed as **velocity defect**

Sept-11

HYDRODYNAMICALLY SMOOTH & ROUGH BOUNDARIES :-

Souvik
2021

- In turbulent flow in pipes, effect of viscosity is maximum near the boundaries, that results in formation of region termed as LAMINAR SUB LAYER (δ').

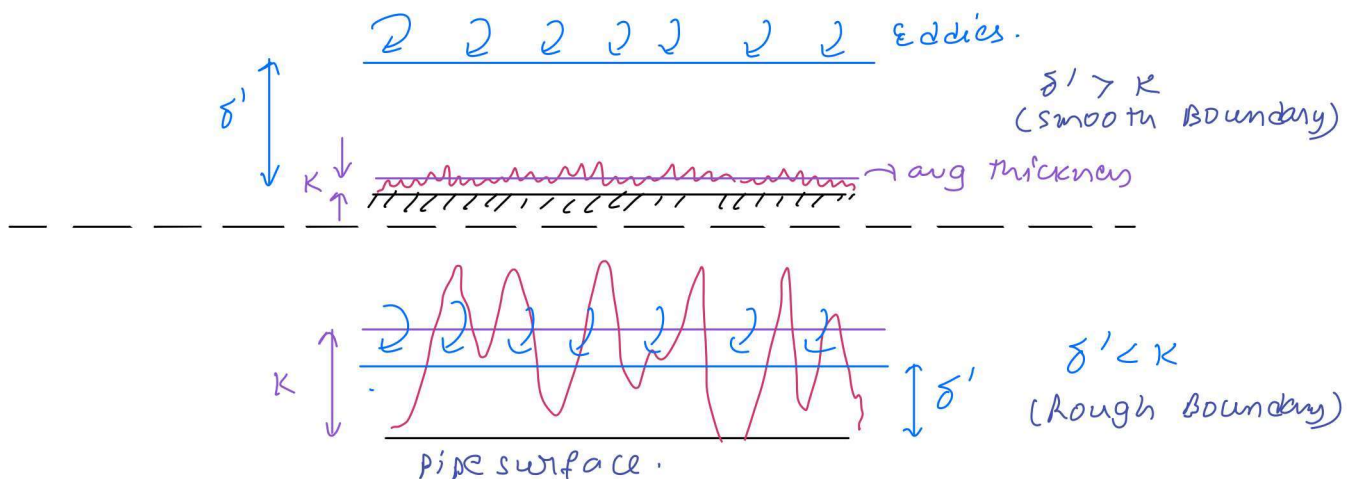
- The thickness (δ') is directly proportional to kinematic viscosity (ν) and $\propto \frac{1}{\text{flow vel. (v)}}$

$$\text{Now, } Re = \frac{vD}{\nu}$$

$$\frac{1}{Re} = \frac{\nu}{vD}$$

hence

$$\delta' \propto \frac{1}{Re}$$



K = avg thickness of Roughness.

- If thickness of laminar sub layer is more, it is not able to reach the boundary and it acts as hydrodynamically smooth boundary whereas if thickness of laminar sub-layer is less eddies are able to penetrate up to the boundary and it act as hydrodynamically rough boundary.

As per NIKURANDHE Experiment

$$\text{if } \frac{k}{\delta'} < 0.25 \rightarrow \text{Smooth boundary}$$

$$\frac{k}{\delta'} > 6 \rightarrow \text{Rough boundary.}$$

$$\text{if } 0.25 < \frac{k}{\delta'} < 6 \rightarrow \text{Transition Boundary.}$$

Also, $\delta' = \frac{11.6 \nu}{u_*}$ $\rightarrow$ Basis on observation $u_* = \sqrt{\frac{\tau_0}{\rho}}$

Hence the above limits can be written in terms of Re_k .

$$\frac{k}{\delta'} = \frac{k u_*}{11.6 \nu} = \frac{Re_k}{11.6}$$

Smooth bound $\frac{k}{\delta'} < 0.25 \Rightarrow \frac{Re_k}{11.6} < 0.25$

$$Re_k < 3$$

Rough bound $\frac{k}{\delta'} > 6 \Rightarrow Re_k > 6 \times 11.6$

$$Re_k > 70$$

Transition bound,

$$3 < Re_k < 70$$

k = avg. thickness of roughness

δ' = thickness of laminar sub layer.

- Q) Find the type of boundary for commercial pipe carrying water with following parameters
 $\rho = 1000 \text{ kg/m}^3$, $\nu = 10^{-6} \text{ m}^2/\text{s}$, $k = 0.12 \text{ mm}$
 $\tau_0 = 600 \text{ N/m}^2$.

Sol $\delta' = \frac{11.6 \nu}{u_*}$ where, $u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{600 \text{ kg m/s}^2 \text{ m}^2}{1000 \text{ kg/s}^2 \text{ m}^2}}$

$$\delta' = \frac{11.6 \times 10^{-6}}{0.6} = 1.93 \times 10^{-2} \text{ mm} \quad u_* = 0.6 \frac{\text{m}}{\text{s}}$$

$$\frac{k}{\delta'} = \frac{0.12}{1.93 \times 10^{-2}} = 6.21 (> 6) \rightarrow \text{Rough.}$$

VELOCITY DISTRIBUTION FOR TURBULENT FLOW IN SMOOTH PIPE:-

$$u = \frac{u^*}{k} \ln y + c \quad k = \text{Karman const.}$$

Due to the presence of Roughness over the boundary surface, vel. is observed to be zero at certain distance (y') away from it.

Hence, if $y = y'$, $u = 0$.

$$0 = \frac{u^*}{k} \ln y' + c$$

$$c = - \frac{u^*}{k} \ln y'$$

$$u = \frac{u^*}{k} \ln y - \frac{u^*}{k} \ln y'$$

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2021

$$u = \frac{u^*}{0.4} \ln (y/y')$$

$$u = 2.5 u^* \ln (y/y')$$

Also, $\frac{u^* y'}{\nu} = 0.108$ (Experimental for smooth pipe)

$$y' = 0.108 \frac{\nu}{u^*}$$

$$u = 2.5 u^* \ln \left(\frac{y u^*}{0.108 \nu} \right)$$

$$\frac{u}{u^*} = 5.75 \left[\log \left(\frac{y u^*}{\nu} \right) - \log (0.108) \right]$$

valid only
for smooth
pipe.

$$\frac{u}{u^*} = 5.75 \log \left(\frac{y u^*}{\nu} \right) + 5.55 \quad \text{--- (A)}$$

Note, $\frac{u^* \delta'}{\nu} = 11.6$ and $\frac{u^* y'}{\nu} = 0.108$.
(Reg1) (Reg1)

$$\frac{y'}{\delta'} = \frac{0.108}{11.6} = \frac{1}{107}$$

$$\frac{y'}{\delta'} = \frac{1}{107}$$

Smooth boundary Klie, y' 107 guna chota hai δ' se. matlab bohot chota hai.

VELOCITY DISTRIBUTION OF TURBULENT FLOW in ROUGH PIPE

(ROUGH BOUNDARY)

From previous,

$$u = 2.5 u^* \ln (y/y')$$

For Rough pipe. $y' = \frac{k}{30}$

[k = avg ht. of roughness]

$$u = 5.75 u^* \log \left(\frac{y}{k} \right)$$

$$\frac{u}{u^*} = 5.75 \log \left(\frac{y}{k} \right) + 5.57 \log (30)$$

$$\boxed{\frac{u}{u^*} = 5.75 \log \left(\frac{y}{k} \right) + 8.5} \quad \text{--- (B)}$$

NOTE:- The above vel. distribution for smooth and rough pipe can be written in terms of avg velocity also.

$$dQ = u \cdot dA$$

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$$Q = \int dQ = \int u \cdot dA \quad \text{--- (i)}$$

Also, $Q = A \cdot V_{avg} = \pi R^2 \cdot \bar{u} \quad \text{--- (ii)}$

From (i) & (ii)

$$\pi R^2 \bar{u} = \int u \cdot dA$$

$$\bar{u} = \frac{1}{\pi R^2} \int u \cdot dA$$

put u for smooth pipe or rough pipe we will get avg velocity.

For smooth pipe.

$$\bar{u} = \frac{1}{\pi R^2} \int_0^R u^* \left[5.75 \log_{10} \left(\frac{u^* y}{\nu} \right) + 5.5 \right] 2\pi r dr$$

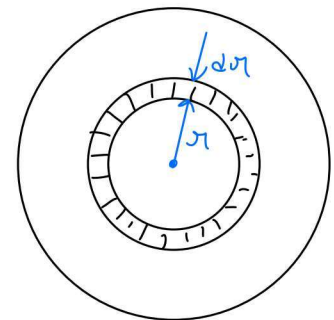
put $y = R - r$

$$\boxed{\frac{\bar{u}}{u^*} = 5.75 \log_{10} \left(\frac{u^* R}{\nu} \right) + 1.75} \quad \text{--- (C)}$$

For rough pipe

$$\boxed{\frac{u}{u^*} = 5.75 \log_{10} \left(\frac{R}{k} \right) + 4.75}$$

--- (D)



From A & C

$$\frac{u}{u^*} - \frac{\bar{u}}{u^*} = 5.75 \log \left(\frac{u^* y}{\nu} \right) + 5.5 - 5.75 \log \left(\frac{u^* R}{\nu} \right) - 1.75$$

$$\boxed{\frac{u - \bar{u}}{u^*} = 5.75 \log_{10} \left(\frac{y}{R} \right) + 3.75}$$
 → valid for both smooth & rough.

Also, @ $y = R$, $u = u_{\max}$

$$\boxed{\frac{u_{\max} - \bar{u}}{u^*} = 3.75} \rightarrow \text{valid for both.}$$

Similarly, (B) - (D)

$$\frac{u}{u^*} - \frac{\bar{u}}{u^*} = \overset{\text{Soniik 2015}}{5.75} \log_{10} \left(\frac{y}{R} \right) + 8.5 - 5.75 \log \left(\frac{R}{R} \right) - 4.75$$

$$\boxed{\frac{u - \bar{u}}{u^*} = 5.75 \log_{10} \left(\frac{y}{R} \right) + 3.75}$$
 → valid for both

For $u = u_{\max}$, $y = R$

$$\boxed{\frac{u_{\max} - \bar{u}}{u^*} = 3.75}$$

NOTE:- velocity distribution in smooth pipe is also given by NIKURADSE power law.

$$\boxed{\frac{u}{u_{\max}} = \left(\frac{y}{R} \right)^{1/7}} \rightarrow \text{only for smooth.}$$

(Remembers in laminar flow, value of α and β was having separate category).

FRICTION FACTOR :-

[NOTE:- For laminar flow, f = function (Re) .]

- Friction factor for turbulent flow depends on type of pipe as follows.

(a) SMOOTH PIPE :-

$$f = \frac{0.316}{Re^{1/4}}$$

$$4000 \leq Re \leq 10^5$$

and

$$f = 0.0032 + \frac{0.221}{Re^{0.237}}$$

$$5 \times 10^4 < Re < 4 \times 10^7$$

For $5 \times 10^4 < \overset{\text{Sawik}}{Re} < 4 \times 10^7$, friction factor for smooth pipe can also be found using NIKURADSE formula.

$$\frac{1}{\sqrt{f}} = 2 \log_{10} (Re \sqrt{f}) - 0.8$$

Conclusion :- In turbulent flow for smooth pipe friction factor is a function of Re .

(b) ROUGH PIPE

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{R}{K} \right) + 1.74$$

K = Avg height of roughness.

R = Radius of pipe

Here friction factor is a function of " K ".

Hence for transition flow, f will be function of both " Re " and " K ".

NOTE:- For both smooth & rough pipe.

$$\frac{u_{\max} - u_{\text{avg}}}{u^*} = 3.75 \rightarrow \text{General Relation.}$$

$$u^* = \sqrt{\frac{\tau_0}{\rho}} = \bar{u} \sqrt{\frac{f}{8}} \quad \text{(derived in laminar flow)} \rightarrow \text{valid for both.}$$

From (i) & (ii)

$$\frac{u_{max}}{u^*} - \frac{\bar{u}}{u^*} = 3.75$$

$$\frac{u_{max}}{u^*} - \sqrt{\frac{8}{f}} = 3.75$$

$$\frac{u_{max}}{u^*} = \frac{3.75 \sqrt{\frac{f}{8}} + 1}{\sqrt{\frac{f}{8}}}$$

$$\frac{u_{max}}{u^*} \sqrt{\frac{8}{f}} = \left(3.75 \sqrt{\frac{f}{8}} + 1 \right) \quad \left[\bar{u} = u^* \left(\frac{8}{f} \right)^{1/2} \right]$$

$$\frac{u_{max}}{\bar{u}} = 1.325 \sqrt{f} + 1$$

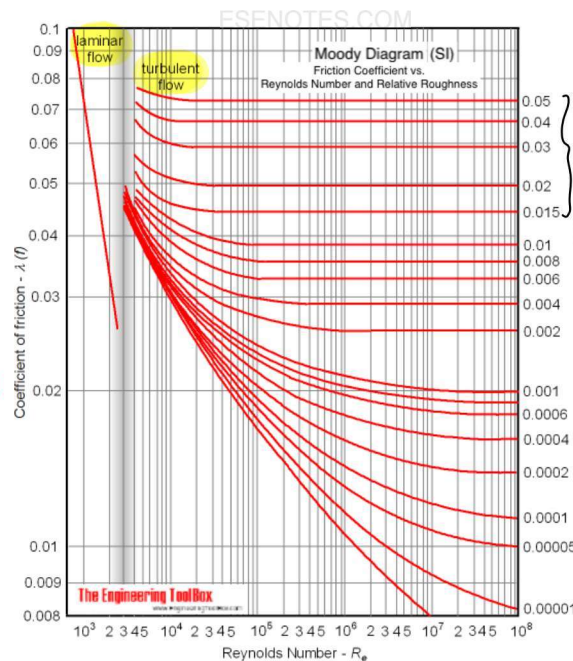
$\Rightarrow$ valid for both.

FRICTION factor for commercial pipe :-

- For commercial pipe friction factor is found using **MOODY'S CURVE**

NOTE:- At large value of Re curve will become horizontal

going upward Reynolds number is increasing.



$\rightarrow$ function of $\left(\frac{k}{D} \right)$

$\rightarrow k =$ Avg height of Roughness.
 $D =$ Diameter

- Moody's curve is given / developed for circular pipe only, But it can be used for other shape also by replacing D with $4R_H$, where $R_H =$ Hydraulic Radius

$$\text{For circular pipe } R_H = \frac{\pi D^2}{4 \pi D} = \frac{D}{4} \Rightarrow D = 4 R_H$$

- As the pipe becomes older, due to wear and tear, its roughness increases i.e k also increases.

$$K = K_0 + \alpha t$$

$t = \text{time}$

$\alpha = \text{const.}$

- Q) For a turbulent flow in pipe of diameter 300mm, find the discharge when the centre line velocity is 2m/s and vel. at point 100mm from centre is 1.6 m/s

Sol

$$d = 300 \text{ mm}$$

$$u_{\max} = 2 \text{ m/s} \quad r = 100 \text{ mm}$$

$$u = 1.6 \text{ m/s}$$

$$\frac{u - \bar{u}}{u^*} = 5.75 \log_{10} \left(\frac{y}{R} \right) + 3.75$$

$$@ y = R, u = 2 \text{ m/s.}$$

$$\frac{2 - \bar{u}}{u^*} = 3.75 \quad \frac{2}{u^*} - \frac{\bar{u}}{u^*} = 3.75 \quad \text{--- (i)}$$

$$y + r = 150$$

$$y = 150 - 100 = 50 \text{ mm}, \quad u = 1.6 \text{ m/s}$$

$$\frac{1.6 - \bar{u}}{u^*} = 5.75 \log_{10} \left(\frac{50}{150} \right) + 3.75$$

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$$\frac{1.6 - \bar{u}}{u^*} = 1 \quad \text{--- (ii)}$$

$$\frac{1.6}{u^*} - \frac{\bar{u}}{u^*} = 1$$

$$\frac{1.6}{u^*} - \frac{\bar{u}}{u^*} + 3.75 = 1$$

$$\frac{0.4}{u^*} = 2.75$$

$$u^* = 0.145 \text{ m/s}$$

$$\therefore \bar{u} = 1.6 - u^*$$

$$\bar{u} = 1.45 \text{ m/s}$$

$$\begin{aligned} \text{Discharge, } Q &= A \times \text{avg} = \frac{\pi}{4} \times (300)^2 \times 1.45 \\ &= 0.102 \text{ m}^3/\text{s} \end{aligned}$$

Q5) Water flows in a 80mm pipe at $Re = 8 \times 10^4$. The pipe is estimated to have an equivalent-sand grain roughness 0.16mm. Determine the head loss expected in 500m length of pipe.

How much head loss is expected in pipe if they were smooth. $\nu = 10^{-6} \text{ m}^2/\text{s}$. Use the following eq,

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log_{10} \left[\frac{k}{d} + \frac{21.25}{Re^{0.9}} \right]$$

k = eq. sand grain roughness.

d = Dia of pipe.

Sol = $d = 80 \text{ mm}$ $k = 0.16 \text{ mm}$
 $Re = 8 \times 10^4$

Head loss ^{Souvik 2021} $h_L = \frac{f L v^2}{2gD}$

Now, $\frac{1}{\sqrt{f}} = 1.14 - 2 \log_{10} \left[\frac{0.16}{80} + \frac{21.25}{(8 \times 10^4)^{0.9}} \right]$

$$\frac{1}{\sqrt{f}} = 6.23$$

$$f = 0.0256$$

Also, $Re = \frac{vD}{\nu}$

$$v = \frac{8 \times 10^4 \times 10^{-6}}{80 \times 10^{-3}} = 1 \text{ m/s.}$$

Now, $h_L = \frac{f L v^2}{2gD} = \frac{0.0256 \times 500 \times 1^2}{2 \times 9.81 \times 0.08} = 8.154 \text{ m}$

(b) For smooth pipe, $k=0$.

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log_{10} \left[\frac{21.25}{(8 \times 10^4)^{0.9}} \right]$$

$$\frac{1}{\sqrt{f}} = 7.31, \quad f = 0.0187$$

$$h_L = \frac{0.0187 \times 500 \times 1^2}{2 \times 9.81 \times 0.08} = 5.95 \text{ m}$$

f generally varies b/w (0.02 - 0.04)

- 8) A 60cm dia rough pipe carries water. The velocity at 4cm from pipe wall is 3m/s and the vel. gradient at same point is 11.25 sec^{-1} . Compute

- (a) mean velocity of flow.
 (b) friction factor
 (c) Avg shear stress at wall.
 (d) Avg ht of roughness.

Sol

$$d = 60 \text{ cm} \quad y = 4 \text{ cm}, \quad v = 3 \text{ m/s}$$

$$\frac{dv}{dy} = 11.25 \text{ s}^{-1}.$$

(a) $\frac{u - \bar{u}}{u^*} = 5.75 \log \left(\frac{y}{K} \right) + 3.75$

$u^* = \frac{K y}{\text{SOLUTION 2021}} \left(\frac{dv}{dy} \right) \quad K = 0.4.$

$u^* = 0.4 \times 0.04 \times 11.25 = 0.18 \text{ m/s}$

$\frac{3 - \bar{u}}{0.18} = 5.75 \log \left(\frac{4}{30} \right) + 3.75$

$\frac{3 - \bar{u}}{0.18} = -1.28$

$\bar{u} = 3.23 \text{ m/s}.$

(b) $u^* = \bar{u} \sqrt{\frac{f}{8}}$

$0.18 = 3.23 \sqrt{\frac{f}{8}} \quad f = 0.198$

(c) $\sqrt{\frac{\tau_0}{\rho}} = u^*$

$\tau_0 = \frac{u^*{}^2 \rho}{1} = 0.18^2 \times 1000 = 32.4 \text{ N/m}^2$

(d) For rough pipe

$\frac{u}{u^*} = 5.75 \log \left(\frac{y}{K} \right) + 8.5$

@ $y = 4 \text{ cm}, \quad u = 3 \text{ m/s}$

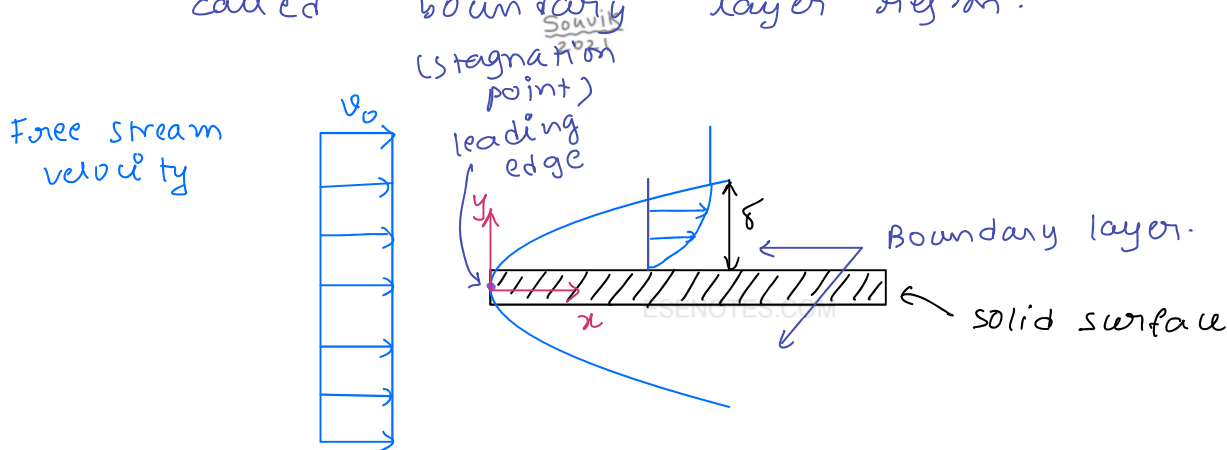
$\frac{3}{0.18} = 5.75 \log \left(\frac{0.04}{K} \right) + 8.5$

$1.42 = \log \left(\frac{0.04}{K} \right)$

$K = 1.52 \text{ mm}.$

BOUNDARY LAYER THEORY

- Boundary layer theory was given by Prandtl and is valid for real fluid flowing over infinitely large medium.
- When a real fluid flows past a solid surface, fluid particles adhere to the boundary due to no-slip condition i.e. if boundary is stationary fluid on it will be stationary and if boundary is moving, fluid attached to it also moves.
- As distance from boundary increases, velocity gradually increases up to certain distance (δ) and then becomes const. This region (δ) is called boundary layer region.



- There exists velocity gradient inside the boundary region and it varies from max at boundary of solid surface to zero at δ from boundary of solid surface.
- Hence shear stress is also maximum at boundary.

$$\left(\tau = \mu \frac{du}{dy} \right)$$

- Essential Boundary conditions :-

- (a) $x=0$, $\delta=0$
- (b) $y=0$, $u=0$
- (c) $y=\delta$, $u=v_0 = \text{const}$
- (d) $y=\delta$, $\frac{du}{dy} = 0$, $\frac{d^2u}{dy^2} = 0$.

8) velocity profile is given by $\frac{u}{v} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$

apply appropriate boundary condition and find the value of κ , if boundary shear stress is $\tau_0 = \kappa\left(\frac{\mu v}{\delta}\right)$

sol $\frac{u}{v} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$

@ $y=0, u=0 \quad 0 = a$

@ $y=\delta, \frac{du}{dy} = 0, \quad \frac{du}{dy} \frac{1}{v} = \frac{b}{\delta} + 2c \frac{y}{\delta} \cdot \frac{1}{\delta} = 0$
 $b + 2c = 0$

@ $y=\delta, u=v, \quad 1 = b(1) + c$

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$$\begin{array}{rcl} b+c & = & 1 \\ b+2c & = & 0 \\ \hline -c & = & 1 \\ c & = & -1 \end{array} \quad b=2$$

$\therefore \frac{u}{v} = 2\left(\frac{y}{\delta}\right) - 1\left(\frac{y}{\delta}\right)^2$

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now,

Boundary shear stress, $\tau_0 = \mu \frac{du}{dy}$

$\tau_0 = \mu \left\{ \frac{2 \cdot}{\delta} - \frac{2y}{\delta^2} \right\}$

$\tau_0 = 2v \frac{\mu}{\delta} \quad [\text{at } y=0]$

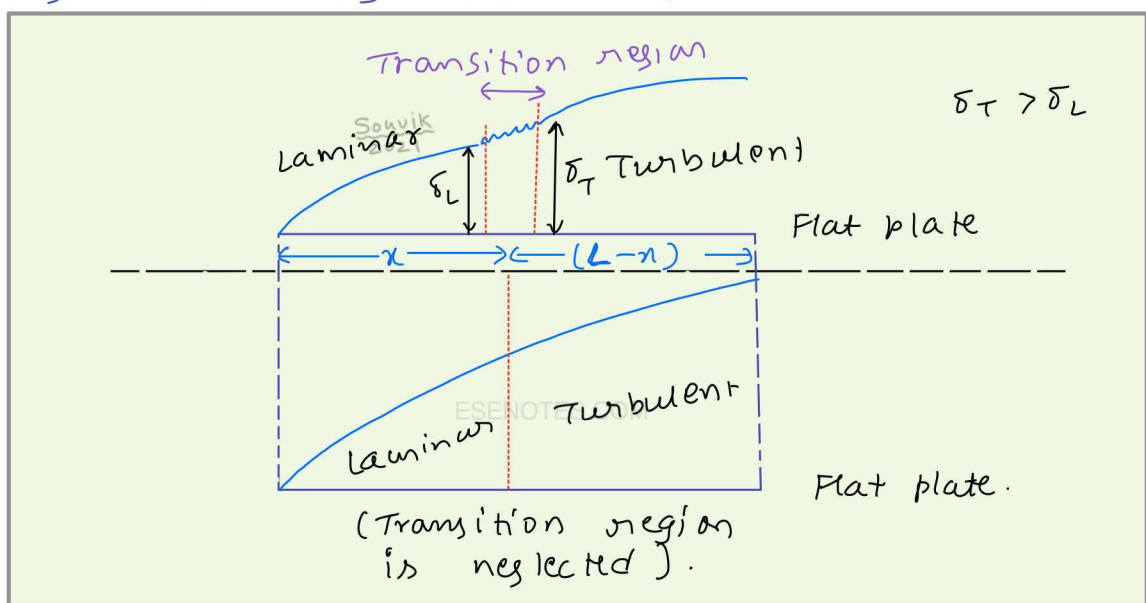
Also, $\tau_0 = \kappa\left(\frac{\mu v}{\delta}\right)$

$\therefore \kappa = 2.$

- When fluid flow past solid surface, the velocity at leading edge is zero and retardation of fluid increases as more and more of the plate is exposed to flow, hence boundary layer thickness increases as the distance from leading edge increases.

NOTE :- The above statement is valid for both laminar & turbulent flow.

- up to a certain distance from leading edge, flow in boundary layer is laminar irrespective of approaching flow nature.
- As depth of laminar boundary layer increases, it cannot dissipate the effect of instability in flow hence transition of flow to turbulent takes place.
- Thereby thickness of turbulent boundary layer is more
- For calculation purpose the presence of transition region is neglected.



- The change of boundary layer from laminar to turbulent is affected by

(a) ROUGHNESS of plate:-

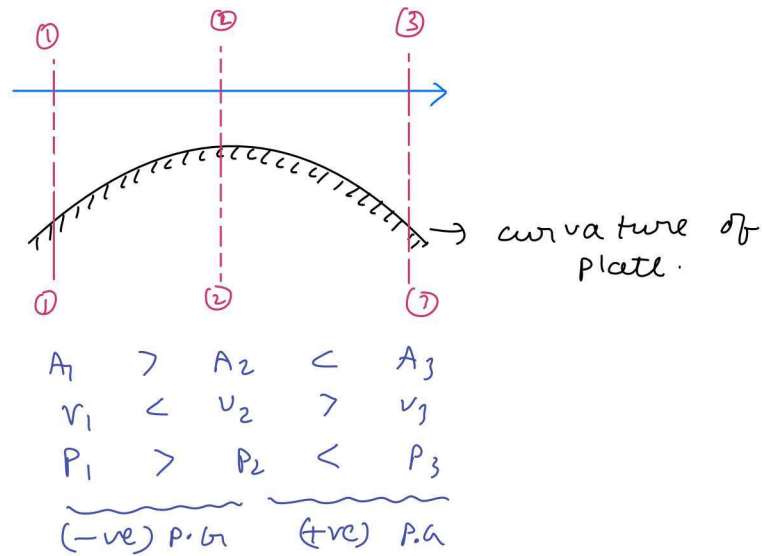
More roughness leads to early formation of transition zone.

(b) INTENSITY & SCALE OF TURBULENCE:-

Greater turbulence leads to early transition.

(c) PRESSURE GRADIENT:- (The pressure gradient (pressure increase in direction of flow) leads to early transition. & -ve pressure gradient leads to delayed transition.

(d) Plate curvature:-



(**)

- Transition of boundary layer from laminar to turbulent is assumed to occur at

$Re = 5 \times 10^5$ (for flat plate)

$$Re_x = \frac{Vx}{\nu}$$

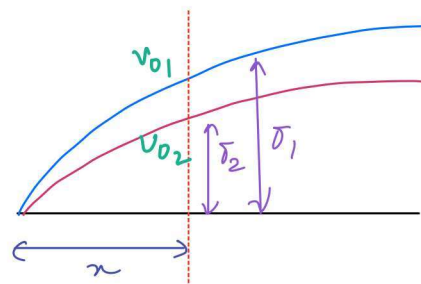
x = distance from leading edge.

V = free stream velocity

ν = kinematic viscosity.

hence, if $Re_x < 5 \times 10^5$ (Boundary layer is laminar).

$Re_x > 5 \times 10^5$ (Boundary layer is turbulent).



$$v_{02} > v_{01}$$

$$\rho = \text{const}$$

$$\text{hence, } \rho v_{02} < \rho v_{01}$$

$$\therefore 1 \cdot \delta_{02} < 1 \cdot \delta_{01}$$

$$\delta_{02} < \delta_{01}$$

- Thickness of boundary layer depends on flow.

(a) velocity of flow :- As $V \uparrow$, $\delta \downarrow$ because $\rho = \text{const}$ hence $A \downarrow$ thus $\delta \downarrow$.

(b) viscosity :- more is the viscosity, more the thickness.

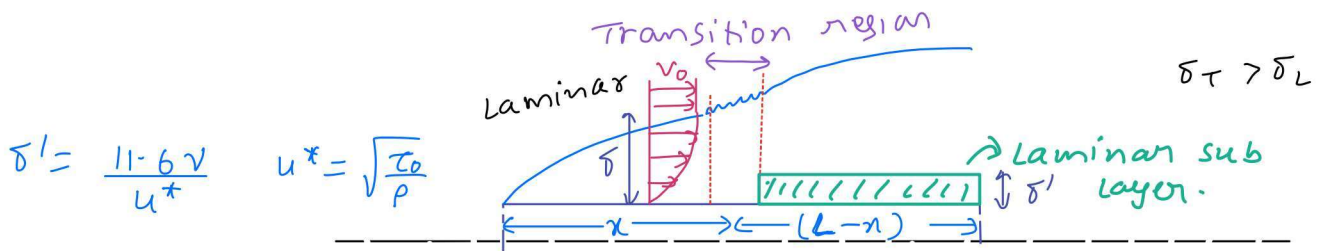
(c) Reynolds Number :- $Re_x = \frac{V_0 x}{\nu}$

Re_x increases, then $\delta \downarrow$

$$\left\{ \begin{array}{l} v_0 \uparrow \rightarrow \delta \downarrow \\ \nu \uparrow \rightarrow \delta \uparrow \\ \frac{v_0}{\nu} \uparrow \rightarrow \frac{\delta}{x} \downarrow \end{array} \right\}$$

(d) Pressure gradient :- More -ve pressure gradient reduces boundary layer thickness ($\rho = \text{const}$).

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- if the plate is smooth, then even in the region of turbulent layer, there is a very thin layer adjacent to the boundary where flow is laminar termed as laminar sub-layer.

- Thickness of laminar sub layer (δ') decreases with increase in Reynolds number. ($\delta' \propto 1/Re$).

NOTE:- velocity profile in laminar boundary layer

$$\frac{u}{v_0} = \left(\frac{y}{\delta}\right)^2 \text{ is parabolic.}$$

velocity profile in turbulent boundary layer.

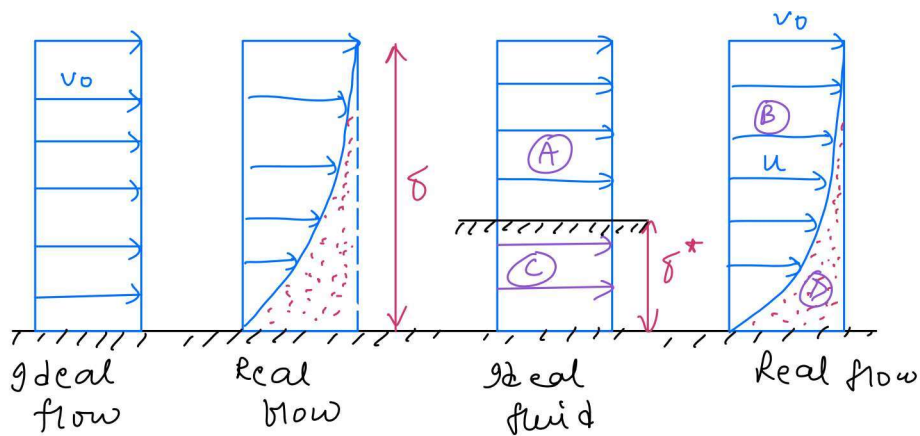
$$\frac{u}{v_0} = \left(\frac{y}{\delta}\right)^{1/7} \text{ (power law) is exponential.}$$

velocity profile in laminar sub-layer is actually parabolic but as thickness is very less, it may be taken as linear.

$$\frac{u}{v_0} = \frac{y}{\delta}$$

② Nominal Boundary layer thickness (δ):- It is the distance from boundary surface in which the velocity reaches 99% of free stream velocity.

Displacement thickness (δ^*):- It is defined as distance by which the boundary should be displaced in order to compensate for the reduction in mass flow rate on account of boundary layer formation.



reduction in mass flow rate on account of boundary layer formation

$\dot{m}$ = mass flow rate (mass of fluid flowing in unit time)

Now,

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 $\dot{m}_c = \dot{m}_d$

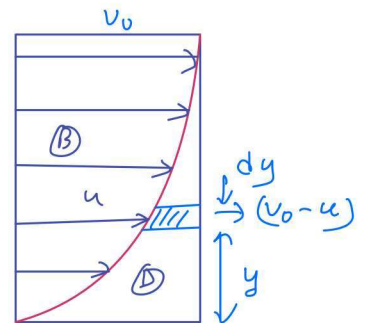
$$\rho A_c = \rho A_d$$

$$\rho A v_c = \rho \int_0^\delta d\delta$$

$$\rho v_0 (\delta^* \cdot 1) = \rho \int_0^\delta dA \cdot v$$

$$\delta^* \cdot v_0 = \int_0^\delta (dy \times 1) (v_0 - u)$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{v_0}\right) dy \Rightarrow \text{this is for plate.}$$



Momentum thickness (θ) :- It is defined as the distance by which the boundary should be shifted in order to compensate for the reduction in momentum on the account of boundary layer formation.

$\dot{m} v$ = moment flow rate.

$$(\dot{m} v)_c = (\dot{m} v)_d$$

$$\downarrow$$

$$\rho A v^2$$

For plate,

$$\theta = \int_0^\delta \left(\frac{u}{v_0}\right) \left(1 - \frac{u}{v_0}\right) dy$$

→ Don't use everywhere

→ only valid for plate.

ENERGY THICKNESS (δ_E) :- It is the distance by which the boundary layer should be displaced in order to compensate for the reduction on account of boundary layer formation.

$$\left(\frac{1}{2} \rho V_C^2\right) = \left(\frac{1}{2} \rho V_D^2\right)$$

$$\delta_E = \int_0^{\delta} \frac{u}{V_0} \left(1 - \frac{u^2}{V_0^2}\right) dy$$

Here, $\frac{\delta^*}{\theta}$ is termed as **shape factor** for boundary layer.

Also, $\delta^* > \delta_E > \theta$

For laminar ^{Souvik} boundary layer on flat plate

$$\delta^* = 0.356, \quad \theta = 0.1356$$

For turbulent boundary layer where $\frac{u}{V_0} = \left(\frac{y}{\delta}\right)^{1/m}$

$$\frac{\delta^*}{\delta} = \frac{1}{m+1}, \quad \frac{\theta}{\delta} = \frac{m}{(m+1)(m+2)}$$

Q) The velocity distribution in boundary layer over the face of spill way was observed to be

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{0.22} \text{ of boundary layer}$$

thickness is 5cm and discharge passing over the spill way was 5 m³/s per metre length of spill way. compute the δ^* , δ_E and θ of energy up to the section under consideration. $u_\infty = 20 \text{ m/s}$.

Sol:-

$$(a) \quad \delta^* = \int_0^{\delta} \left(1 - \frac{u}{u_\infty}\right) dy = \int_0^{\delta} \left(1 - \left(\frac{y}{\delta}\right)^{0.22}\right) dy$$

$$\delta^* = \left[y - \frac{y^{0.22+1}}{1.22 \delta^{0.22}} \right]_0^{\delta}$$

$$\delta^* = \delta - \frac{\delta^{1.22}}{1.22 \delta^{0.22}} = 0.18 \delta = 0.18 \times 5 = 0.9 \text{ cm}$$

$$\begin{aligned}
 (b) \quad \delta_E &= \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \left(\frac{u}{u_{\infty}}\right)^2\right) dy \\
 &= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{0.22} \left(1 - \left(\frac{y}{\delta}\right)^{0.44}\right) dy \\
 &= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{0.22} dy - \int_0^{\delta} \left(\frac{y}{\delta}\right)^{0.66} dy \\
 \delta_E &= 0.217 \delta \\
 \delta_E &= 0.2172 \times 5 = 1.085 \text{ cm.}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad \text{loss of Energy, } E_c &= \frac{1}{2} \dot{m} u_{\infty}^2 = \frac{1}{2} \rho \delta u_{\infty}^2 \\
 E_L &= \frac{1}{2} \rho A u_{\infty}^2 \\
 \text{Solve 2021} \quad E_L &= \frac{1}{2} \rho (\delta_E \cdot 1) u_{\infty}^2 \\
 &= \frac{1}{2} \times 1000 \times 0.01085 \times (20)^3 \\
 E_L &= 43440 \frac{\text{N-m}}{\text{sec}}
 \end{aligned}$$

8) For linear distribution of velocity on boundary layer on flat plate, compute δ^*/θ .

$$\begin{aligned}
 \text{Sol} \quad \frac{u}{u_0} &= \frac{y}{\delta} & \frac{u}{u_0} &= \frac{y}{\delta} \\
 \delta^* &= \int_0^{\delta} \left(1 - \frac{u}{u_0}\right) dy & \theta &= \int_0^{\delta} \frac{u}{u_0} \left(1 - \frac{u}{u_0}\right) dy \\
 &= \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) dy & \theta &= \int_0^{\delta} \left(\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right) dy \\
 \delta^* &= \delta - \frac{\delta}{2} = \frac{\delta}{2} & \theta &= \frac{\delta}{2} - \frac{\delta^3}{3\delta^2} \\
 & & \theta &= \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6} \\
 \therefore \frac{\delta^*}{\theta} &= \frac{\delta}{2} \times \frac{6}{\delta} = 3
 \end{aligned}$$

8) For steady flow in pipe of radius R , find shape factor for boundary layer.

Sol = velocity distribution for pipe of radius R .

$$u = \frac{1}{4\mu} \left(-\frac{\partial P}{\partial x} \right) (R^2 - r^2)$$

$$\text{and } u_{\max} = \frac{1}{4\mu} \left(-\frac{\partial P}{\partial x} \right) R^2$$

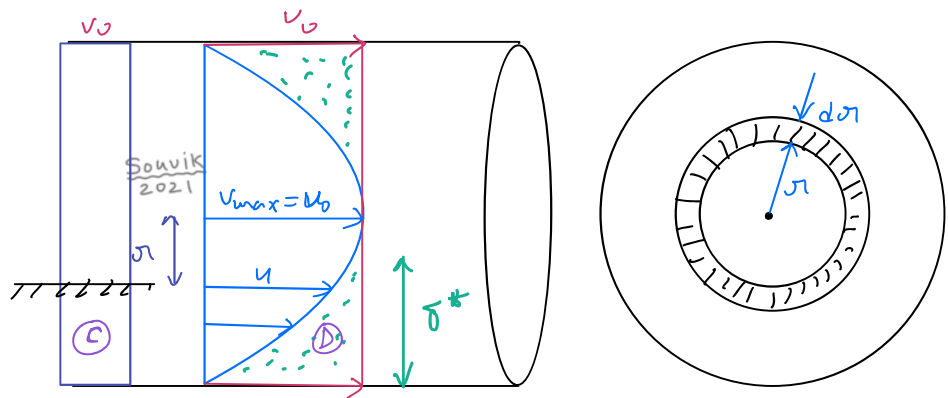
$$\frac{u}{u_{\max}} = \frac{R^2 - r^2}{R^2} = 1 - \frac{r^2}{R^2}$$

For pipe flow,

$$\dot{m}_C = \dot{m}_D$$

$$\theta_C = \theta_D$$

$$A_C \cdot V_0 = A_D \cdot V_D$$



$$A_C = \pi R^2 - \pi (R - \delta^*)^2$$

$$[\pi R^2 - \pi (R - \delta^*)^2] V_0 = \int dA \cdot (V_0 - u)$$

$$= \int 2\pi r \cdot dr \cdot (V_0 - u)$$

$$[-\pi(-2R\delta^* + \delta^{*2})] V_0 = \int 2\pi r (V_0 - V_0(1 - \frac{r^2}{R^2})) dr$$

$$= 2V_0 \int_R^0 r \left(\frac{r^2}{R^2} \right) dr$$

$$= 2V_0 \int_0^R \frac{r^3}{R^2} dr$$

$$[-(-2R\delta^* + \delta^{*2})] V_0 = 2V_0 \frac{R^2}{4} = \frac{V_0 R^2}{2}$$

$$+2R\delta^* - \delta^{*2} = \frac{R^2}{2}$$

Neglecting δ^{*2} .

$$2\pi \delta^* R = \frac{R^2}{2}$$

$$\delta^* = \frac{R}{4}$$

For θ :-

$$(\dot{m}v)_c = (\dot{m}v)_D$$

$$\rho A_c v_0^2 = \rho \int dA (v_0 - u) \cdot u$$

$$u_0^2 [\pi R^2 - \pi (R - \theta)^2] = \int 2\pi r dr (v_0 - u) u$$

$$2R\theta = 2 \int_0^R r \left(u_0 - u_0 \left(1 - \frac{r^2}{R^2} \right) \right) \left(u_0 \left(1 - \frac{r^2}{R^2} \right) \right) dr$$

$$u_0^2 R\theta = \int_0^R r \left(u_0 \frac{r^2}{R^2} \right) \left(u_0 \left(1 - \frac{r^2}{R^2} \right) \right) dr$$

$$u_0^2 R\theta = u_0^2 \int_0^R \frac{r^3}{R^2} \left(1 - \frac{r^2}{R^2} \right) dr$$

$$\theta = \int_0^R \left(\frac{r^3}{R^2} - \frac{r^5}{R^4} \right) dr$$

$$\stackrel{\text{Bouwk 2021}}{=} \frac{1}{R^2} \left[\frac{R^4}{4} - \frac{R^6}{6R^4} \right]$$

$$= \frac{R}{4} - \frac{R}{6} = \frac{R}{12}$$

$$\theta = R/12$$

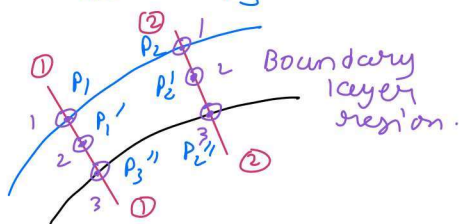
$$\text{Shape factor} = \frac{\bar{u}^*}{\theta} = \frac{R}{4} \times \frac{12}{R} = 3.$$

BOUNDARY LAYER EQUATION :-

(a) conservation of mass :- $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (continuity eqn. in 2D).

$$(b) -\frac{1}{\rho} \left(\frac{\partial p}{\partial y} \right) = 0$$

i.e. [pressure gradient across the boundary layer is constant at a particular section.]



$$p_1 = p_1' = p_1'' \text{ and } p_2 = p_2' = p_2''$$

$$(c) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \left(\frac{\partial p}{\partial x} \right) + \frac{1}{\rho} \left(\frac{d\tau}{dy} \right) \quad \left| \quad \begin{aligned} F &= ma \\ a &= F/m = \frac{P \cdot A}{m} \text{ or } \frac{\tau \cdot A}{m} \\ a &= \frac{P \cdot A}{\rho \cdot V} \text{ or } \frac{\tau \cdot A}{\rho V} \\ a &= \frac{P}{\rho \cdot l} \text{ or } \frac{\tau}{\rho \cdot l} \end{aligned} \right.$$

VON KARMAN MOMENTUM INTEGRAL EQN :-

$$\frac{\tau_0}{\rho v_0^2} = \frac{d\theta}{dx} \rightarrow \text{To find thickness of Boundary layer.}$$

This eqn is valid for following conditions.

- (a) laminar boundary layer [however approx true for turbulent boundary layer also]
- (b) steady flow (c) 2D-flow
- (d) Incompressible flow
- (e) Pressure gradient in direction of flow is zero.

$$\frac{dP}{dx} = 0.$$

DRAW COEFFICIENTS :-

(a) LOCAL DRAG COEFFICIENT :- (C_{fx})

- It is the ratio of wall shear stress at any distance "x" from leading edge to the dynamic pressure.

$$C_{fx} = \frac{\tau_0}{\frac{\rho v_0^2}{2}}$$

(b) Average drag coefficient (C_{favg}). It is the ratio of avg wall shear stress to dynamic pressure

$$C_{favg} = \frac{\tau_{0avg}}{\frac{\rho v_0^2}{2}} = \frac{F_D}{\frac{\rho v_0^2}{2} \cdot A} = \frac{\int \tau_0 \cdot dA}{\frac{\rho v_0^2}{2} \cdot A}$$

Q) For a velocity profile laminar boundary layer

$$\frac{v}{v_0} = \frac{3}{2} \left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^3.$$

compute.

(a) Boundary layer thickness.

(b) Shear stress

(c) Drag force

(d) coeff of drag in terms of Reynolds number.

$$\text{Sol/ (a)} \quad \frac{\tau_0}{\rho v_0^2} = \frac{d\theta}{dx}$$

$$\begin{aligned} \text{also, } \theta &= \int_0^{\delta} \frac{u}{u_0} \left(1 - \frac{u}{u_0}\right) dy \\ &= \int_0^{\delta} \frac{3}{2} \left(\frac{y}{\delta}\right) - \frac{1}{2} \left(\frac{y}{\delta}\right)^3 \left(1 - \frac{3}{2} \left(\frac{y}{\delta}\right) + \frac{1}{2} \left(\frac{y}{\delta}\right)^3\right) dy \\ &= \int_0^{\delta} \frac{3}{2} \frac{y}{\delta} - \frac{9}{4} \left(\frac{y}{\delta}\right)^2 + \frac{3}{4} \left(\frac{y}{\delta}\right)^4 - \frac{1}{2} \left(\frac{y}{\delta}\right)^3 + \frac{3}{4} \left(\frac{y}{\delta}\right)^4 - \frac{1}{4} \left(\frac{y}{\delta}\right)^6 dy \end{aligned}$$

$$= \frac{3}{2} \cdot \frac{1}{2} \cdot \delta - \frac{9}{4} \cdot \frac{1}{3} \delta + \frac{3}{4} \cdot \frac{1}{5} \delta - \frac{1}{2} \cdot \frac{1}{4} \cdot \delta + \frac{3}{4} \cdot \frac{\delta}{5} - \frac{\delta}{4 \times 7}$$

$$\theta = \frac{3\delta}{4} - \frac{9\delta}{12} + \frac{3\delta}{20} - \frac{\delta}{8} + \frac{3\delta}{20} - \frac{\delta}{28}$$

$$\theta = \frac{39\delta}{280} \quad \frac{d\theta}{dx} = \frac{39}{280} \frac{d\delta}{dx}$$

$$\text{now, } \tau_0 = \mu \frac{du}{dy} = \mu \frac{d}{dy} \left[v_0 \left\{ \frac{3}{2} \left(\frac{y}{\delta}\right) - \frac{1}{2} \left(\frac{y}{\delta}\right)^3 \right\} \right]$$

$$\tau_0 = \mu v_0 \left[\frac{3}{2} \cdot \frac{1}{\delta} - \frac{3}{2} \left(\frac{y}{\delta}\right)^2 \cdot \frac{1}{\delta} \right]$$

$$\tau = \mu v_0 \left[\frac{3}{2\delta} - \frac{3y^2}{\delta^3} \right]$$

$$\text{@ } y=0, \tau = \tau_0$$

$$\tau_0 = \frac{3}{2\delta} \mu v_0 \Rightarrow \text{Boundary}$$

putting the value.

$$\frac{\frac{3}{2\delta} \mu v_0}{\rho v_0^2} = \frac{39}{280} \frac{d\delta}{dx}$$

$$\delta d\delta = \frac{3}{2} \mu v_0 \cdot \frac{280}{39} \cdot \frac{1}{\rho v_0^2} \cdot dx$$

$$\delta d\delta = \frac{140}{13} \frac{\mu}{\rho v_0} dx$$

$$\frac{\delta^2}{2} = \frac{140}{13} \frac{\mu}{\rho v_0} x + C$$

$$\text{@ } x=0, \delta=0. \quad 0 = C \quad \therefore \delta = \sqrt{\frac{280}{13} \frac{\mu}{\rho v_0} x}$$

$$\delta = \sqrt{\frac{2 \mu}{3} \cdot \frac{\mu}{\rho V_0 x} \cdot x^2}$$

$$Re = \frac{\rho V D}{\mu}$$

$$\delta = \sqrt{\frac{280}{13} \frac{x^2}{Re}}$$

(***)

$$\delta = \frac{4.64}{\sqrt{Re}} x$$

UVI

(d)

Now,

$$C_{fn} = \frac{\tau_0}{\frac{\rho V_0^2}{2}} = \frac{\frac{3}{2} \frac{\mu V_0}{\delta}}{\frac{\rho V_0^2}{2}} \times \frac{2}{\rho V_0^2} = \frac{3 \mu}{\rho \delta V_0}$$

$$C_{fn} = \frac{3 \mu}{\rho V_0} \cdot \frac{\sqrt{Re} x}{4.64 x}$$

$$C_{fn} \stackrel{\text{Sowik 2021}}{=} \frac{3}{4.64} \frac{\mu}{\rho V_0 x} \frac{\sqrt{Re} x}{4.64}$$

$$C_{fn} = \frac{3 \sqrt{Re} x}{4.64 Re x}$$

$$C_{fx} = \frac{0.646}{\sqrt{Re}}$$

Drag force, $F_D = \int \tau_0 \cdot dA$

$$= \int_0^L \tau_0 \cdot B \cdot dx$$

$$\text{Now, } \tau_0 = \frac{0.646}{\sqrt{Re} x} \times \frac{\rho V_0^2}{2}$$

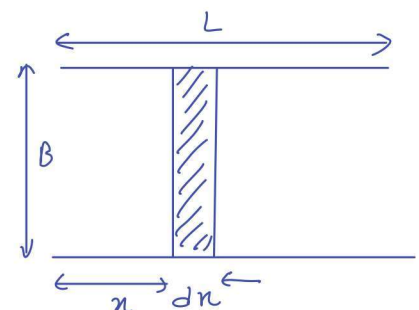
$$F_D = \int_0^L \frac{0.646}{\sqrt{Re} x} \frac{\rho V_0^2}{2} \cdot B \cdot dx$$

$$= \int_0^L \frac{0.646}{(\rho V_0 x)^{1/2}} \mu^{1/2} \frac{\rho V_0^2}{2} B \cdot dx$$

$$= \frac{0.646 \mu^{1/2} \rho V_0^2 B}{2 (\rho V_0)^{1/2}} \int_0^L \frac{1}{\sqrt{x}} dx = \left(2\sqrt{x} \right)_0^L$$

$$= \left(2\sqrt{L} \right)$$

$$F_D = \frac{0.646}{2} \mu^{1/2} \rho^{1/2} V_0^{3/2} B \cdot 2\sqrt{L}$$



Drag Force, $F_D = 0.646 \sqrt{\mu \rho} v_0^{3/2} B \sqrt{L}$

Now,

$$C_{f, \text{avg}} = \frac{F_D}{\frac{\rho v_0^2}{2} \cdot A} = \frac{0.646 \sqrt{\mu \rho} v_0^{3/2} B \sqrt{L}}{\frac{\rho \cdot v_0^2}{2} \cdot (B \cdot L)}$$

$$C_{f, \text{avg}} = \frac{1.292}{\sqrt{Re_L}}$$

In the absence of velocity profile "BLASIUS Expressions" are used.

(a) For laminar Boundary on smooth plate:-

This are applicable when boundary layer is laminar throughout i.e. $Re_L < 5 \times 10^5$

$$\frac{\delta}{x} = \frac{5.08}{\sqrt{Re_x}}$$

$$C_{f, \text{avg}} = \frac{1.328}{\sqrt{Re_L}}$$

$$C_{f, x} = \frac{0.664}{\sqrt{Re_x}}$$

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also, $\delta \propto \sqrt{x}$, $T_o \propto \frac{1}{\sqrt{x}}$

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(b) Turbulent Boundary layer on smooth plate.

$$\frac{\delta}{x} = \frac{0.376}{(Re_x)^{1/5}}$$

$$5 \times 10^5 < Re < 10^7$$

$$\frac{\delta}{x} = \frac{0.22}{(Re_x)^{1/6}}$$

$$10^7 < Re < 10^9$$

NOTE:- $\delta \propto x^{4/5}$, Hence boundary layer thickness increases more rapidly in turbulent boundary layer than in laminar boundary layer.

also,

$$C_{f, x} = \frac{T_o}{\frac{\rho v_0^2}{2}} = \frac{0.059}{(Re_x)^{1/5}}$$

$$5 \times 10^5 < Re < 10^7$$

and

$$C_{f, x} = \frac{T_o}{\frac{\rho v_0^2}{2}} = \frac{0.37}{[\log_{10} Re_x]^{2.58}}$$

$$10^7 < Re < 10^9$$

NOTE:- $\tau_0 \propto 1/x^{0.2}$, Hence boundary shear stress decreases less rapidly in turbulent B.L than in laminar Boundary layer.

$$c_{f \text{ avg}} = \frac{0.074}{(Re_L)^{1/5}} \quad 5 \times 10^5 < Re < 10^7$$

and

$$c_{f \text{ avg}} = \frac{0.455}{[\log_{10}(Re_L)]^{2.58}} \quad 10^5 < Re < 10^7$$

NOTE:- All the above six expressions are applicable only if turbulent Boundary layer is present throughout.

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- 8) Find out the average drag coeff for a rectangular plate on which Boundary layer is partly laminar and partly turbulent and the Reynolds number at trailing edge is less than 10^7 .

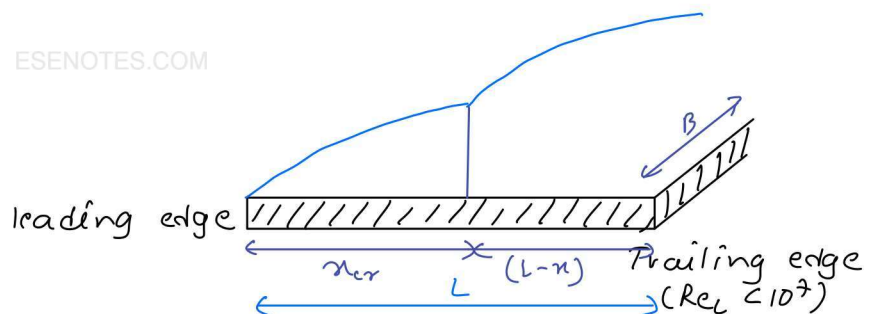
Sol

$$Re_{cr} = 5 \times 10^5$$

$$\frac{v_0 x_{cr}}{\nu} = 5 \times 10^5$$

$$x_{cr} = 5 \times 10^5 \frac{\nu}{v_0}$$

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Here, $c_{f \text{ avg}} = \frac{F_{D1} + F_{D2}}{A \cdot \frac{\rho v_0^2}{2}}$

F_{D1} = drag force at the laminary Boundary layer

$$F_{D1} = \int \tau_0 \cdot dA$$

$$= \int_0^{x_{cr}} \frac{0.664}{\sqrt{\frac{v_0 x}{\nu}}} \frac{\rho v_0^2}{2} B \cdot dx$$

$$F_{D2} = \int \tau_0 \cdot dA = \int_{x_{cr}}^L \frac{0.059}{\left[\frac{v_0 x}{\nu}\right]^{1/5}} \frac{\rho v_0^2}{2} B \cdot dx$$

$$\frac{x^{-1/5+1}}{\frac{\nu}{v_0} 2 \sqrt{x}}$$

$$\frac{v_0 L}{\nu}$$

$$\begin{aligned}
 C_{f \text{ avg}} &= \frac{0.664}{L \cdot \sqrt{\frac{\nu_0}{\nu}}} \int_0^{x_{cr}} \frac{1}{\sqrt{x}} dx + \frac{0.059}{L \left(\frac{\nu_0}{\nu}\right)^{1/5}} \int_{x_{cr}}^L \frac{1}{x^{1/5}} dx \\
 &= \frac{0.664}{L \sqrt{\frac{\nu_0}{\nu}}} \left[2 \sqrt{x_{cr}} \right] + \frac{0.059}{L \left(\frac{\nu_0}{\nu}\right)^{1/5}} \left[\frac{5}{4} L^{4/5} - \frac{5}{4} x_{cr}^{4/5} \right] \\
 &= \frac{0.664 \times 2 \sqrt{5 \times 10^5}}{L \left(\frac{\nu_0}{\nu}\right)} + \frac{0.059}{L \left(\frac{\nu_0}{\nu}\right)^{1/5}} \frac{5}{4} L^{4/5} - \frac{0.059}{L \left(\frac{\nu_0}{\nu}\right)^{1/5}} \frac{5}{4} \frac{(5 \times 10^5)^{4/5}}{\left(\frac{\nu_0}{\nu}\right)^{4/5}} \\
 &= \frac{939.03}{\frac{\nu_0 L}{\nu}} + \frac{0.0737}{\left(\frac{L \nu_0}{\nu}\right)^{1/5}} - \frac{2138.1}{\left(\frac{L \nu_0}{\nu}\right)} \\
 &= \frac{939.03}{Re_L} + \frac{0.0737}{Re_L^{1/5}} - \frac{2672.62}{Re_L} \\
 C_{f \text{ avg}} &= \frac{0.0737}{Re_L^{1/5}} - \frac{1733.59}{Re_L}
 \end{aligned}$$

Hence, avg drag coeff when partly laminar and turbulent boundary layer exist.

$$C_{f \text{ avg}} = \frac{0.074}{Re_L^{1/5}} - \frac{1740}{Re_L}$$

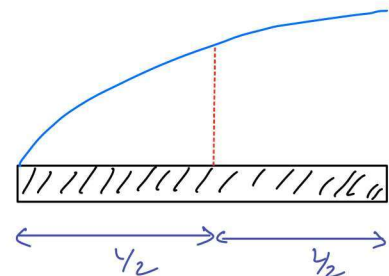
- 8) Find the ratio of drag on first half of the plate to the drag on second half of the plate in case of laminar boundary layer throughout the plate.

$$\text{Sol} \quad \frac{F_{D1}}{F_{D2}} = \frac{F_{D1}}{F_D - F_{D1}}$$

$$F_{D1} = \frac{1.328}{(Re_{x_1})^{1/2}} \frac{\rho \nu_0^2}{2} \cdot \frac{B L}{2}$$

$$\frac{1.328}{(Re_L)^{1/2}} \frac{\rho \nu_0^2}{2} B L - \frac{1.328}{(Re_{x_1})^{1/2}} \frac{\rho \nu_0^2}{2} B L$$

$$\begin{aligned}
 \frac{F_{D1}}{F_{D2}} &= \frac{\sqrt{\frac{\nu}{\nu_0} \cdot \frac{1}{2}}}{\sqrt{\frac{\nu}{\nu_0 L}} - \sqrt{\frac{\nu}{\nu_0 L/2}}} = \frac{\sqrt{\frac{2\nu}{\nu_0 L}}}{\sqrt{\frac{\nu}{\nu_0 L}} [1 - \sqrt{2}]} = \frac{\sqrt{\frac{L}{2}}}{\sqrt{L} - \sqrt{\frac{L}{2}}} \\
 &= 2.414
 \end{aligned}$$



- 8) Calculate frictional drag on a plate of 0.15m wide and 0.45m long placed longitudinally in the stream of oil having free stream velocity of 6m/sec. Also find the thickness of boundary layer and the shear stress at trailing edge.

$$\rho_{oil} = 0.975, \quad \nu = 0.9 \times 10^{-4} \text{ m}^2/\text{s}.$$

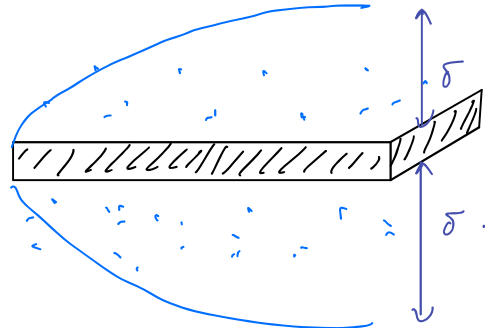
Sol (a) $U_0 = 6 \text{ m/s}$

$$Re_L = \frac{U_0 \cdot L}{\nu} = \frac{6 \times 0.45}{0.9 \times 10^{-4}} = 3 \times 10^4 < 5 \times 10^5$$

hence laminar throughout

$$F_D = 2 \left[\frac{\tau_{avg} \cdot \rho U_0^2}{(Re_L)^{1/2}} \right] BL$$

$$= 2 \left[\frac{1.328 \rho U_0^2}{\sqrt{Re_L}} \times BL \right] = 18.16 \text{ N}.$$



- (b) Thickness of boundary layer.

$$\tau_{0,L} = C_{fL} \frac{\rho U_0^2}{2} = \frac{0.664}{(Re_L)^{1/2}} \frac{\rho U_0^2}{2} = 67.27 \text{ (N/m}^2\text{)}$$

(c) $\frac{\delta}{x} = \frac{5}{(Re_x)^{1/2}}$

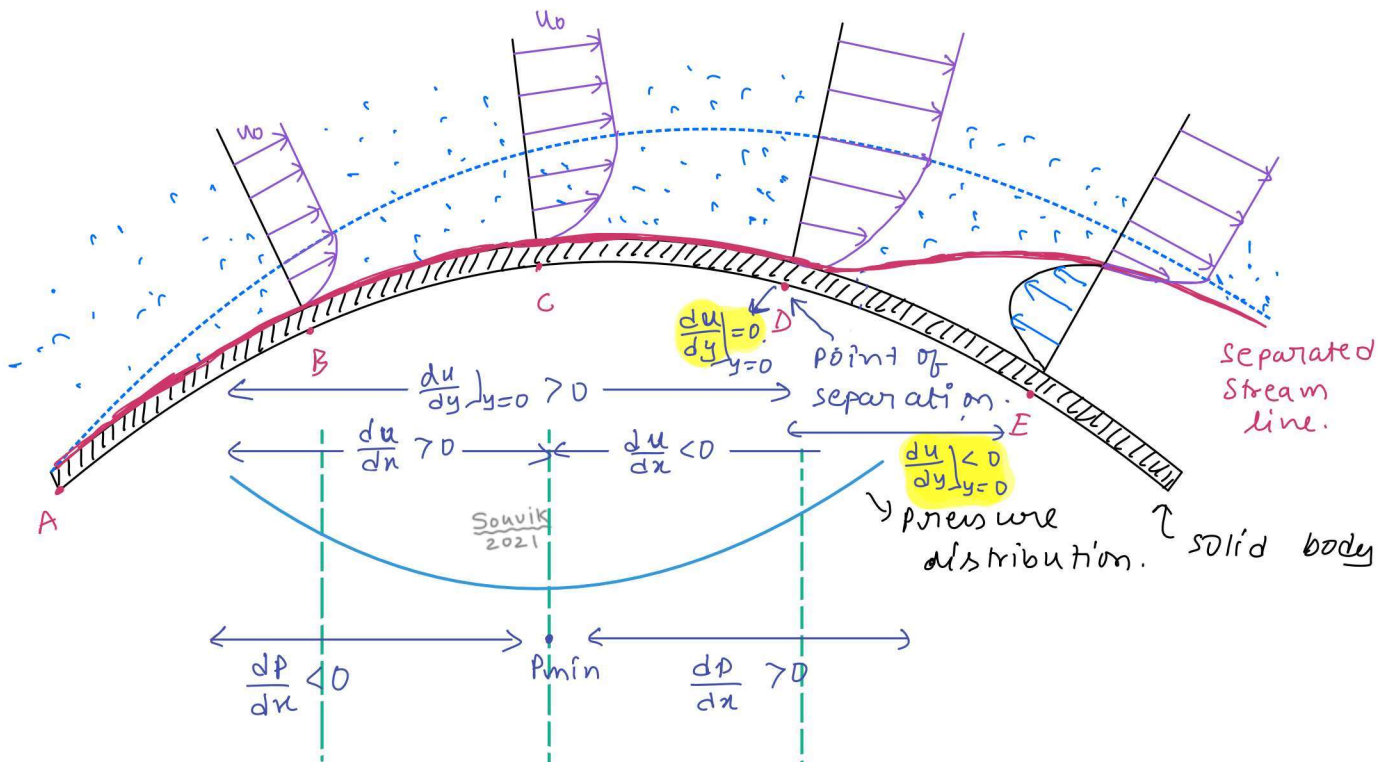
$$\delta = \frac{5L}{Re_L} = 13 \text{ mm}.$$

- 9) Show that if a plate of 10m x 5m is towed through a fluid such that the boundary layer is laminar throughout, the ratio of towing speed. so that the drag force remains constant regardless of whether 10m or 5m side is in flow direction is given by $\frac{U_{10}}{U_5} = 1.26$ and for similar condition

if the layer is turbulent entirely and $Re_L < 10^7$

$$\frac{U_{10}}{U_5} = 1.08.$$

BOUNDARY LAYER SEPARATION :-



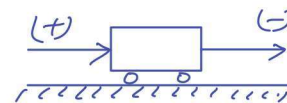
- The boundary layer is formed when fluid flows over the solid surface.
- The vel. in this thin layer of fluid varies from zero to free stream velocity in a direction normal to the surface.
- The fluid layer adjacent to the solid body has to overcome the surface friction at the expense of KE.
- This loss of KE can be recovered by momentum exchange (not possible in laminar B.L but takes place in turbulent boundary layer).
- Thus vel. of the fluid layer goes on decreasing
- At a certain point on solid body a stage may come when the boundary layer may not be able to keep adhering to the solid body. (As at this point KE is not sufficient to overcome the friction offered by the body.)
- This results in separation of B.L from the body surface. This process is called **B.L separation**.

- The point at which B.L is on verge of separation is termed as **POINT OF SEPARATION**.

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- if we consider the flow over curved surface ABCDE.

- In region ABC, Area of flow is reducing. velocity of flow. increase. hence pressure decreases in direction of flow ($\frac{dp}{dx} < 0$).
 - Along region CDE, area of flow increases, hence velocity of flow decrease hence pressure increases in the direction of flow. ($\frac{dp}{dx} > 0$).
 - As vel. of flow decreases in this region, at a point it reduces to such a value that it is not capable of overcoming the resistance offered by ^{Solid} surface.
 - At such pt "D" boundary layer separation takes place and velocity gradient is 0 - $\frac{dy}{dx} = 0$
 - Downstream of pt "D", the flow takes place in the reverse direction & velocity gradient becomes (-)ve ($\frac{dy}{dx} < 0$)
- Boundary layer separation is observed in pumps, FANS, diffusers, Turbine blades, Aerofoil, open channel, Transition etc.
 - BLS leads to increase in pressure drag, magnitude of which is much greater than frictional drag.
 - BLS can be controlled as follows:-
By
 - accelerating the fluid in boundary layer by injecting another same fluid.
 - By creating suction of fluid at the boundary layer.
 - By streamlining the body.



Q) For the following velocity profile determine whether the flow is separated or on verge of separation or remains attached to the surface.

$$(a) \quad \frac{u}{v_0} = \frac{3}{2} \left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^3$$

$$(b) \quad \frac{u}{v_0} = 2 \left(\frac{y}{\delta} \right)^2 - \left(\frac{y}{\delta} \right)^3$$

$$(c) \quad \frac{u}{v_0} = -2 \left(\frac{y}{\delta} \right) - \left(\frac{y}{\delta} \right)^2$$

Ans For flow to be separated $\frac{du}{dy} \Big|_{y=0} > 0$.

$$(a) \quad \frac{u}{v_0} = \frac{3}{2} \left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^3$$

$$\frac{1}{v_0} \frac{du}{dy} = \frac{3}{2\delta} \overset{\text{Sowvik 2021}}{\frac{3}{2}} \left(\frac{y}{\delta} \right)^2 \cdot \frac{1}{\delta}$$

$$\frac{du}{dy} \Big|_{y=0} = \frac{3v_0}{2\delta} > 0 \rightarrow \text{Hence flow is not separated.}$$

$$(b) \quad \frac{u}{v_0} = 2 \left(\frac{y}{\delta} \right)^2 - \left(\frac{y}{\delta} \right)^3$$

$$\frac{1}{v_0} \frac{du}{dy} = 4 \left(\frac{y}{\delta} \right) \cdot \frac{1}{\delta} - 3 \left(\frac{y}{\delta} \right)^2 \cdot \frac{1}{\delta}$$

$$\frac{du}{dy} \Big|_{y=0} = 0 \rightarrow \text{Flow is on verge of separation}$$

$$(c) \quad \frac{u}{v_0} = -2 \left(\frac{y}{\delta} \right) - \left(\frac{y}{\delta} \right)^2$$

$$\frac{du}{dy} \cdot \frac{1}{v_0} = -\frac{2}{\delta} - 2 \frac{y}{\delta} \cdot \frac{1}{\delta}$$

$$\frac{du}{dy} \Big|_{y=0} = -\frac{2v_0}{\delta} < 0 \text{ Hence flow is separated}$$

MODEL ANALYSIS

- For predicting the performance of hydraulic structure like dams, canals, spillways etc. or hydraulic machine like pumps, turbine before construction/manufacturing, model of these structure or machines are made & tests are performed on it to get desired information.

- models are usually of two types :-

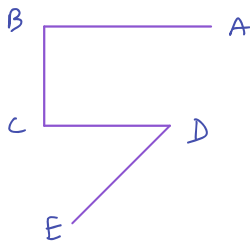
(a) GEOMETRICALLY SIMILAR MODEL

(b) DISTORTED MODEL - In case of harbour, rivers and estuaries.

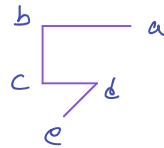
TYPES OF SIMILARITY :-

(a) GEOMETRICAL SIMILARITY :-

For this similarity to exist, the ratio of corresponding length dimension b/w the model and prototype must be same.



Prototype



Model

$$\frac{AB}{ab} = \frac{BC}{bc} = \frac{CD}{cd} = \frac{DE}{de}$$

Geometrical parameters are :- (i) length (ii) width (iii) height (iv) Area (v) volume.

$$\frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{H_m}{H_p} = L_r = \text{length ratio.}$$

NOTE:- Model can be bigger than prototype, depending on type and study.

$$\frac{A_m}{A_p} = L_r^2, \quad \frac{V_m}{V_p} = L_r^3$$

(b) KINEMATIC SIMILARITY :-

- For this similarity to exist ratio of all the kinematic parameter in the model and prototype must be same.
- Kinematic parameters include velocity, discharge, acceleration.

$$\text{vel. ratio} = \frac{V_m}{V_p} = V_r = \frac{L_r}{T_r}$$

$$\text{Accn ratio} = \frac{a_m}{a_p} = \frac{V_r}{T_r} = \frac{L_r}{T_r^2}$$

$$\text{Discharge ratio} = \frac{Q_m}{Q_p} = Q_r = \frac{(L_r)^3}{T_r}$$

- For kinematic similarity, geometric similarity must exist. Souvik 2021
But it doesn't mean that if geometric similarity exist, then kinematic will also exist.
→ (Hence, the above is necessary condition but not sufficient)

(c) DYNAMIC SIMILARITY :-

- For dynamic similarity to exist b/w model and prototype, identical type of force. (Elastic, pressure, viscous etc). must be similar and have same ratio at all corresponding point
- Dynamic similarity are force & power.

$$\text{Force ratio} = \frac{F_m}{F_p} = \rho_r L_r^3 a_r = \rho_r L_r^3 \frac{L_r}{T_r^2} = \frac{\rho_r L_r^4}{T_r^2}$$

$$\text{Power Ratio} = \frac{P_m}{P_p} = F_r \cdot V_r = \frac{\rho_r L_r^5}{T_r^3}$$

- For dynamic similarity to exist, both kinematic and geometric similarity must exist.

(It is necessary but not sufficient condition)

- For complete similarity geometrical, kinematic and dynamic similarity must exist.
- A model can be both larger and smaller than prototype.

DIFFERENT TYPES OF FORCES :-

(a) INERTIA FORCE :- It is the force that resists the change in state of matter.

- It is the product of mass and acceleration.

- It always exists in fluid flow.

$$F_i = m a = \rho L^3 \cdot \frac{v}{t} = \rho L^2 v^2 \quad v = \text{velocity}$$

$$F_i = \rho L^2 v^2$$

(b) Viscous force :- Force due to viscosity of fluid.

$$F_\mu = \tau A = \frac{\mu v}{y} \cdot L^2 = \mu v L$$

$$F_v = \mu v L$$

(c) Gravity force :- Force due to gravity.

$$F_g = mg = \rho L^3 g$$

$$F_g = \rho L^3 g$$

(d) Pressure force :- Due to pressure.

$$F_p = P \cdot A = P L^2$$

$$F_p = P \cdot L^2$$

(e) Surface tension force :- $F_\sigma = \sigma \cdot L$

(f) Elastic force :- $F_k = K A = K L^2$ $K = \text{Bulk Modulus}$

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DIMENSIONLESS NUMBER / NON-DIMENSIONAL NUMBER :-

(a) Reynolds number :- It signifies the dominance of viscous force over the inertial force.

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho L^2 v^2}{\mu v L} = \frac{\rho L v}{\mu}$$

$$Re = \frac{\rho L v}{\mu}$$

(b) Froude's Number :- It signifies the dominance of gravity force over the inertia force.

$$Fr = \sqrt{\frac{\text{Inertial } F}{\text{Gravity } F}} = \sqrt{\frac{\rho v^2 L^2}{\rho L^3 g}}$$

$$Fr = \frac{v}{\sqrt{gL}}$$

(c) Eulers Number :- It signifies the dominance of pressure force over the inertia force. It is defined as.

$$Eu = \sqrt{\frac{\text{Inertial } F}{\text{Pressure } F}} = \sqrt{\frac{\rho v^2 L^2}{p L^2}}$$

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$$Eu = \frac{v}{\sqrt{p/\rho}}$$

(d) Weber Number :- It signifies the dominance of surface tension force over inertia force.

- It is defined as, $We = \sqrt{\frac{\text{Inertial } F}{\text{Surface } F}} = \sqrt{\frac{\rho v^2 L^2}{\sigma L}}$

$$We = \frac{v}{\sqrt{\frac{\sigma}{\rho \cdot L}}}$$

(e) Mach Number :- It signifies the dominance of elastic force over inertia force.

$$M = \sqrt{\frac{\text{Inertial } F}{\text{Elastic } F}} = \sqrt{\frac{\rho v^2 L^2}{K L^2}}$$

$$M = \frac{v}{\sqrt{K/\rho}} = \frac{v}{c}$$

$c = \sqrt{\frac{K}{\rho}}$ is velocity of sound in medium.

- For dynamic similarity b/w model and prototype, ratio of dimensionless number (Re, M, We, Eu, Fr) must be same for model and prototype.

- practically it is not possible to satisfy all the force ratio, hence models are designed on basis of ratio of force which are more dominating amongst all.

- Hence different types of model laws are as follows :-

(a) REYNOLDS MODEL LAW :-

- Four cases where apart from inertial force on viscous force is dominating, Reynolds model law is used.

- According to this Reynolds number of prototype and model must be same.

$$\frac{\rho_m V_m L_m}{\mu_m} = Re_p$$

$$\left(\frac{\rho V L}{\mu} \right)_m = \left(\frac{\rho V L}{\mu} \right)_p$$

$$\frac{\rho_m V_m L_m}{\mu_m} = 1$$

- This law is used in following cases.

(a) Parachute where there is drag.

(b) Flow around submerged objects like submarines, air planes, Automobiles.

(c) Pipe flow.

- According to this law,

$$\frac{\rho_m V_m L_m}{\mu_m} = 1$$

$$V_m = \frac{\mu_m}{\rho_m L_m}$$

$$T_m = \frac{L_m}{V_m} = \frac{\rho_m L_m^2}{\mu_m}$$

$$\text{Accel. } a_m = \frac{V_m}{T_m} = \frac{\mu_m}{\rho_m L_m} \cdot \frac{\mu_m}{\rho_m L_m^2} = \frac{\mu_m^2}{\rho_m^2 L_m^3}$$

$$\text{Force ratio, } \frac{F_m}{F_p} = ma = \frac{\rho_m L_m^3 \mu_m^2}{\rho_m^2 L_m^3}$$

$$F_m = \frac{\mu_m^2}{\rho_m}$$

- Power ratio, $P_m = F_m \cdot v_m = \frac{\mu_m^2}{\rho_m} \cdot \frac{\mu_m}{\rho_m \cdot L_m}$

$$P_m = \frac{\mu_m^3}{\rho_m^2 L_m}$$

- Discharge ratio, $Q_m = \frac{L_m^3}{T_m} = \frac{L_m^3 \mu_m}{\rho_m L_m^2}$

$$Q_m = \frac{L_m \mu_m}{\rho_m} \quad \checkmark$$

(b) FROUDE'S MODEL LAW :-

This law is applicable when apart from inertial force gravity forces are dominating force.

According to this law,

$$F_{rm} = F_{rp}$$

$$\left(\frac{v}{\sqrt{gL}} \right)_m = \left(\frac{v}{\sqrt{gL}} \right)_p$$

$$\frac{v_m}{\sqrt{g_m L_m}} = 1$$

now, $\frac{g_m}{g_p} = 1$ as $g = \text{const.}$

$$\therefore \frac{v_m}{\sqrt{L_m}} = 1$$

- This law is applicable in following cases

(i) Free surface flow as weir, open channels, spillway etc.

(ii) Flow of liquid jet from orifice.

(iii) Flow over notches.

(iv) Flow of ship in turbulent water.

NOTE :- In case of ship, gravity as well as viscous force both are equally dominating. Hence Reynolds and Froude's law both must be used.

- According to this law

$$\frac{v_m}{\sqrt{L_m}} = 1, \quad v_m = \sqrt{L_m}$$

$$T_m = \frac{L_m}{v_m} = \frac{L_m}{\sqrt{L_m}} = \sqrt{L_m}$$

$$a_m = \frac{v_m}{T_m} = 1$$

$$F_m = m a_m = \rho_m L_m^3 \cdot 1 = \rho_m L_m^3$$

$$\text{Power, } P_m = F_m \cdot v_m = \rho_m L_m^3 \sqrt{L_m} = \rho_m \cdot L_m^{7/2}$$

$$g_m = \frac{L_m^3}{T_m} = \frac{L_m^3}{\sqrt{L_m}} = L_m^{5/2}$$

NOTE:- SHIP MODEL Souvik 2021

As ship is partially submerged body, hence drag experienced by it consists of

(a) The wave resistance due to gravity on free surface.

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(b) frictional / viscous resistance offered by water on surface of contact of ship with water.

hence Re and Fr model law should be satisfied.

$$Re_m = \frac{\rho_m v_m L_m}{\mu_m} = 1 \quad \text{and} \quad Fr_m = \frac{v_m}{\sqrt{L_m}} = 1.$$

from (i) & (ii)

$$\boxed{\frac{\rho_m L_m^{3/2}}{\mu_m} = 1.}$$

- Sometimes both the laws becomes difficult to be satisfied hence following analysis is performed.

$$\text{Total resistance } (R_t) = \text{Wave Resistance } (R_w) + \text{viscous Resist } (R_\mu)$$

$$R_t = R_w + R_\mu$$

$$R_\mu = \frac{1}{2} C_D \rho A v^2 \quad (\text{For both model \& proto type}).$$

$$R_w = R_t - R_{\mu}$$

$$\text{now, } R_{wm} = R_{tm} - R_{\mu m}$$

now from Froude's law,

$$\text{force ratio, } F_{or} = R_{wr} = \rho_r L_r^3$$

$$\frac{R_{wm}}{R_{wp}} = \rho_r L_r^3$$

From above relation, wave resistance of prototype can be computed as.

$$R_{wp} = \frac{R_{wm}}{\rho_r L_r^3}$$

$$\text{Now, } R_{tp} = R_{wp} + R_{\mu m}$$

- 8) A 1:20 model of a naval ship having a submerged area of 5 m^2 & length of 8 m have a total drag of 20 N when towed through water at a vel of 1.5 m/sec . Compute the total drag on the prototype when moving at corresponding speed. Use $F_f = \frac{1}{2} C_f \rho A V^2$ for computing the skin resistance and $C_f = \frac{0.0735}{Re^{1/5}}$ $\gamma_{\text{water}} = 0.01 \text{ Stokes}$.

$$\text{sol } Re_m = \frac{V_m L_m}{\gamma_m} = \frac{1.5 \times 8}{0.01 \times 10^{-4}} = 12 \times 10^6 \quad (7.5 \times 10^5) \text{ Turbulent.}$$

$$C_f = \frac{0.0735}{(Re)^{1/5}} = \frac{0.0735}{(12 \times 10^6)^{1/5}} = 2.82 \times 10^{-3}$$

$$F_{f,m} = \frac{1}{2} C_{fm} \rho_m A_m V_m^2 = \frac{1}{2} \times 2.82 \times 10^{-3} \times 10^3 \times 5 \times (1.5)^2$$

$$F_{fm} = R_{\mu m} = 15.86 \text{ N}$$

$$R_{tm} = R_{\mu m} + R_{wm}$$

$$R_{wm} = 20 - 15.86 = 4.14 \text{ (N)}$$

$$\text{Now, from Froude's law, } R_{wr} = \rho_r L_r^3$$

$$\frac{R_{wm}}{R_{wp}} = 1 \times \left(\frac{1}{20}\right)^3 = \frac{1}{8 \times 10^3}$$

$$R_{wp} = R_{wm} \times 8 \times 10^3 = 4.14 \times 20^3 = 33120 \text{ N}$$

$$R_{up} = \frac{1}{2} C_{fp} (\rho_p A_p v_p^2)$$

Now, $v_m = \sqrt{L_m} = \frac{v_m}{v_p} = \sqrt{\frac{1}{20}} \Rightarrow v_p = 1.5 \sqrt{20} = 6.7 \text{ m/s}$

$$A_m = L_m^2 \Rightarrow \frac{A_m}{A_p} = \left(\frac{1}{20}\right)^2 \Rightarrow A_p = 5 \times 20^2 = 2000 \text{ m}^2$$

$$Re_p = \frac{v_p L_p}{\nu_p} = \frac{6.7 \times 8 \times 20}{0.01 \times 10^{-4}} =$$

$$C_{fp} = \frac{0.0735}{(10.72 \times 10^8)^{1/5}} = 1.148 \times 10^{-3}$$

$$R_{up} = \frac{1}{2} \times 1.148 \times 10^{-3} \times 10^3 \times 2000 \times 6.7^2 = 57533.73$$

$$R_{tp} = R_{wp} + R_{up} = 33120 + 57533.73$$

$$R_{tp} = 84653.73 \text{ N}$$

Sep 27 (c) EULER'S MODEL LAW :-

- This law is applicable when pressure force is dominating apart from inertial force.
- According to this law, $\epsilon_{um} = \epsilon_{up}$

$$\left(\frac{v}{\sqrt{\rho/p}}\right)_m = \left(\frac{v}{\sqrt{\rho/p}}\right)_p$$

$$\frac{v_m}{\sqrt{\rho_m/p_m}} = 1$$

- This law can be used in following cases.

- in pipe flow at high pressure. (in which viscous effect are negligible as in turbulent flow).
- in case of cavitation.
- pressure due to sudden closure of valve.

(d) WEBER MODEL LAW :-

- This law is used when surface tension force is dominating apart from inertial force.
- According to this law,

$$We_m = We_p.$$

$$\left(\frac{v}{\sqrt{\sigma/\rho_L}} \right)_m = \left(\frac{v}{\sqrt{\sigma/\rho_L}} \right)_p.$$

$$\frac{v_m}{\sqrt{\frac{\sigma_m}{\rho_m L_m}}} = 1$$

- It can be used ^{Solved 2021} in cases as:-

- (i) capillary rise in narrow passage.
- (ii) Flow of blood in veins & arteries.
- (iii) Flow over weir for small head.
- (iv) Rising bubble in fluid.
- (v) seepage flow through soil.

(e) MACH MODEL LAW :-

- When a part from inertial force, compressibility force is dominating this law is applicable.
- According to this law, $M_m = M_p$.

$$\left(\frac{v}{\sqrt{K \cdot P}} \right)_m = \left(\frac{v}{\sqrt{K \cdot P}} \right)_p.$$

$$\frac{v_m}{\sqrt{K_m P_m}} = 1$$

- This law is applicable in

- (i) water hammer pressure.
- (ii) Under water testing of torpedoes.
- (iii) Aerodynamic testing of missiles, rockets.
- (iv) Flow of gases through pipe at high velocity.

NOTE:- compressibility force are pre-dominating when mach number ≥ 0.3
i.e. $\frac{\text{velocity of movement}}{\text{vel. of sound}} \geq 0.3.$

Now, vel. of sound = 340 m/s.

If $M < 1$, $\rightarrow$ flow is termed as subsonic.
 $M > 1$, $\rightarrow$ flow is " " " " supersonic.
 $M = 1$, $\rightarrow$ " " " " Sonic
 $M \gg 1$, $\rightarrow$ " " " " Hyper sonic.

Q) In a wind tunnel, a truck of the model is tested at $L_m = l_b = \frac{L_p}{L_m}$. Analysis of the model is to be done for speed of vehicles at 25-8 m/s (96.5 km/hr). The density and viscosity of air in the tunnel is same as that of air in which truck is to move. Find the speed of air in the tunnel required for testing. $\rightarrow$ Mach number.

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Sol using Reynold's model law,

$$\frac{\rho_p v_p L_p}{\mu_p} = 1.$$

$$\frac{\rho_p v_p L_p}{\mu_p} = \frac{\rho_m v_m L_m}{\mu_m}$$

$$v_p = 25.8 \text{ m/s.}$$

$$v_m = v_p \times \frac{L_p}{L_m} = 25.8 \times 16 = 429 \text{ m/s.}$$

$$\text{check, } M = \frac{v}{c} = \frac{429}{340} = 1.26 \text{ (70.3)}$$

compressible forces will be dominating,
and the flow is supersonic.

Now, we need to incorporate changes in our model during testing to reduce the effect of compressibility.

$$v_m = \left(\frac{\rho_p}{\rho_m} \right) \left(\frac{\mu_m}{\mu_p} \right) \left(\frac{L_p}{L_m} \right) v_p.$$

Different fluid can be used with higher density eg. water to reduce v_m

Test can be performed at lower temperature to reduce viscosity

wind tunnel can be pressurized. ($\rho_m \uparrow$).

Scale of model can be changed.

Test can be performed at full scale.
($L_m \uparrow = L_p$).

(P) RIVER MODEL LAW :-

- In river models, distorted models are used
- A model is said to be distorted, if it is not geometrically similar.
- For a distorted model different scale ratios for horizontal and vertical directions are used.
- In this if undistorted model is used, then dept of model will be small hence:-
 - (i) Flow may not remain turbulent as in original case.
 - (ii) Difficulty in measurement.
- Here length ratio ^{Souvik 2021} Horizontal direction

$$L_{NH} = \text{scale ratio in horizontal direction} \\ = \frac{L_P}{L_m} = \frac{B_P}{B_m}$$

$$L_{NV} = \text{scale ratio in vertical direction.} \\ = \frac{h_P}{h_m}$$

Also, vel. ratio, $V_r = \sqrt{L_{NV}} = \sqrt{\frac{h_P}{h_m}}$ (Froude's law).

$$\text{Area ratio, } A_r = \frac{B_P \cdot h_P}{B_m \cdot h_m} = L_{NH} \cdot L_{NV}$$

$$Q_r = A_r \cdot V_r = L_{NH} L_{NV} \sqrt{L_{NV}} \\ = L_{NH} \cdot (L_{NV})^{3/2}$$

- Q) The performance of spillway of power project is to be studied by means of model constructed to a scale of 1:9. Neglecting the viscous and surface tension effect. Find.

(a) Rate of flow in model for a prototype discharge of 1000 m³/s.

(b) The dissipation of energy in the prototype hydraulic jump, if the jump in the model studied is 0.4 MP.

Sol

Using Froude's law.

$$F_{rm} = F_{rp}$$

$$V_m = \sqrt{L_m}$$

$$(a) \quad Q_R = A_m \cdot V_m = L_m^{5/2} = \left(\frac{1}{9}\right)^{5/2}$$

$$\frac{Q_m}{Q_p} = \left(\frac{1}{9}\right)^{5/2}, \quad Q_m = 1000 \times \left(\frac{1}{9}\right)^{5/2}$$
$$Q_m = 4.11 \text{ m}^3/\text{s}.$$

$$(b) \quad \text{Power, } P_m = F_m \cdot V_m = \rho_m \cdot L_m^2 \cdot V_m^2 \cdot V_m$$

$$P_m = \rho_m L_m^2 V_m^3 = \rho_m L_m^{7/2} \quad [P_m = 1]$$

$$P_m = L_m^{7/2}$$

$$\frac{P_m}{P_p} = \left(\frac{1}{9}\right)^{7/2} \Rightarrow$$

$$P_p = 9^{7/2} \times 0.4 = 874.8 \text{ HP}.$$

(c) ROTDYNAMIC LAW :-

- This law is applicable for rotodynamic machines.
- For rotodynamic machines

$$\frac{\overset{\text{power}}{P}}{\rho N^3 D^5} = f \left[\frac{Q}{N D^3}, \frac{g H}{N^2 D^2} \right]$$

As per this law,

$$\left(\frac{Q}{N D^3} \right)_m = \left(\frac{Q}{N D^3} \right)_p$$

$$\frac{Q_m}{N_m D_m^3} = 1.$$

$$\left(\frac{g H}{N^2 D^2} \right)_m = \left(\frac{g H}{N^2 D^2} \right)_p$$

$$\frac{H_m}{N_m^2 D_m^2} = 1.$$

$$\text{and, } \left(\frac{P}{\rho N^3 D^5} \right)_m = \left(\frac{P}{\rho N^3 D^5} \right)_p$$

$$\frac{P_m}{\rho_m N_m^3 D_m^5} = 1.$$

- Q) Two geometrically similar pump are running at same speed of 10 rpm and lifting against a head of 25m and 10m respectively. First pump is having an impeller dia of 300mm. Find the impeller dia of 2nd pump.

Sol

$$\frac{H_p}{N_p D_p^3} = 1$$

$$\frac{H_m}{N_m D_m^2} = \frac{H_p}{N_p D_p^2}$$

$$\frac{25}{(0.3)^2} = \frac{10}{D_p^2}$$

$$D_p = 189.73 \text{ mm}$$

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- Q) The light weight ^{Souvik} parachute has diameter 7m and total weight including pay load = 1020N. The design terminal settling velocity (V_t) of parachute is 5.5 m/s. A one-tenth scale model of the parachute is tested in wind tunnel with same air in which parachute is moving.

$$\mu_{\text{air}} = 1.802 \times 10^{-5} \text{ kg/m-sec}, \quad \rho_{\text{air}} = 1.225 \text{ kg/m}^3.$$

- Compute the drag coeff of the prototype.
- At what speed wind tunnel will run for dynamic similarity.
- Aerodynamic drag on model.

Sol (a) $F_D = \left(\frac{1}{2} C_d \rho A V^2 \right)_p$

$$1020 = \frac{1}{2} C_d \times 1.225 \times \frac{\pi}{4} \times 7^2 \times 5.5^2$$

$$C_d = 1.43.$$

- (b) As per Reynold's model law.

$$\frac{\rho V_m V_p}{\mu_m} = 1.$$

$$V_m = \frac{1}{V_p}$$

$$\frac{V_m}{V_p} = \frac{1}{V_p} \Rightarrow V_m = \frac{5.5}{1/10} = 55 \text{ m/s}.$$

(c) Aerodynamic drag-

$$F_D = \left(\frac{1}{2} \rho A v^2 \right)_m$$

Now, $F_{Dm} = \rho_m A_m v_m^2 = \rho_m \cdot \frac{1}{\rho_p^2} \quad (\rho_m = 1)$

$$F_{Dm} = 1.$$

$$\therefore F_{Dm} = F_{Dp} = 1020 \text{ N}.$$

Q) Flow of glycerin of $\nu = 5 \times 10^{-4} \text{ m}^2/\text{sec}$ in a open channel is to be modelled in a laboratory using water of $\nu = 10^{-6} \text{ m}^2/\text{sec}$. If both gravity and viscous force are important find the length scale.

Sol As per Reynolds model law. (viscous).
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$$\frac{\rho_m v_m L_m}{\mu_m} = 1$$

As per Froude's model law (gravity).

$$v_m = \sqrt{L_m}$$

From both, $\frac{\rho_m L_m^{3/2}}{\mu_m} = 1.$ ESENOTES.COM

$$L_m^{3/2} = \mu_m$$

$$L_m = (\mu_m)^{2/3}$$

$$L_m = \left(\frac{\nu_m}{\nu_p} \right)^{2/3} = \left(\frac{10^{-6}}{5 \times 10^{-4}} \right)^{2/3}$$

$$L_m = 1/63.$$

Q) In a model of highway bridge constructed to a scale of 1 in 25 the force of water in the pier was measured as 5 N, what is the force on the prototype pier.

Sol $F_m = \rho_m \cdot L_m^3 \quad \rho_m = 1$

$$\frac{F_m}{F_p} = 1 \cdot \left(\frac{1}{25} \right)^3$$

$$F_p = 25^3 \cdot F_m = 5 \times 25^3 = 78.125 \text{ (N)}$$

8) A river stretch 10 km long and 850 m wide is to be modelled for flood studies in Hydraulics laboratory where the available flume size is 2 m width, 20 m length and 70 cm height. It is decided to use a depth scale of 10. If the peak flood in river is $3000 \text{ m}^3/\text{s}$ at a depth of 6 m. Find the suitable horizontal scale to be adopted for the construction of the model.

- Max available flow is 150 l/s in lab. The horizontal scale should be in the multiple of 10 and such that the model is as big as possible.
- Determine the model dimension, depth and discharge.

Assume the ^{Souvik}hydraulic depth is same as depth of flow in both model and prototype

$$\text{Sol} \quad Q_p = \frac{Q_p}{Q_m} = \frac{A_p V_p}{A_m V_m} = \frac{B_p \cdot H_p \cdot H_p^{1/2}}{B_m M_m M_m^{1/2}} \quad [v = \sqrt{L} = \sqrt{H}]$$

$$\frac{3000}{130 \times 10^{-3}} = L_{xH} \cdot L_{xv}^{3/2}$$

$$2 \times 10^4 = L_{xH} (10)^{3/2}$$

$$L_{xH} = 634.4 \approx 640$$

$$L_{xH} = \frac{B_p}{B_m} \Rightarrow B_m = \frac{B_p}{L_{xH}} = \frac{850}{640}$$

$$B_m = 1.32 \text{ m} < 2 \text{ m} \quad (\text{OK}).$$

$$L_{xH} = \frac{L_p}{L_m} \Rightarrow L_m = \frac{L_p}{L_{xH}} = \frac{10000}{640}$$

$$L_m = 15.625 \text{ m} < 20 \text{ m} \quad (\text{OK}).$$

$$L_{xv} = \frac{H_p}{H_m}, \quad H_m = \frac{H_p}{L_{xv}} = \frac{6 \times 10^2}{10}$$

$$H_m = 60 \text{ cm} (< 70 \text{ cm}) \quad (\text{OK}).$$

$$\text{Discharge, ratio} \quad Q_p = \frac{Q_p}{Q_m} = L_{xH} \cdot L_{xv}^{3/2}$$

$$Q_m = \frac{Q_p}{L_{xH} \cdot L_{xv}^{3/2}} = \frac{3000}{640 \times 10^{3/2}} = 148.23 \text{ l/sec}$$

DIMENSIONAL ANALYSIS

- D.A is a mathematical technique of solving engineering problems. It makes use of the dimensions of the variable on which the problem depends.
- The various physical quantities can be expressed in terms of fundamental quantities.
- Fundamental quantities are (a) Mass (M)
(b) Length (L)
(c) Time (T)
(d) Temp (θ)
- The quantities which are expressed in terms of this fundamental quantities is termed as derived / secondary quantities eg. Area, velocity, acceleration etc.
- The common system of dimensioning a physical quantity is mass, length and time.

NOTE:- DIMENSIONAL HOMOGENEITY in an equation exist when it contains identical powers of each dimension.

For eg:- $h_f = \frac{f L v^2}{2 g d}$

$$[L] = \frac{[L] [L]^2 [T]^{-2}}{[L] [T]^{-2} [L]}$$

$$[L] = [L] \quad \{ \text{Dimensionally equal} \}.$$

- Dimensional Analysis can be done by following mtd.

(A) RAYLEIGH'S Mtd.

- It gives special form of relationship among the dimensionless group.
- It doesnot provide any information regarding the no. of dimensionless group to be obtained.
- It is used for determining the expression for a variable which depends upon 3 or 4 variables.

- In this analysis is done as follows:-

- (i) List all the independent variable likely to influence the value of dependent variable.
- (ii) write the functional relationship b/w dependent and independent variable.

$$y = f(x_1, x_2, \dots, x_n)$$

$$y = K (x_1^a x_2^b x_3^c \dots)$$

K = dimensionless coeff.

- (iii) Express both dependent and independent variables in terms of ^{Solved 2021} fundamental variables.
- (iv) Apply principle of dimensional homogeneity to find the values of const. a, b, c .

Q) Find an exp. for drag force on a smooth sphere of diameter " D " moving with uniform velocity " v " in the fluid of density " ρ " & dynamic visc. " μ ".

sol

$$F_D = f(D, v, \rho, \mu)$$
$$F_D = K [D^a, v^b, \rho^c, \mu^d]$$
$$[M L T^{-2}] = K [L^a] [L T^{-1}]^b [M L^{-3}]^c [M L^{-1} T^{-1}]^d$$

$$1 = c + d \quad - (i)$$

$$1 = a + b - 3(-d) \quad - (ii)$$

$$-2 = -b - d \quad - (iii)$$

cannot be solved,

Since μ is the most significant parameter in analysis of drag force a, b, c is expressed in terms of d .

$$c = 1 - d, \quad b = 2 - d$$

$$a + 2 - d - 3 + 3d - d = 1$$

$$a + d = 2$$

$$a = 2 - d$$

$$F_D = K [D]^{2-d} [v]^{2-d} [\rho]^{-d} [\mu]^d.$$

$$F_D = K D^2 v^2 \rho \left(\frac{\mu}{\rho v D} \right)^d.$$

$$F_D = K D^2 v^2 \rho \left(\frac{\mu}{\rho v D} \right)^d \sim \frac{1}{Re}.$$

- 8) The resistance force R of a supersonic plane during flight depends upon the length of aircraft L , velocity v , air viscosity μ , air density ρ and bulk modulus of air K . Determine the functional relationship b/w these variable & the resisting force.

Sol

$$R = f(L, v, \mu, \rho, K)$$

$$R = K [L^a v^b \mu^c \rho^d K^e]$$

$$[MLT^{-2}] = K [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c [ML^{-3}]^d [ML^{-1}T^{-2}]^e$$

$$1 = c + d + e \quad \text{--- (i)}$$

$$1 = a + b - c - 3d - e \quad \text{--- (ii)}$$

$$-2 = -b - c - 2e \quad \text{--- (iii)}$$

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Since, μ and K are the most significant parameters for supersonic flow: a, b, d is expressed in terms of c and e .

$$d = 1 - c - e$$

$$b = 2 - c - 2e$$

$$1 = a + 2 - c - 2e - c - 3(1 - c - e) - e$$

$$1 = a + 2 - c - 2e - c - 3 + 3c + 3e - e$$

$$a = 2 - c$$

$$R = K [L]^{2-c} [v]^{2-c-2e} [\mu]^c [\rho]^{1-c-e} K^e.$$

$$= K [L^2 v^2 \rho] \left(\frac{\mu}{L v \rho} \right)^c \left(\frac{K}{v^2 \rho} \right)^e$$

$$R = K \rho L^2 v^2 \left[\left(\frac{\mu}{\rho v L} \right) \left(\frac{K}{v^2 \rho} \right) \right]$$

- Q) A circular cylinder of given length / diameter ratio is kept in steady rotation at N rev/sec in a uniform stream of fluid of vel. v . Assuming that the power required to maintain the motion depends on density ρ , kinematic vis. ν , dia. D . Show that

$$P = \frac{\rho v^3}{D} f \left[\frac{vD}{\nu}, \frac{ND^2}{\nu} \right]$$

Sol

$$P = f [\rho, v, \nu, D, N]$$

$$P = f [\rho^a, v^b, \nu^c, D^d, N^e]$$

$$\frac{M}{P} = \frac{M L^{-1} T^{-1}}{M L^{-3}} = L^2 T^{-1}$$

$$[M L^2 T^{-3}] = K [M L^{-3}]^a [L T^{-1}]^b [L^2 T^{-1}]^c [L]^d [T^{-1}]^e$$

$$1 = a$$

$$2 = -3a + b + 2c + d$$

$$-3 = -b - c - e$$

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Since, fluid vel v , and rotational speed, N are significant parameters. Hence expt b, c, d in terms of a and e . (also can take ν)

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$$2 = -3 + b + 2c + d \quad \left| \quad c = 3 - b - e \right.$$

$$2 = -3 + b + b - 2b - 2e + d$$

$$-1 = -b - 2e + d$$

$$d = -1 + b + 2e$$

$$P = K [\rho]^1 [v]^b [\nu]^{3-b-e} [D]^{-1+b+2e} [N]^e$$

$$P = K \frac{\rho \cdot v^3}{D} \left[\left(\frac{vD}{\nu} \right)^b, \left(\frac{ND^2}{\nu} \right)^e \right]$$

$$P = \left(\frac{\rho v^3}{D} \right) \left[f \left(\frac{vD}{\nu} \right) \left(\frac{ND^2}{\nu} \right)^e \right]$$

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(B) BUCKINGHAM'S π Method :-

- The Rayleigh's method of analysis becomes tedious when no. of variables becomes more (>4).
- In such case Buckingham π -method has the advantage that the number of dimensionless group called as $\pi(P_i)$ to be formed are known in advance.
- According to this mtd, if there are " n " number of variables (both dependent and independent) in a dimensionally homogenous eq and these variable contains " m " fundamental dimensions then the number of dimensionless group that can be formed are " $n-m$ " termed as π -terms or groups

$$\text{Sovik 2021 } F_D = f \left[\underbrace{L, v, \rho, \tau}_{\substack{\text{Dep} \rightarrow (1) \quad \text{IN} \rightarrow (4)}} \right]$$

3 - fundamental.

$$\therefore \pi\text{-group} = 5 - 3 = 2.$$

- For eg. consider a phenomenon involving variables

x_i , $i=1, 2, \dots, n$, such that x_1 depends upon independent variable $x_2, x_3, \dots, x_n$, then the general functional relationship can be expressed as

$$x_1 = f(x_2, x_3, \dots, x_n)$$

$$x_1 = K(x_2, x_3, \dots, x_n).$$

$$K(x_1, x_2, x_3, \dots, x_n) = 0$$

$$K(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0.$$

- Here each of π -term may contain max of (mtl) variable, only one of which is required to be changed from term to term.
- These m variable which appears repeatedly in each of π term are called as repeating variables.

NOTE:- For deciding repeating variables following guidelines must be adopted.

- i) Select the first repeating variable from those describing the **geometry** of flow i.e. size & shape. for eg. diameter, length, width, height etc.
 - ii) Select second repeating variable from those representing the **fluid property** for eg. density, viscosity, surface tension, vapour pressure, compress. etc.
 - iii) Select the third repeating variable from those signifying the **fluid motion** such as velocity, acceln, pressure, force, power, discharge etc.
- The repeating variables must not form the non dimensionless parameter amongst themselves.
 - dependent variable must not be selected as repeating variable.

Q) A partially submerged body is stored in water, the resistance R to its motions depends on the density ρ , viscosity ν , length L of the body, velocity of body v and acceln due to gravity. Find the relationship b/w these parameters using Buckingham π -method.

Sol

$$R = f[\rho, \nu, L, v, g]$$

$$K[R, \rho, \nu, L, v, g] = 0.$$

here, Total no. of variable, $n = 6$.

Fundamental variable, $m = 3$

$$\text{No. of } \pi\text{-terms} = n - 3 = 6 - 3 = 3$$

every π -term will be $3 + 1 = 4$
 $\begin{matrix} \uparrow & \uparrow \\ R \cdot \nu & \rightarrow M^2 L^{-1} \end{matrix}$

No. of repeating variable in each π -term $= m = 3$

1st repeating variable = Length (L)
 2nd repeating variable = density, ν uses ρ . (take any).
 3rd repeating variable = velocity, $g = v$.

Now, π -term solution, non repeating left = R, v, g

$$\pi_1 = L^a \cdot p^b \cdot v^c \cdot R$$

$$M^0 L^0 T^0 = [L]^a [ML^{-3}]^b [LT^{-1}]^c [MLT^{-2}]$$

$$b+1=0, \quad b=-1$$

$$0 = a - 3b + c + 1, \quad a + c = -4$$

$$0 = \begin{array}{l} -c - 2 \\ c = -2 \end{array} \quad \left| \quad \begin{array}{l} a - 2 = -4 \\ a = -2 \end{array} \right.$$

$$\pi_1 = L^{-2} p^{-1} v^{-2} R$$

$$\pi_1 = \frac{R}{L^2 p v^2}$$

Now, $\pi_2 = L^a p^b v^c \cdot \gamma$ [change non-repeating variable]

$$M^0 L^0 T^0 = [L]^a [ML^{-3}]^b [LT^{-1}]^c [ML^{-1}T^{-1}]$$

$$b = -1, \quad \begin{array}{l} 0 = a - 3b + c - 1 \\ a + c = -2 \end{array}$$

$$0 = \begin{array}{l} -c - 1 \\ c = -1 \end{array} \quad a = -1$$

$$\pi_2 = \frac{\gamma}{L p v}$$

Now, $\pi_3 = L^a p^b v^c \cdot g$

$$M^0 L^0 T^0 = [L]^a [ML^{-3}]^b [LT^{-1}]^c [LT^{-2}]$$

$$0 = b, \quad \Rightarrow b = 0$$

$$0 = -3b + c + a + 1 \\ a + c = -1$$

$$0 = \begin{array}{l} -c - 2 \\ c = -2 \end{array} \quad \left| \quad a = 1 \right.$$

$$\pi_3 = \frac{g \cdot L}{v^2}$$

$$K [\pi_1, \pi_2, \pi_3] = 0$$

$$K \left[\frac{R}{\rho L^2 v^2}, \frac{\mu}{\rho v}, \frac{\gamma}{v^2} \right] = 0.$$

$$\frac{R}{\rho L^2 v^2} = K \left[\frac{\rho v}{\mu}, \frac{v^2}{\gamma} \right]$$

$$R = \rho L^2 v^2 K_1 [Re, Fr].$$

- Q) In a compressor frictional torque is "T" in the impeller. If the diameter is D, speed N, fluid viscosity μ and density ρ , prove using π -method theorem that

$$T = \frac{\rho N^2 D^5}{\mu} f \left[\frac{\mu}{\rho N D^2} \right]$$

sol $T = f [D, N, \mu, \rho]$

$$n = 5, m = 3, \text{ no. of } \pi \text{ terms} = n - m = 5 - 3 = 2.$$

$$\pi_1 = D^a \cdot \mu^b N^c \cdot \rho$$

$$M^0 L^0 T^0 = [L]^a [ML^{-1}T^{-1}]^b [T^{-1}]^c$$

$$0 = b + 1 \Rightarrow b = -1$$

$$0 = a - b - 3 \Rightarrow a = 2$$

$$0 = -b - c \Rightarrow c = -b = 1$$

$$\pi_1 = \frac{D^2 \rho N}{\mu}$$

$$\pi_2 = D^a, \mu^b N^c, T$$

$$M^0 L^0 T^0 = L^a [ML^{-1}T^{-1}]^b [T^{-1}]^c [ML^2T^{-2}]$$

$$b = -1, \quad a - b + 2 = 0 \\ a + 1 + 2 = 0, \quad a = -3$$

$$0 = -b - c - 2 \\ c = -1$$

$$\pi_2 = \frac{T}{D^3 \mu N}$$

$$K(\pi_1, \pi_2) = 0.$$

$$K\left(\frac{D^2 \rho N}{\mu}, \frac{T}{D^3 \mu N}\right) = 0$$

$$T = D^3 \mu N K_1\left(\frac{\mu}{D^2 \rho N}\right)$$

$$\frac{T}{D^3 \mu N} = K_3\left(\frac{\mu}{D^2 \rho N}\right)$$

$$T = D^3 \mu N K_3\left(\frac{\mu}{D^2 \rho N}\right)$$

$$v = \omega r$$

$$v = \frac{2\pi N}{60} r$$

$$v = \frac{2\pi N d}{120}$$

$$v = N D$$

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FLOW THROUGH PIPES

- Most of the pipe flow problems are turbulent in nature.
- Flow in pipe is turbulent when $Re > 4000$
- when water flows through pipe loss in it head takes place. which is due to following

(a) Major loss

(b) Minor loss

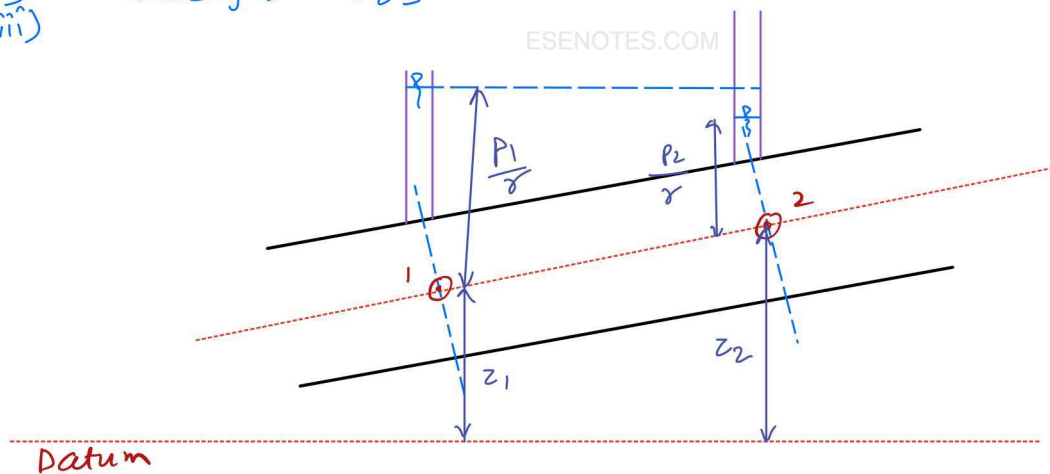
→ due to friction (80-90)% of total loss

→ due to pipe fittings (10-20)% of total loss.

MAJOR LOSS :-

- major loss in pipe is due to friction.
- This loss is given by

- (i) Darcy - weisbach eqn.
- (ii) Chezy's eqn
- (iii)



Using Energy eqn at section ① & ②

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + z_2 + \frac{V_2^2}{2g} + h_L$$

$$\left(\frac{P_1}{\rho} + z_1\right) - \left(\frac{P_2}{\rho} + z_2\right) = h_L$$

As per Darcy weisbach eqn; $h_L = \frac{f L V^2}{2g D}$

f = friction factor,
 f' = coeff of friction.

$$h_L = \frac{4 f' L V^2}{2g D}$$

$$f = 4f'$$

$$\text{Also, } h_L = \frac{f L Q^2}{2g D A^2} = \frac{f L Q^2}{2g D \left(\frac{\pi}{4} D^2\right)^2} = \frac{16 f}{2g \pi^2} \frac{Q^2 L}{D^5}$$

$$h_L = \frac{f L Q^2}{12.1 D^5} \quad \text{--- (A)}$$

For laminar flow, $f = \frac{64}{Re}$

↳ Turbulent flow, $f = \frac{0.316}{Re^{1/4}}$ $4000 < Re < 10^5$
in smooth pipe.

& $f = 0.0032 + \frac{0.221}{(Re)^{0.237}}$ $10^5 < Re < 4 \times 10^7$

For turbulent flow in:
Rough pipe: $\frac{1}{\sqrt{f}} = 2 \log_{10} \left(\frac{R}{K} \right) + 1.74$

Note:- In general for metallic pipe $f = (0.02 - 0.04)$

As per chezy's eqn $v = c \sqrt{RS}$

$$v = c \sqrt{\frac{A}{P} \cdot \frac{h_f}{L}}$$

$$v = c \sqrt{\frac{D}{4} \cdot \frac{h_f}{L}}$$

$$\frac{v^2}{c^2} = \frac{D}{4} \cdot \frac{h_f}{L}$$

$$h_f = \frac{4 L v^2}{c^2 D} \quad \text{--- (B)}$$

From (A) & (B)

$$\frac{f L v^2}{2g D} = \frac{4 L v^2}{c^2 D}$$

$$c = \sqrt{\frac{8g}{f}}$$

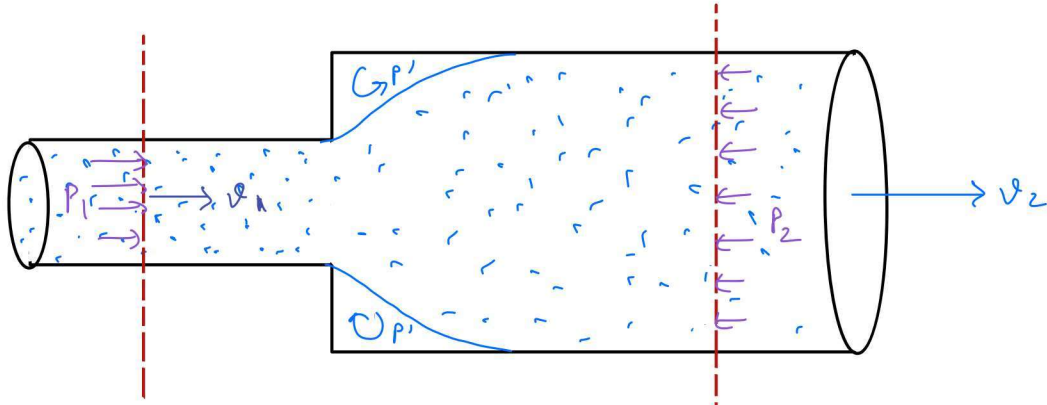
f = friction factor.

Ques

MINOR LOSS

- Minor losses are due to the pipe fittings and are as follows :-

(a) loss due to sudden expansion :-



Applying momentum eqn in x-direction.

$$P_1 A_1 + P' (A_2 - A_1) - P_2 A_2 = \rho g v_2 - \rho g v_1$$

Assuming $P' = P_1$

$$P_1 A_1 + P_1 (A_2 - A_1) - P_2 A_2 = \rho g (v_2 - v_1)$$

$$(P_1 - P_2) A_2 = \rho g (v_2 - v_1) \quad \text{--- (i)}$$

Using Energy eqn b/w (i) & (ii).

$$\frac{P_1}{\rho} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$h_L = \frac{P_1 - P_2}{\rho} + \frac{v_1^2 - v_2^2}{2g} \quad \text{--- (ii)}$$

From (i) & (ii)

$$h_L = \frac{\rho g (v_2 - v_1)}{A_2 \rho} + \frac{v_1^2 - v_2^2}{2g}$$

$$= \frac{v_2 (v_2 - v_1)}{g} + \frac{v_1^2 - v_2^2}{2g}$$

$$h_L = \frac{1}{2g} [v_1^2 + v_2^2 - 2v_1 v_2]$$

$$h_L = \frac{(v_1 - v_2)^2}{2g}$$

Also,

$$h_L = \frac{v_1^2}{2g} \left(1 - \frac{v_2}{v_1}\right)^2$$

$$\text{Now, } Q = A_1 v_1 = A_2 v_2$$

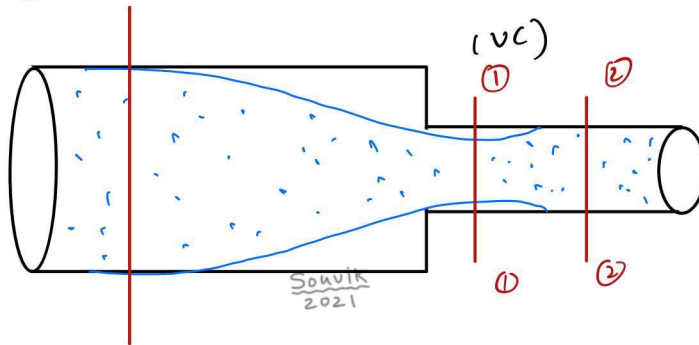
$$\frac{v_2}{v_1} = \frac{A_1}{A_2}$$

$$\therefore h_L = \frac{v_1^2}{2g} \left(1 - \frac{A_1}{A_2}\right)^2$$

$$h_L = K \frac{v_1^2}{2g} \quad \text{where } K = \left(1 - \frac{A_1}{A_2}\right)^2$$

(b) loss due to sudden contraction :-

- The losses due to sudden expansion are much higher than due to sudden contraction.



$$h_L = \frac{(v_1 - v_2)^2}{2g} = \frac{(v_c - v_2)^2}{2g}$$

$$= \frac{v_2^2}{2g} \left(\frac{v_c}{v_2} - 1\right)^2$$

$$h_L = \frac{v_2^2}{2g} \left(\frac{A_2}{A_c} - 1\right)^2$$

$$h_L = \frac{v_2^2}{2g} \left(\frac{1}{C_c} - 1\right)^2$$

$$h_L = K \frac{v_2^2}{2g}$$

[Loss in expansion region]
(neglecting loss due to contraction).

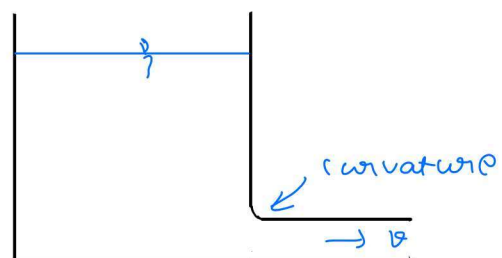
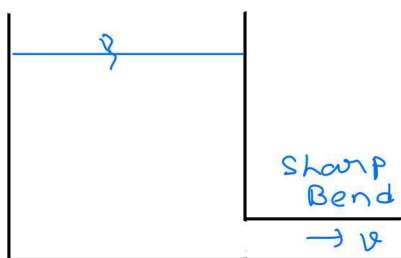
$$Q = A_c v_c = v_2 A_2$$

$$\frac{v_c}{v_2} = \frac{A_2}{A_c}$$

$$\frac{v_c}{v_2} = \frac{1}{C_c} = \frac{A_2}{A_c}$$

$$K = \left(\frac{1}{C_c} - 1\right)^2 \approx 0.5 \text{ (approx).}$$

(c) loss at entry into the pipe :-



$$h_L = K \frac{v^2}{2g}$$

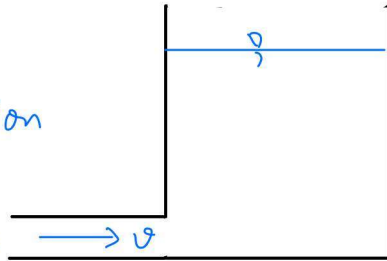
$$K = 0.5$$

- However, the value of K varies with curvature (if not given take $K=0.5$).

(d) loss due to exit from the pipe :-

$$h_L = K \frac{v^2}{2g}$$

$K=1$



- Here K is equal to K.E correction factor.

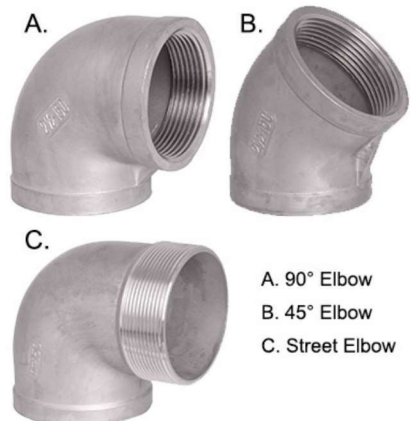
- Since most of the time flow is turbulent in pipe, hence its value is taken to be 1.

(e) loss due to Bend and Pipe fittings :-

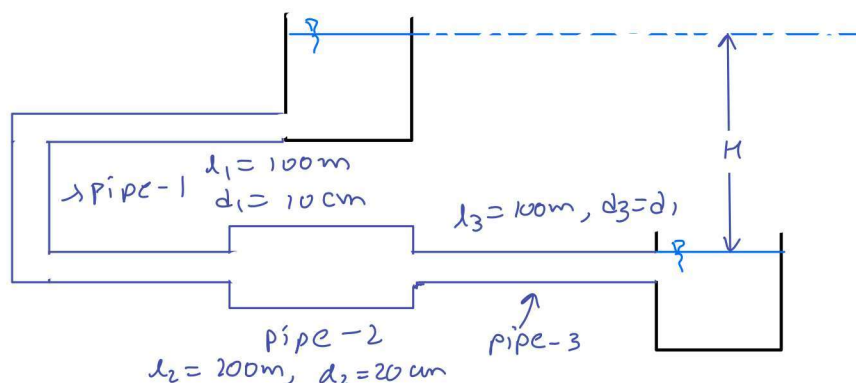
$$h_L = K \frac{v^2}{2g}$$

- Here K depends upon type of pipe fitting as follows.

Type of fitting	K
- Angle valve	5
- Gate valve	5.5
- Foot valve	1.5
- 90° sharp bend	1.2
- 45° elbow	0.4
- standard elbow	0.9
- standard	1.8



- 5) water flow from a reservoir through a series of pipe joints as shown. Find the % error in discharge if minor losses are neglected. Assume $K=1$ for bend, $f=0.02$ for all the pipes. Available head of 20m is used in overcoming losses and providing discharge.



sol

Total loss = Major loss + Minor loss.

Major loss :-

$$h_L = \frac{f L V^2}{2g D} = \frac{f L Q^2}{12.1 D^5}$$

Total Major head loss, $h_L = \frac{f Q^2}{12.1} \left(\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} \right)$

$$h_L = \frac{0.02 \times Q^2}{12.1} \left[\frac{100}{(0.1)^5} + \frac{200}{(0.2)^5} + \frac{100}{(0.1)^5} \right]$$

$$h_L = 34090.9 Q^2$$

Minor head loss

Total minor $h_L =$ Entry ^{Souvik} ₂₀₂₁ ^{Pipe} + Bent ① + Bent ② + sudden exp. + Sudden contr. + Exit from pipe.

$$= K_1 \frac{V^2}{2g} + K_2 \frac{V^2}{2g} + K_2 \frac{V^2}{2g} + K_3 \frac{V^2}{2g} + K_4 \frac{V^2}{2g} + K_5 \frac{V^2}{2g}$$

$$V = \frac{Q}{A}$$

$$= \frac{Q^2}{2g A^2} [K_1 + 2K_2 + K_3 + K_4 + K_5]$$

$$= \frac{Q^2}{19.62 \times \left(\frac{\pi}{4} \times (0.1)^2 \right)^2} \left[0.5 + 2 \times 1.2 + \left(1 - \frac{10^2}{20^2} \right)^2 + 0.5 + 1 \right]$$

$$= 4100 Q^2$$

=

If minor head loss is neglected then

$$H_1 = 34090.9 Q^2, \quad 20 = 34090.9 Q^2$$

If minor head loss is included then $Q = 0.02422 \text{ m}^3/\text{s}$

$$H_2 = (4100 + 34091) Q^2 = 38191 Q^2$$

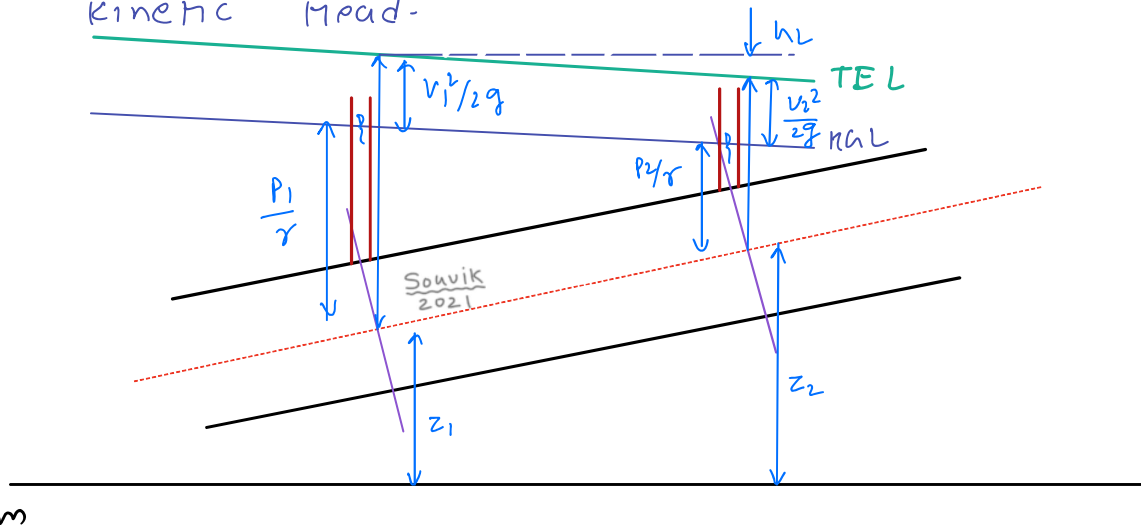
$$\text{Now, } H_2 = 20 = 38191 Q^2$$

$$Q = 0.0228 \text{ m}^3/\text{s}$$

$$\% \text{ error} = \frac{0.02422 - 0.0228}{0.0228} = 6.22\%$$

HYDRAULIC GRADIENT LINE & TOTAL ENERGY LINE :-

- Hydraulic gradient line is the line which joins the hydraulic / piezometric head at various points in flow.
- TEL is the line which joins the total head at various points in flow.
- Hence the difference b/w TEL & HGL is velocity or kinetic head.



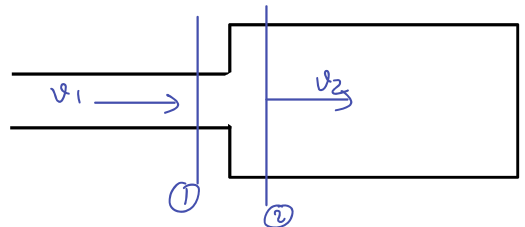
- TEL always falls down, whereas HGL may rise or fall

NOTE:-) Sudden rise or fall in TEL occurs due to pump and turbine respectively.
 as due to turbine mechanical energy is added and
 due to turbine mechanical " " extracted.

- 2) HGL can rise suddenly in case of sudden expansion.

using energy eqn at ① and ②.

$$\frac{p_1}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho} + z_2 + \frac{v_2^2}{2g} + \underbrace{\frac{(v_1 - v_2)^2}{2g}}_{h_L}$$



$$\left(\frac{p_1}{\rho} + z_1\right) - \left(\frac{p_2}{\rho} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + \frac{v_1^2}{2g} - \frac{2v_1v_2}{2g} + \frac{v_2^2}{2g}$$

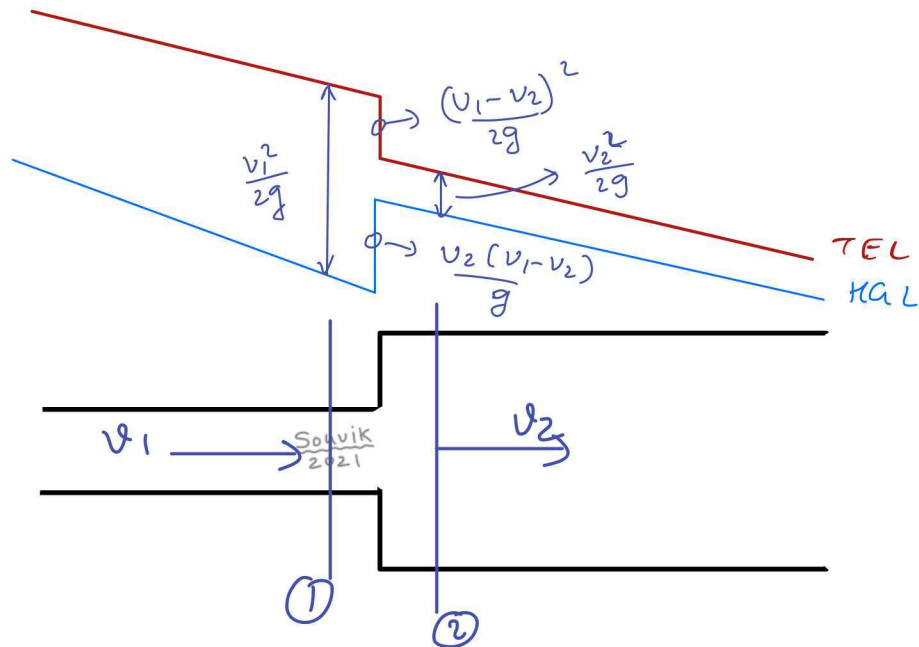
$$= \frac{1}{2g} [2v_2^2 - 2v_1v_2]$$

$$= \frac{v_2(v_2 - v_1)}{g} (< 0)$$

$$\left(\frac{p_2}{\gamma} + z_2 \right) - \left(\frac{p_1}{\gamma} + z_1 \right) = v_2 \left(\frac{v_1 - v_2}{g} \right) \quad (70).$$

$$(PH)_2 - (PH)_1 > 0.$$

hence HGL rises at the section of sudden expansion but TEL will fall.



NOTE (3):- For uniform flow condition, slope of HGL i.e. Hydraulic gradient (i) is given by.

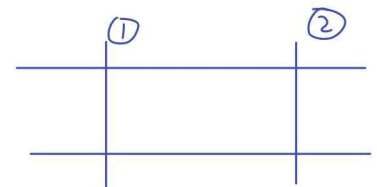
$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_L$$

$$\text{Since } v_1 = v_2$$

$$\left(\frac{p_1}{\gamma} + z_1 \right) - \left(\frac{p_2}{\gamma} + z_2 \right) = h_L$$

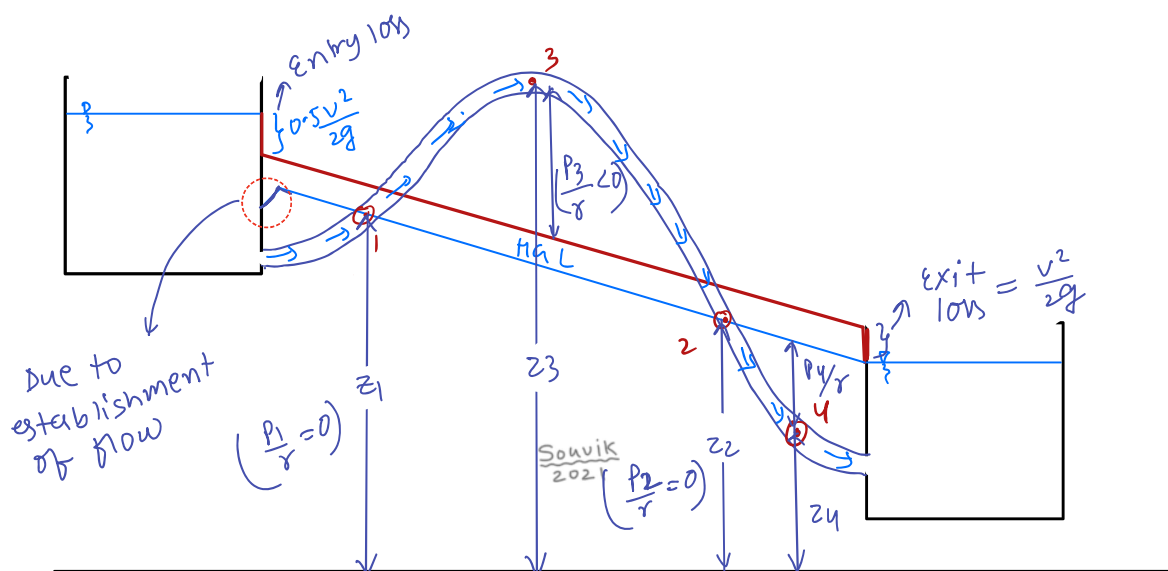
$$\text{Now, } i = \frac{h_L}{L} = \frac{\Delta \left(\frac{p}{\gamma} + z \right)}{L} = \frac{f L v^2}{12 \cdot 1 D^5} \cdot \frac{1}{L}$$

$$i = \frac{f v^2}{12 \cdot 1 D^5}$$



- TEL is horizontal for Ideal flow condition.
- HGL is horizontal in above case when velocity is constant.
i.e. velocity head is const.
- For open channel flow, HGL coincides with free surface.
- At pipe exit HGL coincides with pipe surface as pressure head is zero (gauge pressure).

- Gauge pressure is zero at location where HGL intersect the fluid surface.
- Pressure in flow section that lies above HGL is negative and positive where flow is below HGL.



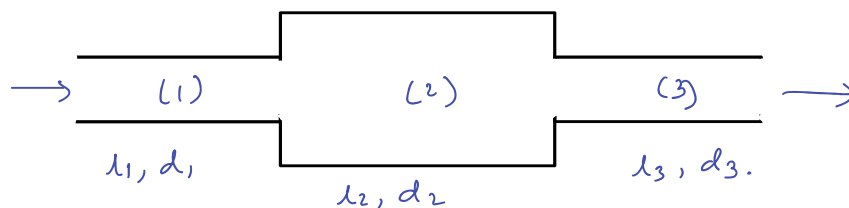
- TEL & HGL for pipe can never intersect each other, yes they will come together when velocity head is 0.

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Ques 6

PIPE CONNECTION :-

(A) SERIES CONNECTION :-



$$\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_3 + \dots + \Sigma_n$$

$$h_L = h_{L1} + h_{L2} + h_{L3} + \dots + h_{Ln}$$

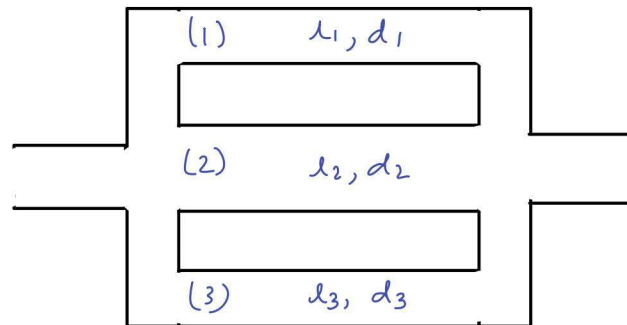
(B) PARALLEL CONNECTION :-

- Here head loss through each pipe will be same.

$$L = L_1 = L_2 = L_3 \dots = L_n$$

$$h_L = h_{L1} = h_{L2} = h_{L3}$$

$$Q = Q_1 + Q_2 + \dots + Q_n$$



EQUIVALENT PIPE :-

- hydraulically equivalent pipe is a pipe which can replace existing compound pipe while carrying same discharge under same discharge.

$$h_L = \text{same}$$

$$(h_L)_{cp} = (h_L)_{EP}$$

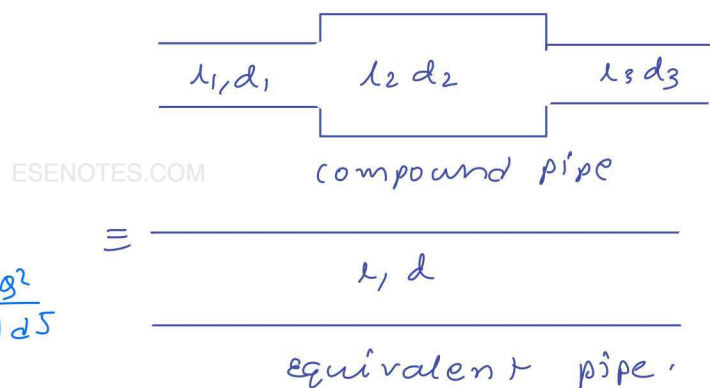
$$h_{L1} + h_{L2} + h_{L3} = (h_L)_{EP}$$

$$\frac{f L_1 Q^2}{12 \cdot L_1 d_1^5} + \frac{f L_2 Q^2}{12 \cdot L_2 d_2^5} + \dots = \frac{f L Q^2}{12 \cdot L d^5}$$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\boxed{\frac{L}{d^5} = \sum_{i=1}^n \frac{L_i}{d_i^5}}$$

→ Minor losses neglected
 $f = \text{const.}$



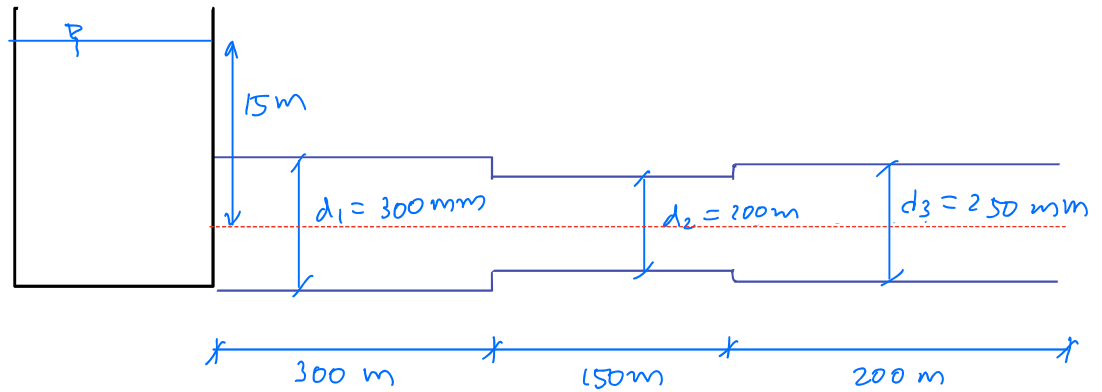
Q) Three pipes 300m long, 300mm dia, 150m long, 200mm dia and 200m long & 250mm dia are connected in series in same order. pipe having 300mm dia is connected to the reservoir. water level in the reservoir is 15m above the pipe axis which is horizontal. The respective f for three pipes are 0.018, 0.02 & 0.019 compute.

(a) Flow rate

(b) magnitude of loss in each pipe section.

(c) The dia when the three pipes are replaced by a single pipe of $f = 0.016$ to give same Q .

Sol



(a) Flow rate, $Q = Q_1 + Q_2 + Q_3$

Total head loss, $H = h_{L1} + h_{L2} + h_{L3}$

$$15\text{m} = \frac{f L_1 Q^2}{12.1 D_1^5} + \frac{f L_2 Q^2}{12.1 D_2^5} + \frac{f L_3 Q^2}{12.1 D_3^5}$$

Souvik
2021

$$15 = \left\{ \frac{0.018 \times 300}{12.1 \times 0.3^5} + \frac{0.02 \times 150}{12.1 \times 0.2^5} + \frac{0.019 \times 200}{12.1 \times 0.25^5} \right\} Q^2$$

$$15 = \{ 183.65 + 774.8 + 321.58 \} Q^2$$

$$Q = 0.108 \text{ m}^3/\text{s}$$

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(b) $h_{L1} = 2.142 \text{ m}$

$h_{L2} = 9.037 \text{ m}$

$h_{L3} = 3.75 \text{ m}$

(c) when a single pipe is replaced,

$$(h_L)_{eq} = h_{L1} + h_{L2} + h_{L3}$$

$$\frac{f L Q^2}{12.1 D^5} = 2.142 + 9.037 + 3.75$$

$$D^5 = \frac{0.016 \times 650 \times 0.108^2}{12.1 \times 14.999}$$

$$D = 0.2317 \text{ m}$$

$$D = 231.7 \text{ mm}$$

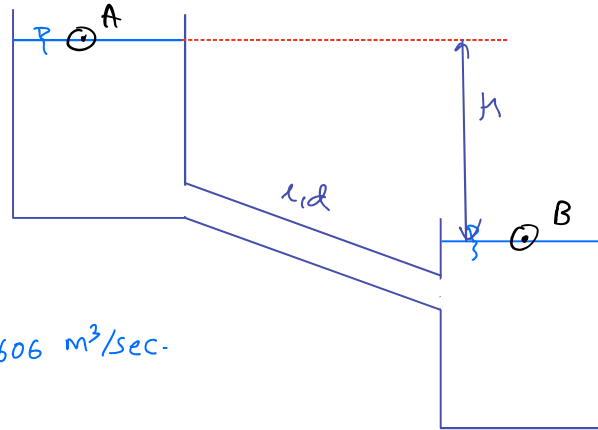
- 8) Two reservoirs with 15m diff in their water levels are connected by a 300mm dia pipeline of 3000 m length. compute the discharge also if a parallel pipeline of 300mm dia is attached to the last 1500 m length of the existing pipe. determine the modified discharge Q . Neglect minor losses. $f = 0.04$.

Sol. (a) using energy eqn b/w A & B

$$\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{P_B}{\gamma} + z_B + \frac{V_B^2}{2g} + h_{Lf}$$

$$h_{Lf} = z_A$$

$$15 = \frac{f L Q^2}{12.1 D^5}, \quad Q = 0.0606 \text{ m}^3/\text{sec.}$$



(b)

$$Q_1 = Q_2 + Q_3$$

$$h_{Lf2} = h_{Lf3}$$

$$\frac{f L_2 Q_2^2}{12.1 D_2^5} = \frac{f L_3 Q_3^2}{12.1 D_3^5}$$

$$Q_2 = Q_3 \quad \text{--- (i)}$$

$$\text{From (i) and (ii)} \quad Q_2 = Q_3 = \frac{Q_1}{2}$$

using energy eqn

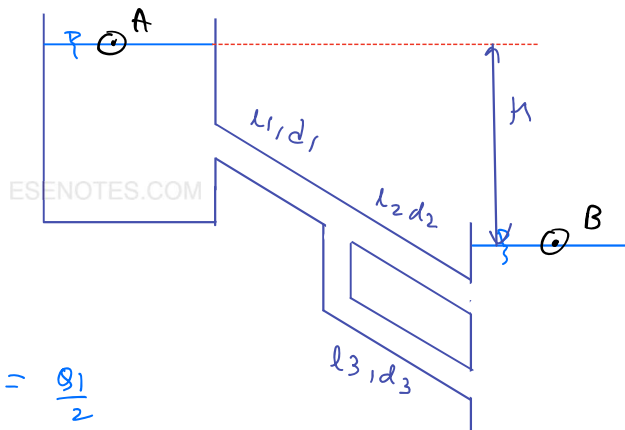
$$h = h_{Lf1} + h_{Lf2}$$

$$15 = \frac{f L_1 Q_1^2}{12.1 D_1^5} + \frac{f L_2 Q_2^2}{12.1 D_2^5} = \frac{f L_1 Q_1^2}{12.1 D_1^5} \left(1 + \frac{1}{4}\right)$$

$$Q_1 = 0.0766 \text{ m}^3/\text{s} \quad [\text{discharge is increased}]$$

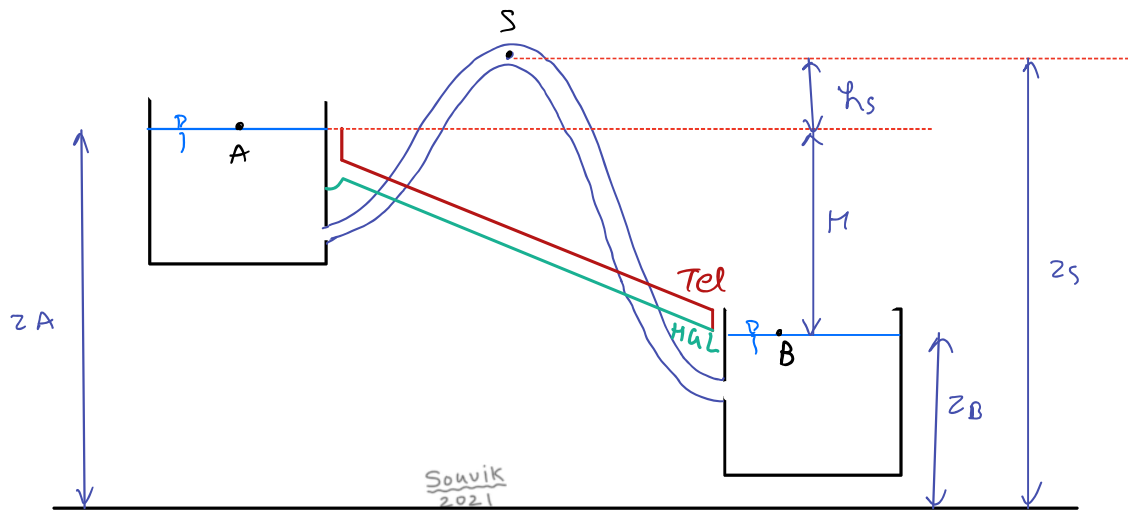
$$\text{Note:- } \% \text{ incr. in } Q = \frac{0.0766 - 0.0606}{0.0606} \times 100 = 26.4 \%$$

This is the adv. of parallel connection.



FLOW THROUGH SYPHON :-

- Siphon is a pipe/system used to drain or carry the fluid under the action of gravity.



For computation of flow rate

Applying energy eqn b/w A & B.

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_B}{\gamma} + z_B + \frac{v_B^2}{2g} + h_L$$

$$h_L = z_A - z_B = H \quad \text{--- (i)}$$

Also $h_L = h_{Lf} + \text{minor loss}$ → neglect if mentioned.

$$H = \frac{fLv^2}{2gd} + 0.5 \frac{v^2}{2g} + \frac{v^2}{2g}$$

Find v and thus Q .

- For cavitation not to occur.

pressure at top $\geq$ vapour pressure.
most p_t

using energy eqn b/w A and S

$$\frac{p_A}{\gamma} + z_A + \frac{v_A^2}{2g} = \frac{p_s}{\gamma} + z_s + \frac{v^2}{2g} + h_{Lf} + h_{L_{exit}}$$

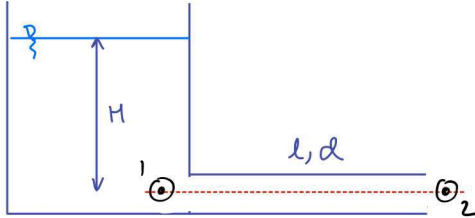
$$\frac{p_{atm}}{\gamma} = \frac{p_s}{\gamma} + h_s + \frac{v^2}{2g} + h_{Lf} + h_{Le}$$

$$\frac{p_{atm}}{\gamma} = \frac{p_s}{\gamma} + h_s + \frac{v^2}{2g} + h_{Lf} + h_{Le}$$

For no vapourisation.

P_s & Vapour pr (Propom).

Qd7 # POWER TRANSMISSION THROUGH PIPE :-



Head available @ 1 = H

Power " @ 1 = $\gamma g H$

head available @ 2 = $H - h_L$

$h_L = h_{Lf} + \text{minor loss.}$

Power available @ 2 = $\gamma g (H - h_L)$

efficiency of ^{Souvik 2022} power transmission = $\frac{\gamma g (H - h_L)}{\gamma g H} \times 100$

$$\% \text{ eff} = \frac{H - h_L}{H} \times 100$$

For max^m power transmission, $\frac{dP}{dQ} = 0$.

$P = \gamma g (H - h_L)$ neglecting minor loss.

$$P = \gamma g \left(H - \frac{f L Q^2}{12.1 d^5} \right)$$

$$P = \gamma g H - \frac{\gamma f L Q^3}{12.1 d^5}$$

$$\frac{dP}{dQ} = \gamma H - \frac{\gamma f L}{12.1 d^5} 3 Q^2 = 0$$

$$H = \frac{f L 3 Q^2}{12.1 d^5}$$

$$H = 3 h_{Lf}$$

$$\therefore h_{Lf} = \frac{H}{3}$$

$$\% \text{ eff.} = \frac{H - H/3}{H} \times 100 = \frac{2}{3} \times 100 = 66.67\%$$

$$\% \text{ eff for max power trans} = 66.67\%$$

- Q) Power is to be transmitted hydraulically along a distance of 80 km, through a number of 10cm dia pipe laid in parallel to each other. The pressure at the disch. end is maintained const at 6650 kPa. Determine the min. no. of pipes required to ensure an eff. of atleast 90%, when the power delivered is 150 kW. $f' = 0.0075$ for all pipes and neglect minor losses.

Sol

$$\eta = 90\%$$

$$\text{head avail. at discharge end, } h = H - h_L = H - h_{Lf} = \frac{P}{\gamma}$$

$$H - h_L = \frac{6650 \text{ kN/m}^2}{9.81 \text{ kN/m}^3} = 677.87 \text{ m}$$

$$\eta = \frac{H - h_f}{H} \quad \text{Sowik 2021}$$

$$H = \frac{H - h_f}{0.9} = \frac{677.87}{0.9} = 753.18 \text{ m}$$

$$h_f = H - 677.87 = 75.31 \text{ m.}$$

$$\text{Power delivered at disch. end, } P = \gamma Q (H - h_f). \\ 150 \times 10^3 = 9810 Q (677.87)$$

$$\text{Total disch. } Q = 0.0225 \text{ m}^3/\text{s.}$$

$$\text{Since pipes are in parallel, } h_{Lf1} = h_{Lf2} = \dots = h_{Lfn}$$

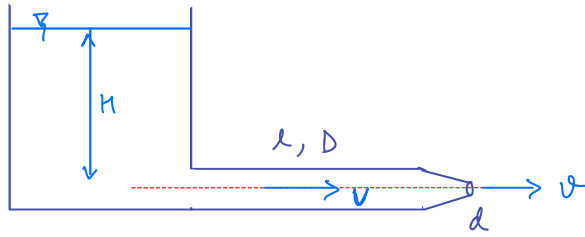
$$h_f = \frac{f L Q_i^2}{12.1 d^5}$$

$$75.31 = \frac{0.0075 \times 80000 \times Q_i^2}{12.1 \times (0.1)^5}$$

$$\text{individual discharge } Q_i = 0.00194 \text{ m}^3/\text{s}$$

$$\therefore \text{No. of pipe, } N = \frac{Q}{Q_i} = \frac{0.0225}{0.00194} = 11.59 \approx 12.$$

POWER AVAILABLE FROM NOZZLE :-



v = vel. thru nozzle
 V = " " pipe.

If minor losses are neglected, $H = h_f + \frac{v^2}{2g}$

$$H = \frac{f l V^2}{2g D} + \frac{v^2}{2g}$$

$$A V = a v$$

$$V = \frac{a}{A} v$$

$$H = \frac{f l \left(\frac{a v}{A}\right)^2}{2g D} + \frac{v^2}{2g}$$

vel. through nozzle, $v = \sqrt{\frac{2g H}{1 + \frac{f l a^2}{A^2 D}}}$

Power available at nozzle head

$$= \gamma Q (H - h_f)$$

$$= \gamma Q \left(\frac{v^2}{2g} \right)$$

$$P = \gamma Q \left[\frac{H}{1 + \frac{f l a^2}{A^2 D}} \right]$$

Eff. of power trans, $\eta = \frac{\gamma Q (H - h_f)}{\gamma Q H} \times 100$

$$\eta = \frac{H - h_f}{H} \times 100$$

$$\eta = \left[\frac{1}{1 + \frac{f l a^2}{A^2 D}} \right] \times 100$$

For max power transmission, $h_f = H/3$

$$\eta_{\max} = 66.67\%$$

$$\frac{2}{3} \times 100 = \frac{1}{1 + \frac{f l a^2}{A^2 D}} \times 100$$

$$1 + \frac{f l a^2}{A^2 D} = 1.5$$

$$\frac{f l a^2}{A^2 D} = 0.5$$

$$\frac{f l d^4}{D^5} = 0.5$$

$$d = \left[\frac{D^5}{2 f l} \right]^{1/4}$$

- 8) A fire engine supplies water to a hose pipe 75 m and 0.075 m in dia. at a pressure of 3 kg/cm² (gauge). The discharge end of the hose pipe has a nozzle of dia "d" fixed to it. If $f = 0.032$, determine dia of nozzle, so that the momentum of the issuing jet may be max.

sol = velocity through the nozzle.

$$v = \sqrt{\frac{2 g H}{1 + \frac{f l a^2}{A^2 D}}}$$

momentum through nozzle, $= \rho g v = \rho a v^2$

$$M = \rho a \frac{2 g H}{1 + \frac{f l a^2}{A^2 D}} = \rho \pi \frac{d^2}{4} \cdot \left(\frac{2 g H}{1 + \frac{f l d^4}{D^5}} \right)$$

$$M = \frac{\rho \pi d^2 g H D^5}{2 (D^5 + f l d^4)} = K \left(\frac{d^2}{D^5 + f l d^4} \right)$$

For max. momentum. $\frac{dM}{dd} = 0$.

$$\frac{dM}{dd} = K \left[\frac{2d(D^5 + f l d^4) - d^2(f l 4d^3)}{(D^5 + f l d^4)^2} \right] = 0$$

$$2d D^5 + 2 f l d^5 = 4 f l d^5$$

$$f l d^5 = d D^5$$

$$f l d^4 = D^5$$

$$d^4 = \frac{D^5}{f l}$$

$$d = \left(\frac{D^5}{f l} \right)^{1/4}$$

$$\text{Now, } d = \left(\frac{(0.075)^5}{0.032 \times 75} \right)^{1/4} = 0.0315 \text{ m} = 3.15 \text{ cm.}$$

LOSS OF HEAD DUE TO FRICTION IN TAPERED PIPE :-

$$Dx = D_1 - \left[\frac{D_1 - D_2}{L} \right] x$$

$$Dx = D_1 - Kx$$

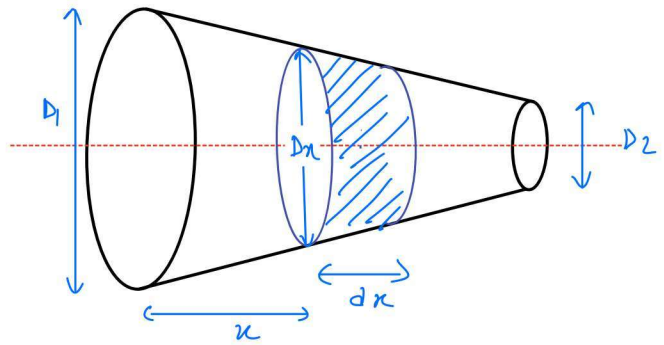
$$dh_f = \frac{f dx Q^2}{12 \cdot 1 D x^5}$$

$$h_f = \int dh_f = \int \frac{f dx Q^2}{12 \cdot 1 D x^5}$$

$$= \frac{f Q^2}{12 \cdot 1} \int \frac{1}{(D_1 - Kx)^5} dx$$

$$h_f = \frac{f Q^2}{12 \cdot 1} \left[\frac{(D_1 - Kx)^{-4}}{-4(-K)} \right]_0^L = \frac{f Q^2}{12 \cdot 1} \left[\frac{1}{4K(D_1 - KL)^4} - \frac{1}{4K D_1^4} \right]$$

$$h_f = \frac{f Q^2}{48 \cdot 4 (D_1 - D_2)} \left[\frac{1}{D_2^4} - \frac{1}{D_1^4} \right]$$



LOSS OF HEAD THROUGH A PIPE IN WHICH WATER IS DISCHARGED FROM SIDES AT A UNIFORM RATE AT REGULAR INTERVAL WITH ENDS CLOSED

Let, the water is discharged per unit length at rate of "q", $q = \frac{Q}{L}$

Water discharged over dist. x

$$= qx$$

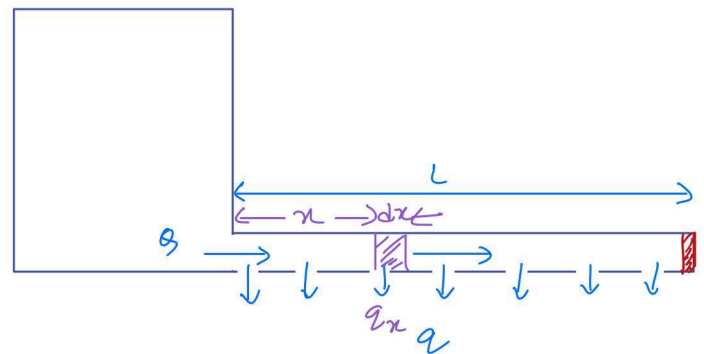
Flow available at distance x

$$Q_x = Q - qx$$

$$dh_f = \frac{f dx Q_x^2}{12 \cdot 1 D^5}$$

$$h_f = \int_0^L \frac{f dx Q_x^2}{12 \cdot 1 D^5} = \frac{f Q^2}{12 \cdot 1 D^5} \int_0^L \left(1 - \frac{x}{L}\right)^2 dx$$

$$h_f = \frac{f Q^2}{12 \cdot 1 D^5} \cdot \frac{L}{3} = \frac{1}{3} \frac{f L Q^2}{12 \cdot 1 D^5}$$



$$\text{head loss} = \frac{1}{3} \times \text{head loss of const. discharge}$$

Q.8

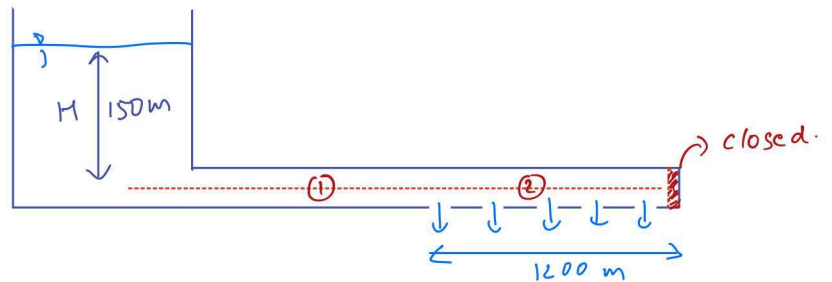
- Q.8) A pipe 0.6m dia takes off from a reservoir 150m above the datum. The pipe is 5000m long and is laid completely at the datum. For the last 1200m water is drawn by service pipe at a uniform rate of 0.088 cumec/300m. find the pressure at the end of the pipe line. consider the other end is dead end i.e. velocity is 0.

Sol

$$Q = \frac{0.088 \text{ m}^3/\text{s}}{300} = 2.93 \times 10^{-4}$$

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$$Q = 9 \times 1200 = 2.93 \times 10^{-4} \times 1200 = 0.352 \text{ m}^3/\text{s}$$



$$h_{L1} = \frac{f L Q^2}{12.1 D^5} = \frac{0.04 \times 3800 \times (0.352)^2}{12.1 \times (0.6)^5} = 20.02 \text{ m.}$$

$$h_{L2} = \frac{1}{3} \times \text{head loss when } Q \text{ is const} = \frac{1}{3} \left[\frac{f L_2 Q^2}{12.1 D^5} \right] =$$

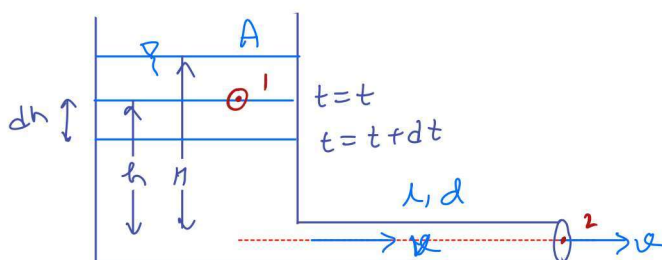
$$= \frac{1}{3} \times \frac{0.04 \times 1200 \times (0.352)^2}{12.1 \times (0.6)^5} = 2.107 \text{ m.}$$

Using Energy conservation: $0 + 150 + 0 = \frac{P}{\rho} + 0 + 0 + h_{L1} + h_{L2}$

$$P = (150 - 20.02 - 2.107) \times 9.81$$

$$P = 1254.44 \text{ KPa}$$

TIME REQ FOR EMPTYING A RESERVOIR THROUGH A PIPE DISCHARGED IN OPEN AIR



using Energy cons b/w 1 & 2 at any time t

$$\frac{P}{\rho} + z_1 + \frac{v_1^2}{2g} = \frac{P}{\rho} + z_2 + \frac{v_2^2}{2g} + h_L$$

$$0 + h + 0 = 0 + 0 + \frac{v^2}{2g} + \left(0.5 \frac{v^2}{2g} + \frac{f L v^2}{2g D} \right)$$

$$h = \frac{v^2}{2g} \left[1.5 + \frac{f L}{D} \right]$$

vol. of water drained in time interval "dt"

$$Q dt = -A dh \quad (dh \downarrow, dt \uparrow)$$

$$Q \cdot a dt = -A dh$$

A = Area of tank,
a = " " pipe

$$\sqrt{\frac{2gh}{1.5 + \frac{fL}{d}}} a dt = -A dh$$

$$\sqrt{\frac{2g}{1.5 + \frac{fL}{d}}} \left(\frac{a}{A}\right) dt = -\frac{A}{\sqrt{h}} dh$$

$$K \int dt = -\int \frac{A}{\sqrt{h}} dh$$

$$Kt = -2\sqrt{h} + C$$

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$$@ t=0, h=H$$

$$0 = -2\sqrt{H} + C \quad ; \quad C = 2\sqrt{H}$$

$$Kt = 2(\sqrt{H} - \sqrt{h})$$

Time req by the container to empty upto h height.

$$t = \frac{2}{K} (\sqrt{H} - \sqrt{h})$$

- Time req to empty the container, $h=0$

$$t = \frac{2}{K} \sqrt{H}$$

- Time req for half empty. $h = H/2$

$$t = \frac{2}{K} (\sqrt{H} - \sqrt{\frac{H}{2}})$$

- Time req to empty the second half of reservoir

$$t = (t)_{full tank} - (t)_{first half}$$

$$t = \frac{2}{K} \sqrt{H} - \frac{2}{K} (\sqrt{H} - \sqrt{\frac{H}{2}})$$

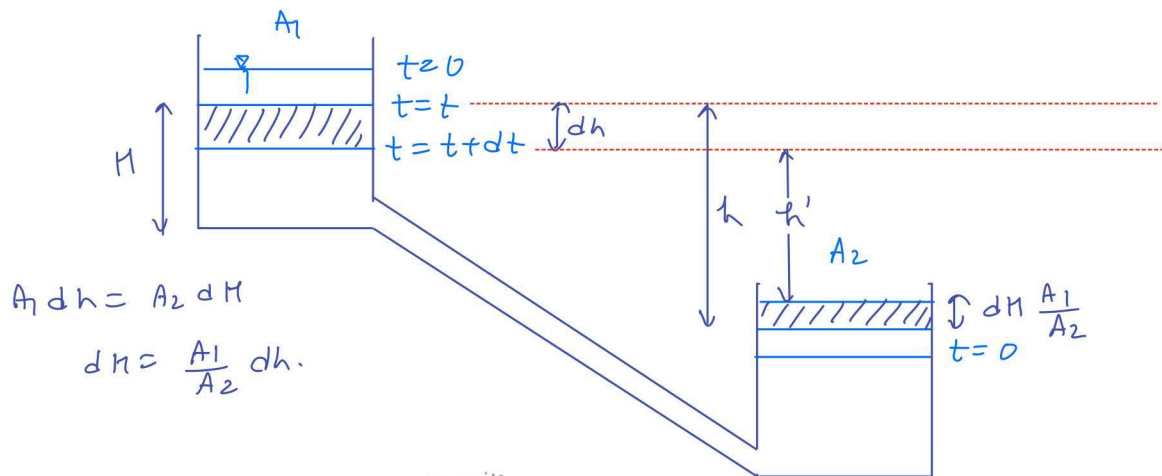
$$t = \frac{2}{K} \sqrt{\frac{H}{2}}$$

$$\text{Ratio of Time req to empty 1st half} = \frac{\frac{2}{K} [\sqrt{H} - \sqrt{\frac{H}{2}}]}{\frac{2}{K} \sqrt{\frac{H}{2}}} = \sqrt{2} - 1$$

$$= 0.414$$

NOTE:- Time req to empty the second half is more than first half as energy is reduced when tank gets half empty.

TIME req to reduce the level diff b/w two reservoirs:-



- change in level diff in time dt , i.e $dh = h' - h$

$$dh = h - dH - dH \frac{A_1}{A_2} - h$$

$$dh = -dH \left(1 + \frac{A_1}{A_2}\right)$$

vol. of water drained in time interval " dt ".

$$-A_1 dH = Q dt$$

$$\frac{dh}{\left(1 + \frac{A_1}{A_2}\right)} A_1 = Q dt \quad \text{--- (i)}$$

Now, $h = h_f + \text{entry loss} + \text{exit loss}$

$$h = \frac{f l v^2}{2 g d} + 0.5 \frac{v^2}{2 g} + \frac{v^2}{2 g}$$

$$\frac{1.5 v^2}{2 g} + \frac{f l v^2}{2 g d} = h$$

$$\frac{v^2}{2 g} \left[1.5 + \frac{f l}{d}\right] = h$$

$$v = \sqrt{\frac{2 g h}{1.5 + \frac{f l}{d}}}$$

$$\text{Now, } Q = a \cdot v$$

From eq (i)

$$\frac{A_1 A_2 dh}{\sqrt{A_1 + A_2}} = \frac{\pi d^2}{4} \sqrt{\frac{2gh}{1.5 + \frac{fl}{d}}} dt$$

$$K \left(\frac{A_1 A_2}{A_1 + A_2} \frac{4}{\pi d^2} \sqrt{\frac{1.5 + \frac{fl}{d}}{2g}} \right) dh = dt$$

$$K \int_{h_1}^{h_2} \frac{dh}{\sqrt{h}} = \int_0^t dt$$

$$2K (\sqrt{h_1} - \sqrt{h_2}) = t$$

time req to reduce the water level from h_1 to h_2 .

- **) Compute the ratio of time req to empty a cylindrical tank which is half full initially when the tank is vertical and when tank is horizontal. Dia = 2.5m, length = 5m. orifice at bottom of tank is of dia 7.5cm, $C_d = 0.6$.

Sol (a) when tank is vertical.

$$Q dt = -A dh$$

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$$C_d \cdot a \cdot \sqrt{2gh} dt = -A dh$$

$$C_d \cdot a \sqrt{2g} dt = -A \frac{dh}{\sqrt{h}}$$

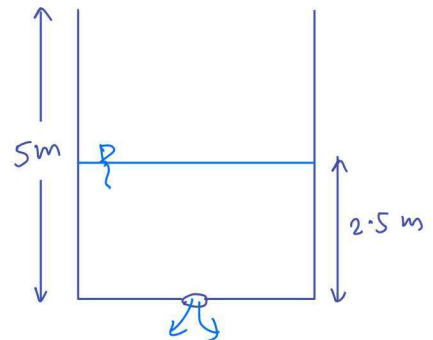
$$\text{Let, } K = C_d \left(\frac{a}{A} \right) \sqrt{2g} = 0.0797$$

$$K \int_0^t dt = \int_{2.5}^0 - \frac{dh}{\sqrt{h}}$$

$$K t = \int_0^{2.5} \frac{dh}{\sqrt{h}}$$

$$K t = [2\sqrt{h}]_0^{2.5}$$

$$t_v = \frac{1}{0.0797} \times 2\sqrt{2.5} = 1322 \text{ sec.}$$



(b) when tank is horizontal.

$$g dt = -A dh$$

$$\text{Now, } A = 5 \times 2x \\ = 10x$$

$$\text{also, } R^2 = x^2 + (R-h)^2$$

$$x = \sqrt{R^2 - (R-h)^2}$$

$$x = \sqrt{2Rh - h^2}$$

$$A = 10 \sqrt{2Rh - h^2}$$

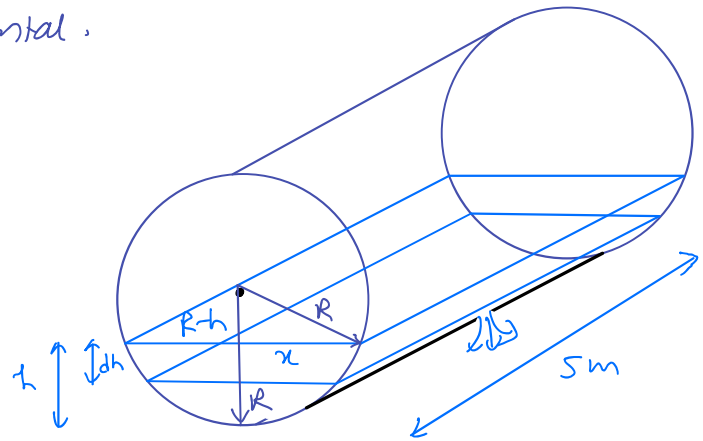
$$\therefore c_d a \sqrt{2gh} dt = -10 \sqrt{2Rh - h^2} dh$$

$$c_d \cdot a \cdot \sqrt{2gh} dt = -10 \sqrt{2Rh - h^2} dh$$

$$\frac{c_d a \sqrt{2g}}{10} \int_0^t dt = - \int_{1.25}^0 \sqrt{2R - h} dh$$

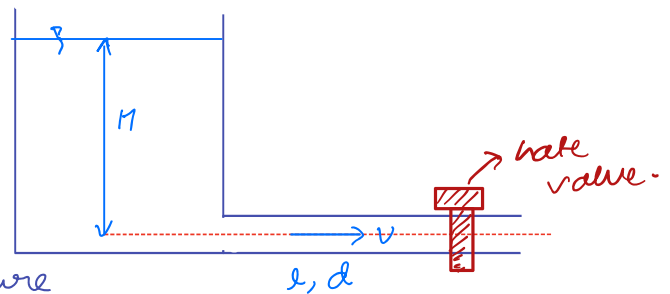
$$t_H = 1451 \text{ sec.}$$

(c) Now $\frac{t_v}{t_H} = 0.911$



WATER HAMMER PRESSURE :-

- when gate valve is open in pipe, water flows with the velocity of V in it, now if valve is suddenly closed, momentum of flowing water is destroyed and consequently a high pressure wave will setup.



- This pressure wave will travel at the speed of sound & causes hammering action in the pipe and is termed as water hammer/knocking.

- If the pipe is rigid, the pressure wave travels with the velocity of sound (c).

$$c = \sqrt{\frac{K}{\rho}}$$

K = Bulk mod. of liq.

ρ = density of liq.

- If the pipe material is elastic the velocity of pressure wave is given by

$$c = \sqrt{\frac{K}{\rho} \left[\frac{1}{1 + \left(\frac{D}{t}\right) \left(\frac{K}{E}\right)} \right]^{\frac{1}{2}}}$$

D = Diameter of pipe
 t = thickness of pipe.
 E = Modulus of elasticity.

RIGID PIPE

- The pressure wave developed due to sudden closure of valve undergoes reflection at the reservoir as well as at the valve end, time period of which is given by.

$$T_0 = \frac{4L}{c}$$

L = length of pipe b/w valve & reservoir.
 c = speed of wave/sound.

- If time of closure, $T < T_c \rightarrow$ it is termed as "rapid closure."

where, $T_c = \text{critical time} = \frac{T_0}{2} = \frac{2L}{c}$

In such case, water hammer pressure is given by

$$P_h = \rho c \Delta V.$$

$$\Delta V = V_2 - V_1$$

- $V_1 + V_2$ is the velocity of flow before & after closure of valve.
- If flow in pipe is completely stopped $V_2 = 0$.

$$P_h = -\rho c V_1$$

- If time of closure $T > T_c$, then it is termed as "slow closure"

- In such case water hammer pressure is given by.

(a) $T_c < T < 1.5T_c$: $\frac{P_{hs}}{P_{hr}} = \frac{T_c}{T}$

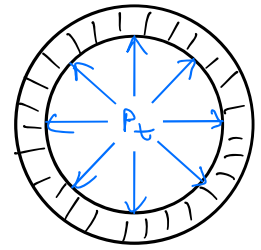
(b) $T > 1.5T_c$: $P_{hs} = \frac{\rho L V}{T}$

- For designing of the pipes prone to water hammer pressure, total pressure is considered to be resisted by it.

$$P_t = P_s + P_h$$

P_s = steady state pressure.

P_h = water hammer pressure.



- For no failure $\sigma_{hoop} \leq \sigma_{Design}$

$$\Rightarrow \frac{(P_s + P_h)D}{2t} \leq \sigma_D$$

- Q) compute the pressure due to water hammer caused by sudden closure of the pipe if (a) pipe is elastic (b) pipe is rigid. [Assume $\mu = 0.5$]

Solved 2021

Sol (a) Pipe is elastic :-

KE of liquid = strain energy of liquid + strain energy of pipe.

$$KE \text{ of liquid} = \frac{1}{2}mv^2 = \frac{1}{2}PALV^2 \quad \text{--- (i)}$$

$$SE \text{ of liquid} = \frac{1}{2}P\epsilon_v \cdot vol$$

$$= \frac{1}{2} \frac{P^2}{K} (AL)$$

$$K = \frac{P}{\left(\frac{dv}{v}\right)} = \frac{P}{\epsilon_v}$$

$$SE \text{ of pipe} = \frac{1}{2E} [\sigma_L^2 + \sigma_h^2 - 2\mu\sigma_L\sigma_h]$$

$$\sigma_L = \frac{Pd}{4t}, \quad \sigma_h = \frac{Pd}{2t}$$

$$\text{Hence, } SE \text{ of pipe} = \frac{1}{2E} \left[\left(\frac{Pd}{4t}\right)^2 + \left(\frac{Pd}{2t}\right)^2 - \frac{Pd}{2t} \times \frac{Pd}{4t} \right] \times vol$$

$$= \frac{1}{2E} \frac{P^2 d^2}{t^2} \left[\frac{1}{16} + \frac{1}{4} - \frac{1}{8} \right] \times vol$$

$$= \frac{1}{2E} \frac{P^2 d^2}{t^2} \times \frac{1}{4} = \frac{P^2 d^2}{8Et^2} \times vol$$

$$\text{Total SE of pipe material} = \frac{P^2 d^2}{8Et^2} \times vol = \frac{P^2 d^2}{8Et^2} \times \pi d t L$$

$$= \frac{P^2}{2Et^2} \left(\frac{\pi d^3}{4} \right) d t L$$

$$= \frac{P^2 A d L}{2Et}$$

$$\therefore \frac{1}{2} P A v^2 = \frac{P^2 A L}{2K} + \frac{P^2 d L A}{2 E t}$$

$$P v^2 = P^2 \left(\frac{1}{K} + \frac{d}{E t} \right)$$

$$P = \sqrt{\frac{P v^2}{\frac{1}{K} + \frac{d}{E t}}}$$

(b) If pipe is rigid, $E = \infty$.

$$P = v \sqrt{PK} = v P \sqrt{\frac{K}{\rho}} = v P c$$

BRANCHING OF PIPE :-

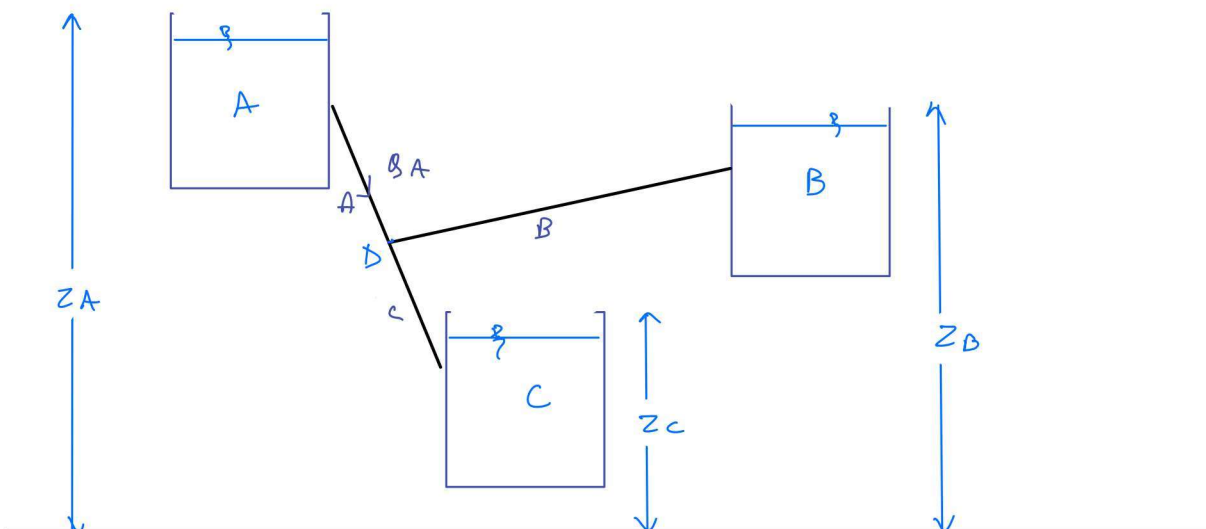
- It is the process of drawing multiple pipe from a single pipe

- Analysis of branching of pipe can be done by using.

- (a) Continuity eqn
- (b) Energy eqn
- (c) Darcy weisbach eqn

- Following types of cases are to be analysed in this case:-

Type - (I) :- f, L, d of all the pipes are given.
 Z_A, Z_B, θ_A are given.
 θ_A, θ_C, Z_C are to be found.



Sol :-

Step ① :- find Total head at D. (Junction pt).

Step ② :- find Q_B .

③ :- find Q_C .

④ :- find Z_C .

8)
$$\begin{array}{lll} l_A = 1.2 \text{ km} & d_A = 0.3 \text{ m} & z_B = 38 \text{ m} \\ l_B = 0.6 \text{ km} & d_B = 0.2 \text{ m} & z_A = 40 \text{ m} \\ l_C = 0.8 \text{ km} & d_C = 0.3 \text{ m} & Q_A = 60 \text{ l/s} = 0.06 \text{ m}^3/\text{s} \\ f = 0.024 & Q_B = ?, Q_C = ?, Z_C = ? \end{array}$$

Sol

loss in pipe A, $h_{LA} = \frac{f L Q_A^2}{12.1 d^5}$

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$$h_{LA} = \frac{0.024 \times 1200 \times (0.06)^2}{12.1 \times (0.3)^5}$$

$$h_{LA} = 3.52 \text{ m}$$

head at D, $H_D = 40 - 3.52 = 36.47$

Now, $H_D < z_B$ Hence water will flow from B to D.

head loss of pipe B = $38 - 36.47 = 1.53$

$$h_{LB} = \frac{0.024 \times 600 \times Q_B^2}{12.1 \times (0.2)^5}$$

$$Q_B = 0.0164 \text{ m}^3/\text{s}$$

$$Q_C = Q_A + Q_B = 0.0764$$

$$h_{LC} = \frac{0.024 \times 800 \times (0.0764)^2}{12.1 \times (0.3)^5}$$

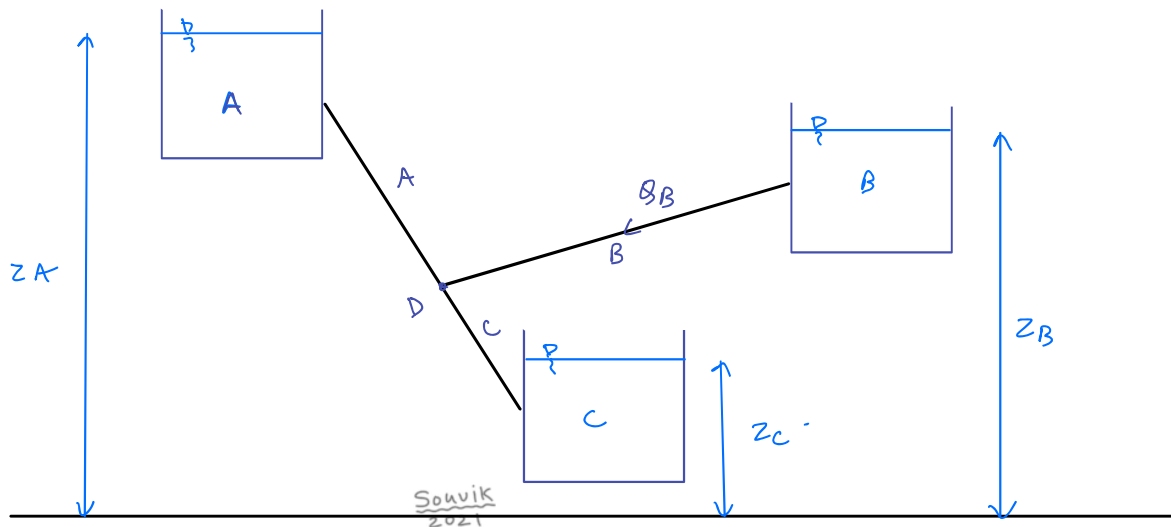
$$h_{LC} = 3.811$$

Head of reservoir C = $36.47 - 3.811$
 $= 32.65$

Oct 10

Type (2):- f, L, D of all the pipes are given.
 z_A, z_B, Q_B is also given.

To find z_B, Q_A, Q_C .



- Sol =
- (a) $z_A - z_C = h_{fAD} + h_{fDC}$.
- (b) Assume head at "D" let say H_D .

$$z_A - H_D = h_{fAD}$$

$$H_D - z_C = h_{fDC}$$

- (3) compute $Q_A + Q_C$ with the help of above eqs.
- (4) The value of H_D for which $Q_B - |Q_A - Q_C| = 0$ is the correct H_D .

- 8) For the three reservoirs $z_1 = 100m, z_3 = 20m, Q_2 = 0.5 m^3/s$
 Assume, $n_1 = 10, n_2 = 40, n_3 = 20$ and $n = 2$ for the relation $h_f = n Q^n$. Find Q_1, Q_3, z_2 .

Sol =

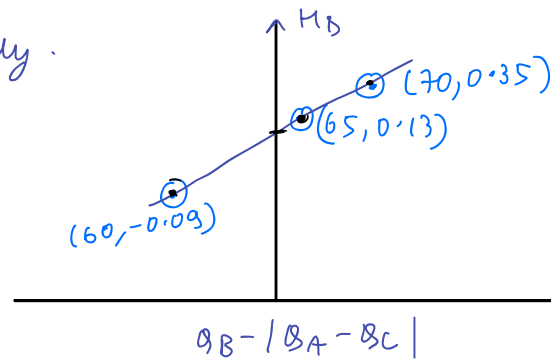
$$z_A - H_D = h_{fAD} = n_1 Q_A^n = 10 Q_A^2$$

$$\therefore Q_A = \sqrt{\frac{z_A - H_D}{10}}$$

$$H_D - z_C = h_{fDC} = n_3 Q_C^n = 20 Q_C^2 \Rightarrow Q_C = \left(\frac{H_D - z_C}{20} \right)^{1/2}$$

H_D (m)	Q_A (m^3/s)	Q_C (m^3/s)	$Q_B - Q_A - Q_C $
70 (Assume)	1.73	1.53	0.35
60 (Assume)	2	1.41	-0.09
65	1.87	1.5	0.13
62	1.95	1.45	0. ✓

can also be found graphically.



$$\text{Now, } h_{fBD} = \pi_2 Q_B^n = 40 (0.5)^2$$

$$Z_B - H_D = h_{fBD}$$

$$Z_B = H_D + h_{fBD} = 62 + 10 = 72 \text{ m.}$$

NOTE:- In general, $h_f = \pi Q^n$

(a) Darcy's theory ^{Souvik}, $h_f = \frac{f l Q^2}{12.1 D^5} = \pi Q^n$

$$\pi = \frac{f l}{12.1 D^5}, n=2.$$

(b) Manning's theory, $v = \frac{1}{n} R^{2/3} S^{1/2}$.

$$v = \frac{1}{n} \left(\frac{D}{4} \right)^{2/3} \left(\frac{h_f}{L} \right)^{1/2}$$

$$h_f^{1/2} = \frac{v n L^{1/2}}{\left(\frac{D}{4} \right)^{2/3}}$$

$$h_f = \frac{n^2 Q^2 L}{A^2 \left(\frac{D}{4} \right)^{4/3}} = \pi Q^n.$$

$$\pi = \frac{n^2 L}{A^2 \left(\frac{D}{4} \right)^{4/3}}$$

(c) chezy's theory :- $v = c \sqrt{RS}$

$$\frac{Q}{A} = c \left(\frac{D}{4} \right)^{1/2} \left(\frac{h_f}{L} \right)^{1/2}$$

$$h_f = \frac{Q^2}{A^2} \frac{L}{c^2 D} = \pi Q^n$$

$$\pi = \frac{L}{A^2 c^2 D}, n=2$$

Type (3):- f, L, D of all pipes are given.

z_A, z_B, z_C are given

to find : Q_A, Q_B, Q_C .

① Assume the total head at junction point D
 $= H_D = z_B$.

$$z_A - H_D = z_A - z_B = h_{fAD}$$

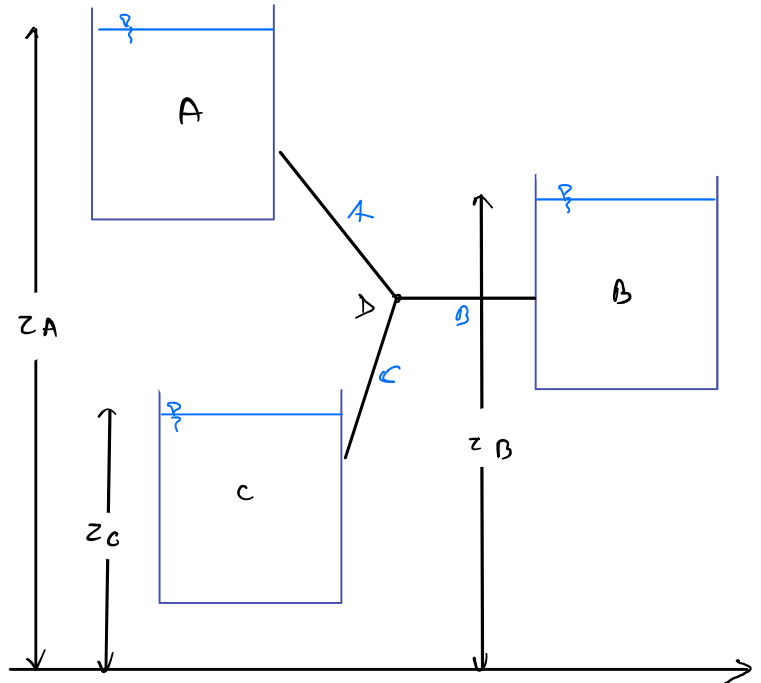
$$z_A - z_B = \frac{f_A L_A Q_A^2}{12.1 D_A^5}$$

$$Q_A = \checkmark$$

$$H_D - z_C = z_B - z_C = h_{fDC} \quad \text{Sowvik 2021}$$

$$z_B - z_C = \frac{f_C L_C Q_C^2}{12.1 D_C^5}$$

$$Q_C = \checkmark$$



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Now, if $Q_A > Q_C$

means, $H_D > z_B$

if $Q_A < Q_C \Rightarrow H_D < z_B$

- Now assume H_D as per the above condition and find $h_{fAD}, h_{fBD}, h_{fDC}$.

$$h_{fAD} = z_A - H_D = \frac{f_A L_A Q_A^2}{12.1 D_A^5} \quad Q_A = \checkmark$$

$$h_{fBD} = |H_D - z_B| = \frac{f_B L_B Q_B^2}{12.1 D_B^5} \Rightarrow Q_B = \checkmark$$

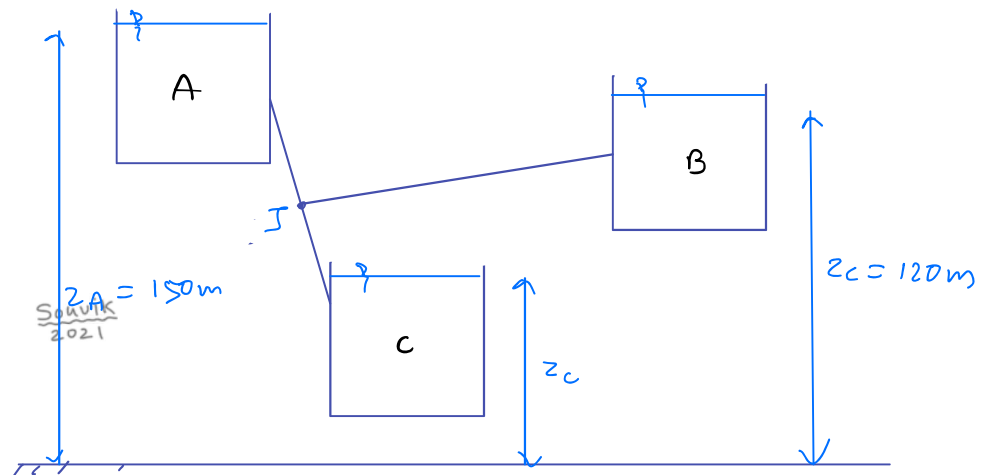
$$h_{fDC} = |H_D - z_C| = \frac{f_C L_C Q_C^2}{12.1 D_C^5} \Rightarrow Q_C = \checkmark$$

This is checked (Q_A, Q_B, Q_C) must satisfy continuity eqn at D.

$$Q_B - |Q_A - Q_C| = 0.$$

(a) Three tanks A, B, C supply water to a common junction J through pipes AJ, BJ, CJ and the characteristics of flow through these pipes is given as -

$AJ = 4Q^2$, $BJ = 7Q^2$, $CJ = 9Q^2$. The water surface elevation in the three reservoir are 150 m, 120 m, 40 m. Find the flow through each of pipes.



Sol

Assume $H_J = z_B = 120$

$$h_{fAJ} = 4Q_A^2 = z_A - H_J = 150 - 120 = 30$$

$$Q_A = 2.73 \text{ m}^3/\text{s}$$

$$h_{fJC} = 9Q_C^2 = H_J - z_C = 120 - 40 \Rightarrow Q_C = 2.98 \text{ m}^3/\text{s}$$

Since $Q_C > Q_A \Rightarrow H_J < z_B$.

Now, $h_{fAJ} = 4Q_A^2 = z_A - H_J$

$$Q_A = \left(\frac{15 - H_J}{4} \right)^{1/2}$$

$$h_{fBJ} = 7Q_B^2 = z_B - H_J \Rightarrow Q_B = \left(\frac{120 - H_J}{7} \right)^{1/2}$$

$$h_{fJC} = 9Q_C^2 = H_J - z_C \Rightarrow Q_C = \left(\frac{H_J - 40}{9} \right)^{1/2}$$

$H_f (m)$	$Q_A (m^3/s)$	$Q_B (m^3/s)$	$Q_C (m^3/s)$	$Q_B - Q_A - Q_C $
115	2.95	0.84	2.88	0.77
118	2.83	0.53	2.94	0.41
119	2.78	0.37	2.94	0.18
119.5	2.76	0.267	2.97	0.0

Hence,

$$Q_A = 2.76 \text{ m}^3/\text{s}$$

$$Q_B = 0.267 \text{ m}^3/\text{s}$$

$$Q_C = 2.97 \text{ m}^3/\text{s}$$

PIPE NETWORK :-

- Analysis of pipe network is done using Hardy cross method. Souvik 2021

(a) According to which assume flow in each pipe satisfying continuity eqn at each joint.

(b) clockwise flow is +ve.
Anti clockwise " " -ve.

(c) Apply correction over these assumed flow.

$$\Delta = - \frac{\sum H_L}{n \sum \frac{H_L}{Q}}$$

upto an extent flow in pipes in two iteration are same.

NOTE:- The above correction is computed using the fact that $\sum H_L$ in a loop = 0.

let Assumed flow be Q_a and correction be Δ . Then actual flow in pipe
 $Q = Q_a + \Delta$.

$$H_L = K Q^n = K (Q_a + \Delta)^n$$

$$H_L = K [Q_a^n + n Q_a^{n-1} \Delta + \cancel{\Delta^n} + \cancel{H.L.}]$$

$$H_L = K Q_a^n \left(1 + \frac{n\Delta}{Q_a}\right)$$

$$\text{Now, } \sum H_L = 0.$$

$$\sum n Q_a^n \left(1 + \frac{n\Delta}{Q_a}\right) = 0.$$

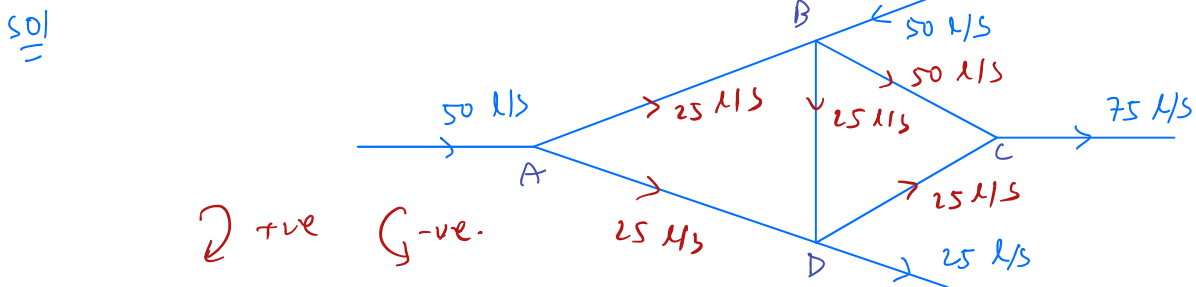
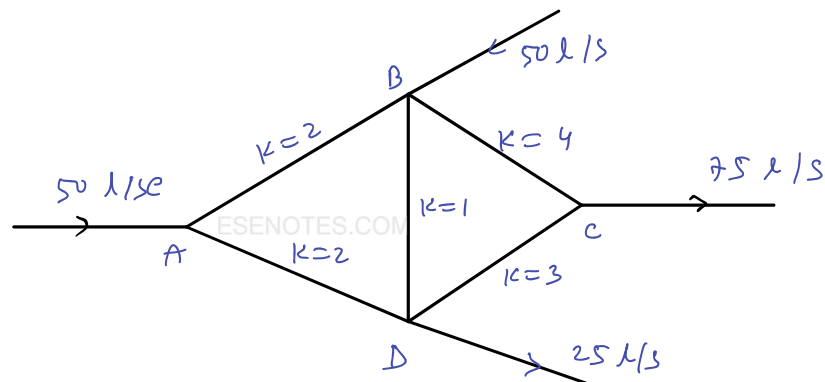
$$\sum n Q_a^n + n\Delta \sum n Q_a^{n-1} = 0.$$

$$\Delta = \frac{-\sum n Q_a^n}{n \sum n Q_a^{n-1}}$$

$$\Delta = - \frac{\sum H_L}{n \sum \frac{H_L}{Q_a}}$$

Oct 11

8) Determine the distribution of the flow in a pipe network head loss may be assumed as KQ^x . The flow is turbulent and the pipes are rough. The value of K is indicated in the fig. Use Hardy cross method.



For turbulent flow. $x=2$
1st iteration :-

Form loop ABDA

Pipe	K	Assumed Q_a	$H_L = K Q_a^2$	H_L / Q_a	corrected Q
AB	2	25	1250	50	$25 - 2.5 = 22.5$
DA	2	-25	-1250	50	$-25 - 2.5 = -27.5$
BD	1	25	625	25	$25 - 2.5 + 2.5 = 25$
			$\sum H_L = 625$	$\sum \frac{H_L}{Q_a} = 125$	$= 35$

negative direction.

$$\Delta = - \frac{\sum M_L}{n \sum \left(\frac{M_L}{Q_a} \right)} = - \frac{625}{2 \times 125} = -2.5$$

For loop BCDB

Pipe	K	(Q _a)	M _L = K Q _a ⁿ	M _L /Q _a	Q _c
BC	4	50	10000	200	50 - 12.5 = 37.5
CD	3	-25	-1875	75	-37.5
DB	1	-25	-625	25	-25 - 12.5 + 2.5 = -35
			$\sum M_L = 7500$	$\sum \frac{M_L}{Q_a} = 300$	

$$\Delta = - \frac{\sum M_L}{n \sum \frac{M_L}{Q_a}} = - \frac{7500}{2 \times 300} = -12.5$$

Souvik
2021

IInd iteration

For loop ABDA.

Pipe	K	Q _a	M _L = K Q _a ⁿ	M _L /Q _a	Q _c
AB	2	22.5	1012.5	45	19.82
DA	2	-27.5	-1512.5	55	-30.18
BD	1	35	1225	35	35 - 2.68 + 0.30 = 32.62
			$\Sigma = 725$	$\Sigma = 135$	

$$\Delta = - \frac{725}{2 \times 135} = -2.68$$

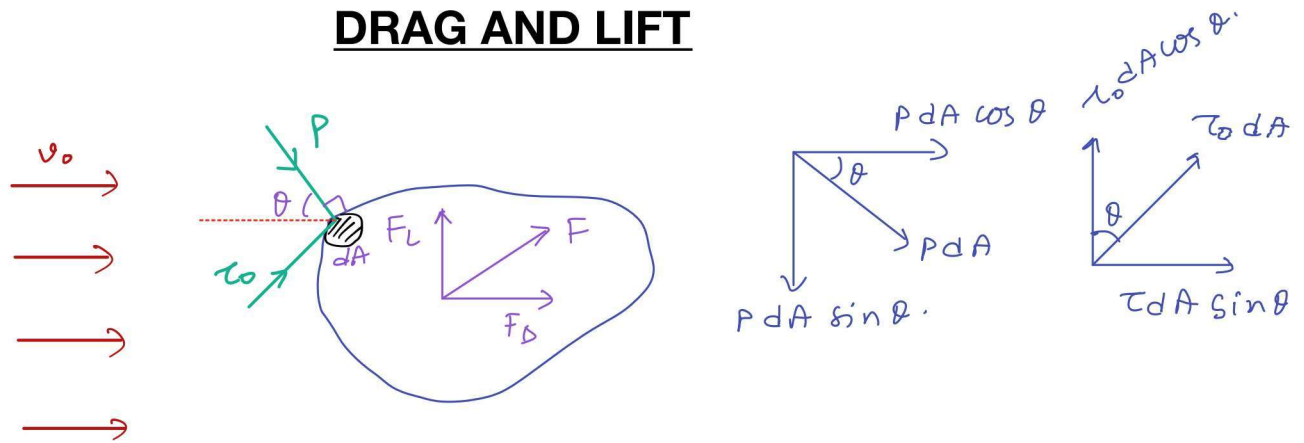
For loop BCDB

Pipe	K	Q _a	M _L = K Q _a ⁿ	M _L /Q _a	Q _c
BC	4	37.5	5625	150	37.5 - 0.3 = 37.2
CD	3	-37.5	-4218.75	112.42	-37.5 - 0.3 = -37.8
DB	1	-35	-1225	35	-35 - 0.30 + 2.68 = -32.62
			$\Sigma = 181.25$	$\Sigma = 297.42$	

$$\Delta = - \frac{181.25}{2 \times 297.42} = -0.304$$

Keep on iterating until Q_a and Q_c are same.

DRAG AND LIFT



- when a body is held in flow of uniform velocity (V_0), the force (F) acting on the body can be resolved into two components.
- The force acting in the direction of motion is termed as Drag force.
- And the force acting $\perp$ to the direction of flow is termed as LIFT FORCE (F_L).
- Since the velocity at different at different points near the surface of body are different, application of pressure at this points also varies.
- At any point on the surface of the body, two types of forces are acting.

(a) Shear stress (τ_0) :- Acting in tangential dirⁿ.

(b) Pressure force (P), Acting in Normal direction to the surface.

- Hence drag force acting in direction of motion is given by the summation of the components of shear force and pressure force acting over the entire surface of body

Drag force,

$$F_D = \int_A \tau_0 dA \sin \theta + \int_A P dA \cos \theta.$$

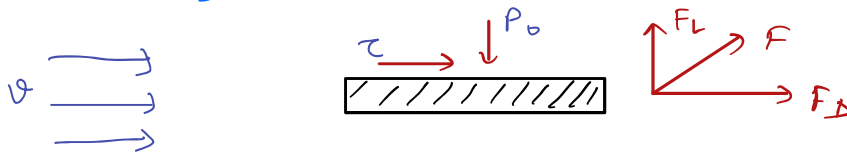
- And lift force acting in direction of flow is given by

$$F_L = \int_A \tau_0 dA \cos \theta + \int_A P dA \sin \theta.$$

- The relative contribution of each of these components to total drag and lift force depends on.

- (a) shape of immersed body.
- (b) characteristics of flow.
- (c) " " fluid.

- For Eg. if flat plate is held parallel to the direction of flow, the contribution of P -force in total drag is zero & it is only due to shear / friction / viscous drag.



(b) on the other hand if the plate is held $\perp$ to the flow, the contribution of shear in total drag is zero.

Oct 13

ANALYSIS OF DRAG FORCE :-

- The total drag is the sum of DEFORMATION Drag & FORM DRAG.
- The deformation drag exists at very small velocity i.e. when Re is very small. ($Re < 0.2$)
- At such small value of Re , the contribution of viscous force is much higher than inertial force.
- The deformation drag consists of FRICTION / SURFACE / SHEAR DRAG at the surface boundary and the PRESSURE DRAG due to pressure variation caused by wide spread deformation.
- In case of the sphere, the relative magnitude of this two components of deformation drag is observed to be $\frac{1}{3}$ rd of the total drag due to pressure diff and $\frac{2}{3}$ rd of the total drag due to friction.

$$F_D = \frac{1}{2} C_d P A v^2$$

For laminar flow, $C_d = 24/Re$

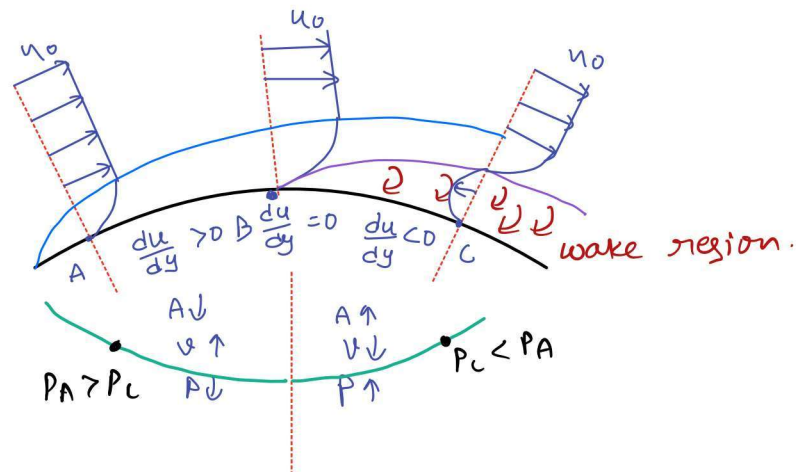
$$F_D = 3\pi\mu Dv$$

$$\text{now, friction drag,} = \frac{2}{3} \cdot 3\pi\mu Dv = 2\pi\mu Dv$$

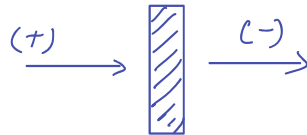
$$\text{pressure drag} = \frac{1}{3} \cdot 3\pi\mu Dv = \pi\mu Dv$$

- As velocity increases Re also increases and Boundary layer separation occurs and the reformation zone becomes smaller and gets combined to the boundary layer only.
- separation of Boundary layer i.e. streamline leaves the boundary surface and results in development of (-)ve velocity gradient or flow reversal, due to which WAKE formation (Pens) ^{Souvik 2021} in the region of separation takes place.
- In the separated zone (wake region), pressure drops and hence due to difference in pressure b/w front and back a drag is formed termed as FORM / PRESSURE DRAG.

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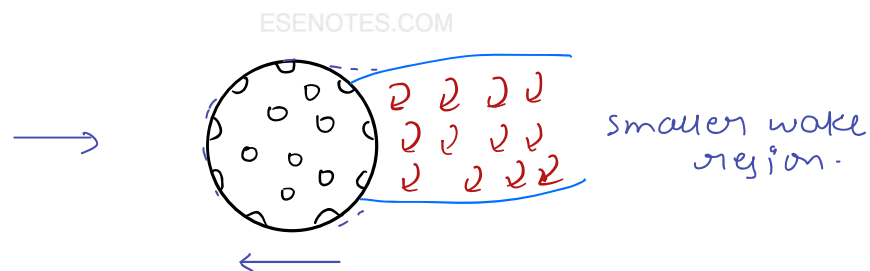
- now, in order to reduce the "FORM DRAG", wake zone is to be reduced for which boundary layer separation can be delayed.
- It can be achieved as follows:-
 - (a) Stream lining the body.
 - (b) By accelerating the fluid in boundary layer.
 - (c) By sucking the retarding fluid from boundary.



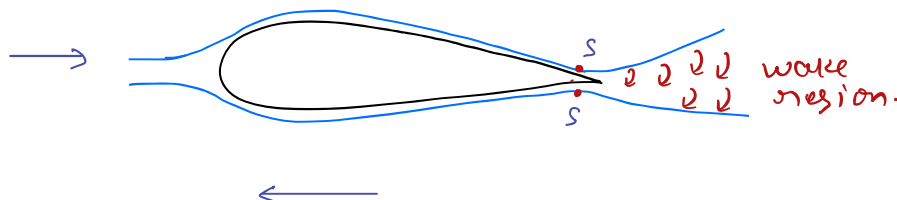
- Also laminar boundary layer is more prone to early separation than turbulent boundary layer, as there is no momentum transfer b/w diff layers of fluid.
- If delayed separation occurs, wake zone becomes smaller, form drag reduces, but friction drag increases.
- However overall drag would increase or decrease depending upon type of object under consideration (shape).

^{Souvik}
~~F₂₀₂₁~~ $F_{\text{form}} + F_{\text{friction}}$.

eg:- (a) In case of blunt objects (golf ball, cylinder). overall drag reduces.
(form drag ↓, friction drag ↑). Hence overall drag reduces.



(b) In case of stream-lined objects eg. aerofoil; sub marine. The overall drag increases.
(form drag ↓ + friction ↑↑)



Hence drag force depends on

- shape of object.
- Reynolds number (velocity).
- Surface roughness.

NOTE:- Drag force in some cases depends upon Froude's number and Mach number.

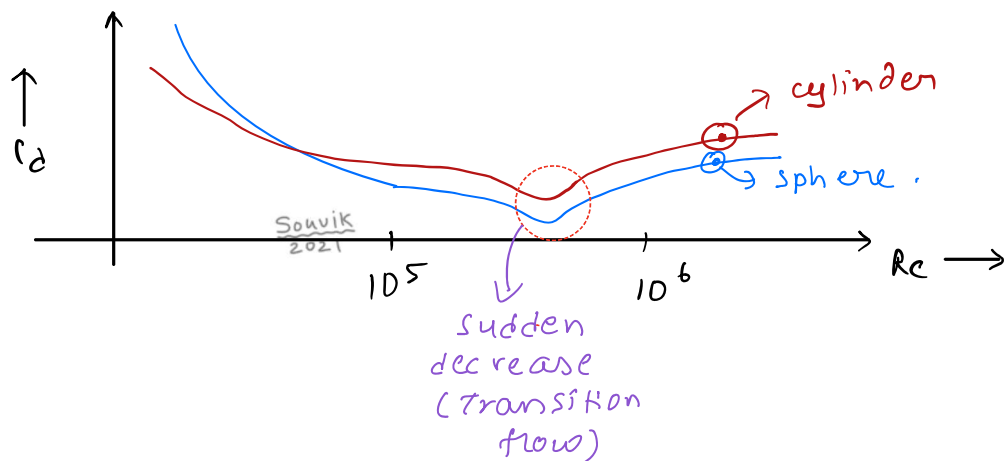
For eg:- In case of ship or pier in water. Drag force is due to gravity.

(2) If $Ma \leq 0.5$, drag is independent of elastic force.

but if $0.5 < Ma < 1$, drag incr. significantly

Also, drag force is given by, $F_D = \frac{1}{2} C_d \rho A U^2$

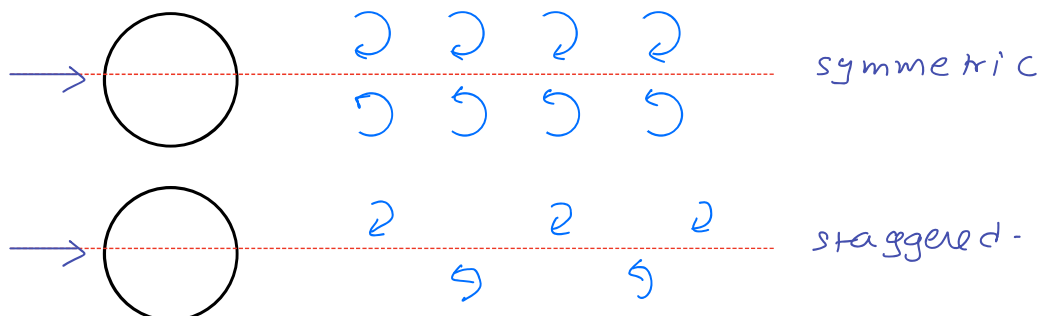
A: Frontal area / projected area.



Oct 15

VORTEX SHEDDING :- (KARMAN).

- As the Reynolds number increases and becomes more than 30, the two vortices behind the body (cylinder) get stretched and becomes unstable.
- As a result of this, they are transformed down & another two vortices are created in place of these transformed vortices.
- The process continues and is termed as "KARMAN VORTEX TRAIL".
- According to Karman, there are two possible combination of vortex pair.



NOTE:- VORTEX is a rotating region of fluid around a body.

- These vortices are shed from one side of the cylinder with the frequency given by

$$f = 0.198 \frac{u_0}{D} \left\{ 1 - \frac{19.7}{Re} \right\}$$

u_0 = velocity flow, D = dia of cylinder.

- Due to shedding of the vortices a periodic force is experienced in lateral direction (direction $\perp$ to flow.)
- If the frequency of this lateral force becomes equal to natural frequency of the cylinder large lateral ^{Sonik 2021} forces are developed, which may lead to the collapse.
- This process plays imp role in design of tall chimneys, suspension bridges, telephone / electric wires installation.
- The singing of telephone wire is due to "Karman vortex Trail".

NOTE:- Here, $\frac{fD}{u_0}$ is termed as **STROUHL number**.

which represents the shedding of vortex

- Q) A standard cricket ball of dia 7.13 cm is bowled at speed of 100 km/hr. Estimate the drag force on ball by taking the following values of C_d and fluid properties for a sphere.

$$C_d = 0.5 \quad \text{if} \quad 10^4 \leq Re \leq 3 \times 10^5$$

$$C_d = 0.2 \quad \text{if} \quad Re > 3 \times 10^5$$

$$\rho_{air} = 1.2 \text{ kg/m}^3, \quad \mu_{air} = 1.8 \times 10^{-5} \text{ N-sec/m}^2.$$

Sol/

$$F_D = \frac{1}{2} C_d \rho A v^2, \quad \text{Now} \quad Re = \frac{\rho v D}{\mu}$$

$$= 1.2 \times \frac{100 \times 10^3}{3600} \times \frac{7.13 \times 10^{-3}}{1.8 \times 10^{-5}}$$

$$Re = 1.32 \times 10^5 (< 3 \times 10^5)$$

Hence, $C_d = 0.5$

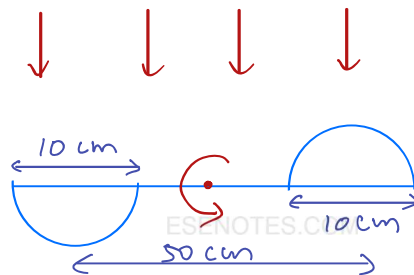
$$F_D = \frac{1}{2} \times 0.5 \times 1.2 \times \frac{\pi}{4} (7.13 \times 10^{-3})^2 \times \left(\frac{100 \times 10^3}{3600} \right)^2$$

$$F_D = 0.9242 \text{ N.}$$

- Q) A cup anemometer used to measure wind velocity consist of two hemispherical cup mounted in opposite direction on a horizontal connecting rod. which can turn freely on a vertical axis mid way. what torque would be required to hold the rod from rotating in a wind of velocity (a) 45 km/hr (b) 60 km/hr.

(d) for hemispherical bodies : $C_{d \text{ up stream}} = 1.33$
 $C_{d \text{ down stream}} = 0.34$

Sol



$$r = 25 \text{ cm.}$$

$$T = F_{D1} \times r - F_{D2} \times r = (F_{D1} - F_{D2}) r$$

$$T = \left(\frac{1}{2} C_{d1} \rho A v^2 - \frac{1}{2} C_{d2} \rho A v^2 \right) r$$

$$(a) \quad T = (1.33 - 0.34) \frac{1}{2} \times 1.2 \times \frac{\pi}{4} \times 0.1^2 \times \left(\frac{45 \times 1000}{3600} \right)^2 \times 0.25$$

$$T = 0.182 \text{ N-m.}$$

$$(b) \quad T = (1.33 - 0.34) \frac{1}{2} \times 1.2 \times \frac{\pi}{4} \times 0.1^2 \times \left(\frac{60 \times 1000}{3600} \right)^2 \times 0.25$$

$$T = 0.323 \text{ N-m}$$

- Q) A spherical balloon weighs 135 N and contains helium with the density of 0.22 kg/m^3 . The surrounding air has a density of 1.22 kg/m^3 .

(a) what dia of the balloon permits a terminal rising velocity of 2 m/s.

(b) If the balloon is tied to the ground by a cable. what would be the inclination.

of the cable when a wind of 10 km/hr blow past the balloon.

$$v_{air} = 1.5 \times 10^{-5} \text{ m}^2/\text{s}, \quad 10^4 < Re < 3 \times 10^5, \quad C_d = 0.5$$

$$Re > 3 \times 10^5, \quad C_d = 0.2.$$

Sol (a) To attain Terminal rising velocity, accel must be zero.

$$\Sigma F_y = 0$$

Here we need to know C_d for that Re should be known and thus velocity, which is unknown hence we need to Assume

Assume $Re > 3 \times 10^5$, $C_d = 0.2$

$$\Sigma F_y = 0:-$$

$$F_B = F_D + W_B + W_H$$

$$v_{air} \times \rho \times \text{Vol} = \frac{1}{2} C_d \rho A v^2 + 135 + v_{air} \times \rho \times \text{Vol}$$

$$v_{air} = v_{rel} = v_{ball}$$

$$\frac{\pi}{6} \times D^3 \times 9.81 (1.22 - 0.22) = \frac{1}{2} \times 0.2 \times 1.22 \times \frac{\pi}{4} D^2 \times 2^2 + 135$$

$$D = 3 \text{ m.}$$

$$\text{check, } Re = \frac{\rho v D}{\mu} = \frac{v_0 D}{\gamma} = \frac{3 \times 2}{1.5 \times 10^{-5}} = 4 \times 10^5$$

hence our assumption was correct.

(b) From force eq.

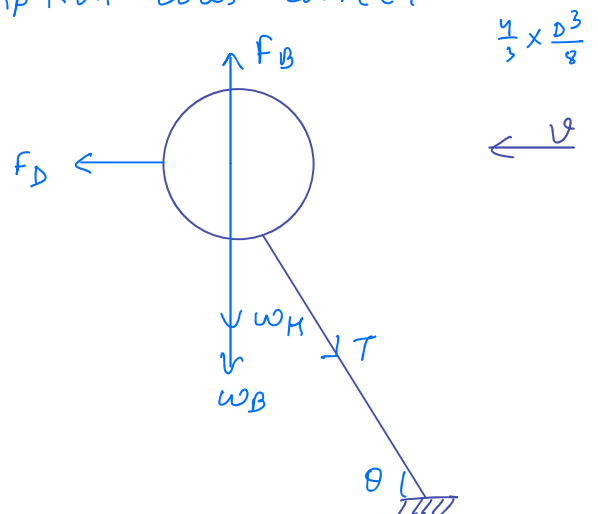
$$T \cos \theta = F_D = \frac{1}{2} C_d \rho A v^2$$

$$T \sin \theta + W_B + W_H = F_B$$

$$T \cos \theta = \frac{1}{2} \times 0.2 \times 1.22 \times \frac{\pi}{4} \times 3^2 \times \left(\frac{10}{3.6}\right)^2$$

$$T \cos \theta = 6.65 \quad - (i)$$

$$\begin{aligned} \text{and } T \sin \theta &= F_B - W_H - W_B \\ &= \frac{\pi}{6} \times D^3 \times g (\rho_{air} - \rho_{ne}) - 135 \\ &= 5.136 D^3 - 135 = 3.68 \end{aligned}$$



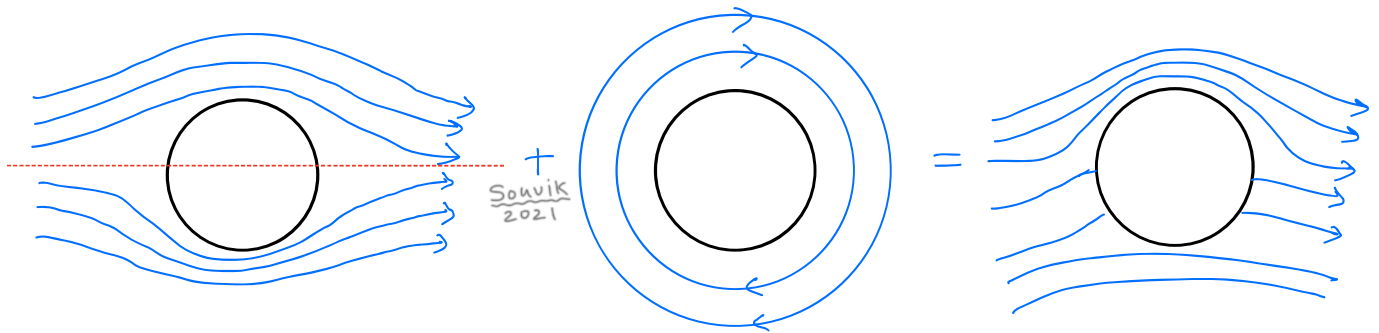
$$\tan \theta = 0.55$$

$$\theta = 28.89^\circ$$

$$T = 7.58 \text{ (N)}$$

LIFT :-

- It is the force which acts $\perp^r$ to the direction of flow on the body.
- This force is generated due to diff of pressure on the two sides of the body.



- If a irrotational flow is superimposed by flow with circulation, velocity on both sides of object varies.

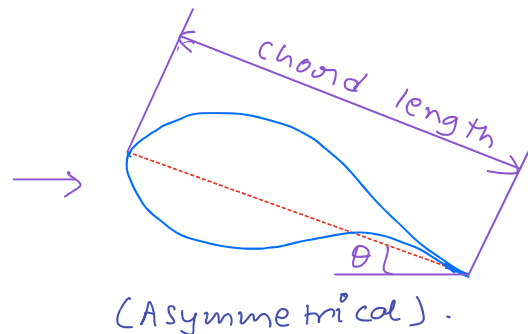
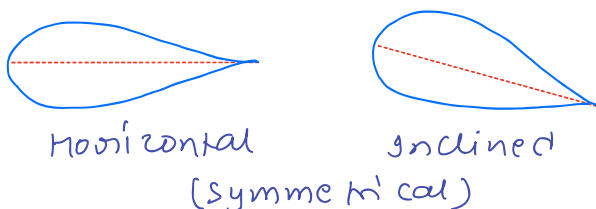
[For eg:- In previous case, velocity at top increase at bottom decreases]

that cause pressure difference due to which object experiences lift force.

- This effect is also termed as **MAGNUS EFFECT**.

Oct 18

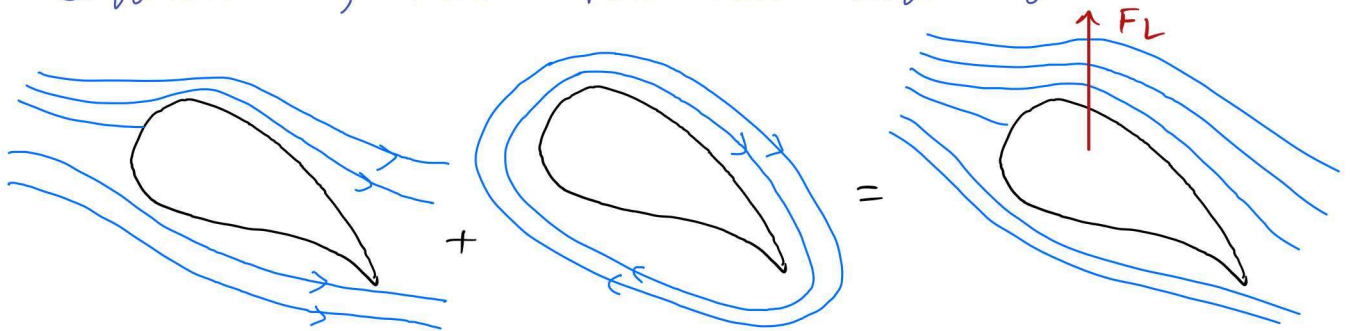
LIFT on AEROFOIL :-



$\theta = \text{Angle of attack.}$

- In Aero plane wings termed as Aero foils are used to produce the lift forces as they move through fluid (air).

- when a circulating flow around an aerofoil is superimposed over an irrotational flow around the same aerofoil, causes pressure difference, that produces lift force.



- These aerofoils can be symmetrical as well as asymmetrical and are characterised by GEOMETRIC (CHORD C) & THE angle of ATTACK (θ).
b/w geometric chord and the direction of flow.
- If angle of attack is zero, no lift will be produced and if it is increased beyond a value then also lift disappears, due to boundary layer separation & this stage is termed as STALLING.

- In order to make the streamline at the trailing edge, tangential to the trailing edge, the circulation Γ and lift force is given by

$$\Gamma = \pi C u_0 \sin \theta \quad \text{--- (i)}$$

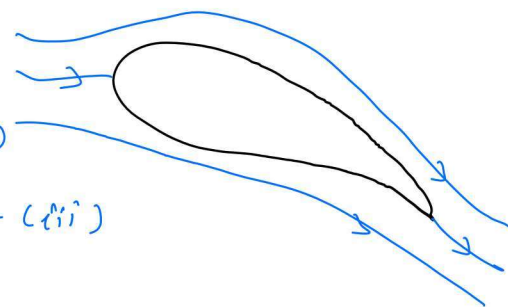
$$F_L = \pi C \rho u_0^2 \sin \theta \quad \text{--- (ii)}$$

$$\text{also, } F_L = \frac{1}{2} C_L \rho (CL) u_0^2 \quad \text{--- (iii)}$$

from (ii) & (iii)

$$\frac{1}{2} C_L \rho (CL) u_0^2 = \pi C u_0 \sin \theta \cdot L$$

$$C_L = 2\pi \sin \theta$$



- Q) An aeroplane wing 22000N has a wing area of 22m^2 and span of 12m. What is the lift coeff if it is travels at speed of 360 km/hr in horizontal direction. Also compute the theoretical value of circulation & angle of attack.

Sol (a) $\Sigma F_y = 0$:-

$$W = F_L$$

$$W = F_L = \frac{1}{2} \times C_L \rho A u_0^2$$

$$22000 = 0.5 \times C_L \times 1.2 \times 22 \times \left(\frac{360}{3.6}\right)^2$$

$$C_L = 0.166$$

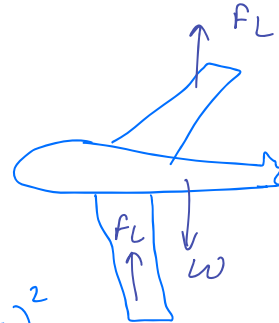
(b) $C_L = 2\pi \sin \theta$

$$\sin \theta = \frac{0.166}{2\pi} \quad \theta = 1.57^\circ$$

(c) $\Gamma = \pi C u_0 \sin \theta = \pi \times \frac{A}{L} \times u_0 \sin \theta$

$$\Gamma = 3.14 \times \frac{22}{12} \times \frac{360}{3.6} \sin (1.57)$$

$$\Gamma = 15.23 \text{ m}^2/\text{s}$$



- Q) A kite in the form of rectangular aerofoil with chord length 60 cm and width of 45 cm & weight of 0.8N. It is maintained at an angle 10° to the horizontal & the string makes an angle of 30° to the vertical. If the wind speed is 15 km/hr and $C_D = 0.25$. Estimate the tension in the string & the lift coeff.

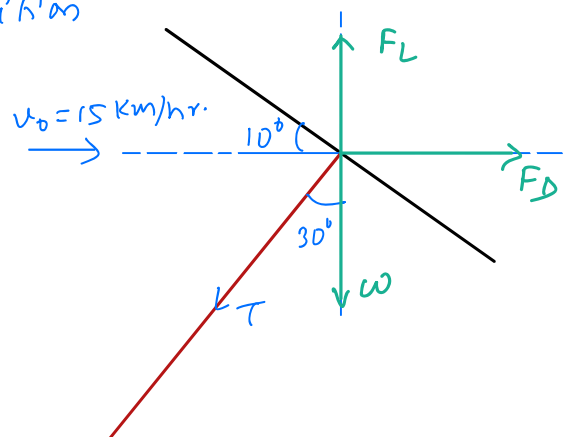
Sol From static Eq. condition

$$T \sin 30^\circ = F_D \quad \text{--- (i)}$$

$$T \cos 30^\circ + W = F_L \quad \text{--- (ii)}$$

$$F_D = \frac{1}{2} C_D \rho A u^2$$

$$F_L = \frac{1}{2} C_L \rho A u^2$$



$$T = \frac{0.5 \times 0.25 \times 1.2 \times (0.6 \times 0.45)}{\sin 30} \times \left(\frac{15}{3.6}\right)^2$$

$$T = 1.406 \text{ (N)}$$

$$F_L = 1.406 \times \frac{\sqrt{3}}{2} + 0.8$$

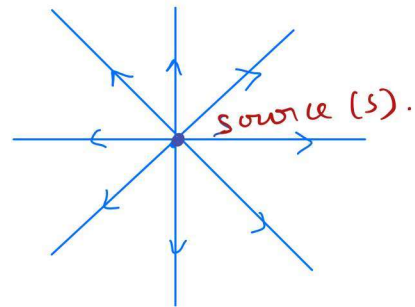
$$F_L = 2.017$$

$$C_L = \frac{2.017 \times 2}{1.2 \times 0.6 \times 0.45 \times \left(\frac{15}{3.6}\right)^2} = 0.717$$

SOURCE & SINK

Source flow :- The source flow is the flow coming from a point source and moving out radially in all directions of a plane at uniform rate.

- The strength of the source flow is defined as vol. rate per unit depth coming out from it (i.e.) $[m^3/m/sec]$.



- Here radial velocity

$$u_r = \frac{q}{2\pi r} \quad \text{and} \quad u_\theta = 0.$$

(As flow is in radial dirⁿ only).

- stream fn and velocity fn for source flow are given by:

$$u_r = \frac{1}{r} \frac{d\psi}{d\theta}, \quad u_\theta = -\frac{\partial \psi}{\partial r}$$

$$\frac{q}{2\pi r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

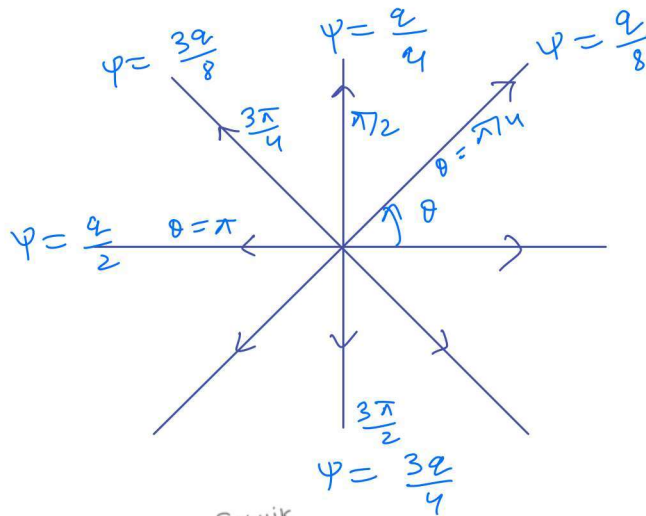
$$\partial \psi = \frac{q}{2\pi} d\theta$$

$$\int d\psi = \int \frac{q}{2\pi} d\theta$$

$$\psi = \frac{q}{2\pi} \theta + C$$

if $\theta = 0$, $\psi = 0 \Rightarrow \psi = 0$.

$$\psi = \frac{q}{2\pi} \theta$$



Sovik
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$$u_r = \frac{\partial \psi}{\partial r} \text{ and } u_\theta = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

$$\frac{q}{2\pi r} = \frac{\partial \psi}{\partial r}$$

$$\partial \psi = \frac{q}{2\pi} \frac{\partial r}{r}$$

$$\int \partial \psi = \frac{q}{2\pi} \int \frac{dr}{r}$$

$$\psi = \frac{q}{2\pi} \ln r$$

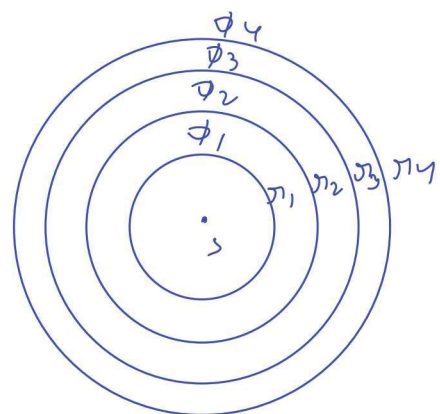
Now, using energy eqn at any pt (A) at a distance of "r" from point source & a pt (B) at infinity.

$$\frac{P_A}{\rho g} + \frac{u_A^2}{2g} = \frac{P_B}{\rho g} + 0.$$

$$P_A - P_B = -\frac{\rho}{2} u_A^2$$

$$= -\frac{\rho}{2} \left(\frac{q}{2\pi r} \right)^2$$

$$\Delta P = -\frac{\rho q^2}{8\pi^2 r^2}$$



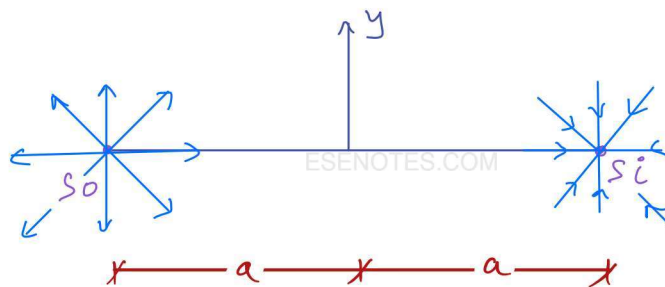
(b) Sink Flow :-

- It is the flow in which fluid moves radially inwards a point where it disappears at a const rate.
- It is also called as negative source.
- Hence the relation used for source flow replace " q " with " $-q$ " for sink flow

SUPERIMPOSED FLOW :-

(a) SOURCE & SINK PAIR :-

- In this flow a source of strength (q) and sink of strength ($-q$) are placed at equal distance apart from reference pt.



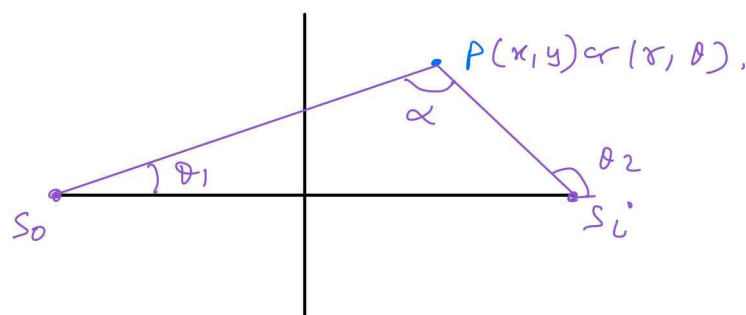
- In this case stream fn at any pt "P" is given by

$$\psi = \psi_1 + \psi_2$$

$$\psi = \frac{q \theta_1}{2\pi} + \left(-\frac{q}{2\pi}\right) \theta_2$$

$$\psi = \frac{q}{2\pi} (\theta_1 - \theta_2)$$

$$\psi = \frac{q}{2\pi} \alpha$$

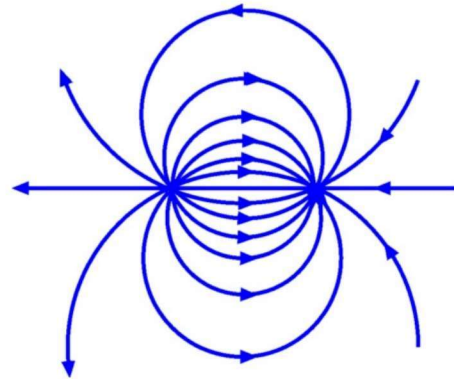


- On the same line vel. potential is given by

$$\phi = \phi_1 + \phi_2$$

$$= \frac{q}{2\pi} \ln r_1 + \frac{(-q)}{2\pi} \ln r_2$$

potential $\leftarrow$ $\phi = \frac{q}{2\pi} \ln \left(\frac{r_1}{r_2} \right)$



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stream $\rightarrow$ for source & sink pair.

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(6) DOUBLET :-

It is a special case of source & sink pair in which source and sink of same strength, approaches towards each other such that $(q \cdot 2a) = \text{const.}$
i.e. termed as double strength (μ)

$$\mu = q \cdot 2a$$

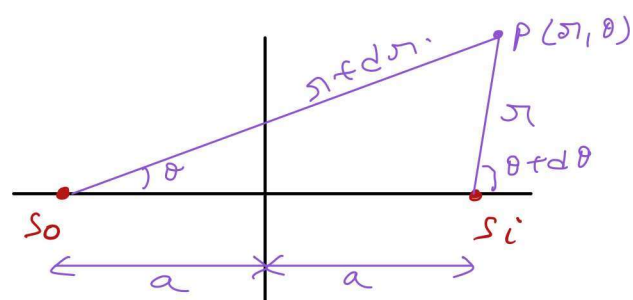


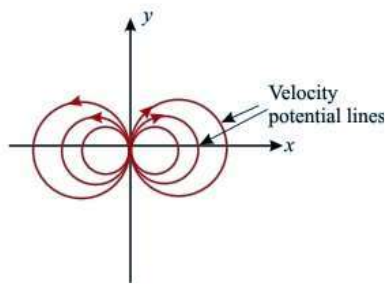
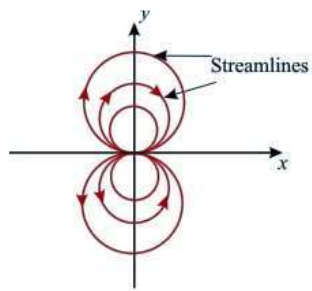
- Here stream ψ is given by -

$$\psi = -\frac{\mu}{2\pi} \frac{\sin \theta}{r}$$

and velocity potential (potential ϕ) is given by

$$\phi = \frac{\mu}{2\pi} \frac{\cos \theta}{r}$$





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